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Abstract

This paper presents a new state-space framework to jointly estimate the elasticity and factor-augmenting technical change. Technical change is specified as a smooth process that captures a long run trend and persistent shifts during transition periods. In a simulation study, this approach outperforms the widely applied linear and Box-Cox trend assumptions. The framework is subsequently applied to a dataset containing 16 OECD countries. For all countries, the estimated elasticity is significantly below unity. In the long run, technical change is labor-augmenting, but with several persistent shifts during transition periods, in particular in the 90'es and after the financial crisis.

Keywords: Factor-Augmenting Technical Change, Medium Run, Constant Elasticity of Substitution, State-space Models.

JEL: C32, E25, O33.

1 Introduction

The elasticity of substitution between capital and labor is a central parameter in macroeconomic models. It influences factors such as the long run growth rate of GDP, the effects of monetary policy, the effect of

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a technology shock on hours worked, and variations in the labor share.[1] While an important parameter, the identification of the elasticity is difficult as it requires factor price movements that are unrelated to factor-augmenting technical change, the so-called Diamond et al. (1978) non-identification result.

The non-identification result has led researchers to impose parametric assumptions on factor-augmenting technical change (i.e. technical change that affects the efficiency of capital and labor differently) in the time dimension. Although many researchers have employed a linear trend assumption (Antras, 2004; Leon-Ledesma et al., 2010; Mallick, 2012; Leon-Ledesma et al., 2015), it is known to be problematic for at least two reasons: First, Knoblach et al. (2020) find that the main determinant of the major cross-study difference in the estimated elasticity is the lack of incorporating a time-varying and non-linear process of factor-augmenting technical change. Second, it has recently been shown that the labor share is best described by having transition periods of 10-30 years, the so-called "medium run cycle", particularly driven by persistent fluctuations in factor-augmenting technical change (Blanchard, 1997; Comin and Gertler, 2006; Growiec et al., 2018; Leon-Ledesma and Satchi, 2019; Charpe et al., 2020; Oberfield and Raval, 2021). Consequently, technical change is far from constant in periods of transition and involves several persistent periods of shifts in the average growth rate of the relative factor-augmenting technical change. This motivated a series of papers to apply a Box-Cox transformation of the time trend to describe factor-augmenting technical change (Klump et al., 2007, 2008; McAdam and Willman, 2013; Muck, 2017; Stewart and Li, 2018). While the Box-Cox transformation can account for accelerating, decelerating, or constant growth rates, it cannot reproduce several persistent transition periods. When the typical transition period is 10-30 years, several periods are likely present when studying longer samples. Therefore, a more flexible estimation approach is needed.

In this paper, we propose a new state-space framework to jointly estimate the elasticity of substitution and a process of factor-augmenting technical change. The framework allows for a long run trend and several persistent transition periods in factor-augmenting technical change. A likelihood-driven approach is applied to determine whether these periods are present as well as their persistence. The only assumption imposed is that the process of factor-augmenting technical change has to be sufficiently smooth to not capture short run fluctuations generated by factors such as markup variations and labor market frictions. Hence, except for the estimation of the noise-to-signal ratio, our approach can be characterized as a non-parametric estimation of technical change. This way, we present a framework that is less restrictive than the widely applied linear or Box-Cox trend, which both represent a full parametric specification of technical change in the time dimension. The approach is fairly easy to implement as an improvement to models with parametric assumptions on factor-augmenting technical change and is made publicly available through the statistical software program R.[2] In addition to the estimation of the capital-labor

[1] Papers that emphasize the role of the elasticity of substitution include among many de La Granville (1989); Ky-Hyang (1991); Blanchard (1997); Klump and de La Granville (2000); Mallick (2012); Cantore et al. (2014); Karabarbounis and Neiman (2014); Piketty (2014); Cantore et al. (2015, 2017); Leon-Ledesma and Satchi (2019); Gechert et al. (2021); Oberfield and Raval (2021).

[2] We refer readers to Kastrup et al. (2021) for a tutorial on how to apply the package. We are happy to take comments and suggestions.

elasticity, the framework also applies to other studies estimating CES functions, e.g. the substitution between different skill types of labor or consumption goods.

In line with the majority of the literature, our estimation framework starts from the relative first order conditions. The share of capital relative to labor in total income is determined by two factors: The relative factor payments and factor-augmenting technical change. Their relative importance is determined by the elasticity of substitution. Whereas the relative factor payments are observed data, factor-augmenting technical change is unobserved and refers to all persistent factors that affect the relative factor shares in total income for given factor payments. We specify factor-augmenting technical change to satisfy three requirements: i) A long run trend in factor-augmenting technical change should be allowed. ii) The process must be sufficiently flexible to capture persistent fluctuations in factor-augmenting technical change during several transition periods. Yet, as a very flexible process has a tendency to ascribe all variations in factor shares to technical change, placing less emphasis on price changes, the process should not capture the year-to-year errors between the model and the data. iii) The process should be related to the existing trend assumptions applied in the literature and potential deviations from these determined by a likelihood-driven approach. A process that fits these three requirements is an I(2) process of factor-augmenting technical change. The smoothness of the process depends on the inverse signal-to-noise ratio (i.e. the noise-to-signal ratio, Leon-Ledesma et al., 2010), defined as the measurement error variance relative to variance of factor-augmenting technical change. A high estimated value of the noise-to-signal ratio implies that the process converges to a linear trend. Oppositely, a low estimated value implies that most of the variations in relative factor shares (not described by the relative factor payments) are ascribed to factor-augmenting technical change. Thus, the resulting process lies somewhere between a linear trend assumption and a dynamic calibration of technical change (i.e. where all year-to-year errors between the model and the data are described as technical change).

A potential concern is that when the noise-to-signal ratio is freely estimated, the ratio with the highest likelihood results in a dynamic calibration, which is against how we intend to measure factor-augmenting technical change. Importantly, we find that the model is generally misspecified for low noise-to-signal ratios based on an autocorrelation test and a filter consistency test. In these cases, we supplement the free estimation with a grid search procedure and implement the value that maximizes the likelihood conditional on being well specified.

In summary, we here provide a framework that allows the identification of the elasticity of substitution jointly with a process of factor-augmenting technical change that contains a long run trend with potentially persistent fluctuations during transition periods. We emphasize that our goal is not to prove if medium run cycles of technical change are present as this is already well documented in the literature using spectral analysis.[3]

We start by analyzing the performance of our state-space framework in a simulation study. Data is

[3] Comin and Gertler (2006); Leon-Ledesma and Satchi (2019); Growiec et al. (2018) assume that the medium term component is variations in the frequency domain of 32 to 200 quarters. Charpe et al. (2020) assume that the medium term is 8 to 32 years.

simulated according to Leon-Ledesma et al. (2010, 2015) with the following extensions: We simulate the factor-augmenting technical change according to three different deterministic trends: A constant growth rate assumption, a Box-Cox transformation of the growth rates, and a time-varying growth rates assumption which implies a long run trend in factor-augmenting technical change and several persistent transition periods. In addition, we update the variances of factor demands and value-added to match the US data applied in the empirical analysis. The performance of the state-space framework is compared to two system estimators where the first assumes a linear trend and the second a Box-Cox transformation of the growth rates. For all trend assumptions in the data generating process, the state-space framework reproduces the true estimate on the median. Oppositely, we find that both system estimators are biased towards unity, in particular when the deterministic trend becomes increasingly non-linear. Importantly, this shows that a flexible trend assumption is important for obtaining unbiased estimates on the median, in particular when several persistent fluctuations of technical change are present in the sample.

After the methodological contribution of the paper, we subsequently apply the state-space framework to data from the Penn World Tables version 10 (PWT, Feenstra et al., 2015). We estimate the elasticity of substitution for 16 OECD countries along with the factor-augmenting technical change. The estimated elasticity of substitution is significantly below unity and above zero for all countries, rejecting the Cobb-Douglas and Leontief production functions. The obtained weighted average is 0.42 and the estimates range from 0.11 (Norway) to 0.65 (Korea). Importantly, the US estimate is 0.54 and thus close to the range of consensus estimates in meta regression studies (Chirinko, 2008; Knoblach et al., 2020; Gechert et al., 2021) where they correct for different sources of bias. On the contrary, using the system estimator with Box-Cox trend we obtain an estimate that falls outside the range of consensus estimates.

For all countries, the estimated process of factor-augmenting technical change supports the assumption that technical change is labor-augmenting in the long run, but with a declining growth rate relative to capital over time in most countries. Overall, the long run trend in the estimated processes with the state-space framework is similar to the processes estimated with a Box-Cox trend. However, the processes estimated with the state-space framework indicate that there are persistent fluctuations during transition periods, in particular in the 90'es and after the financial crisis, which are not captured by the Box-Cox trend. This highlights that while the Box-Cox transformation and state-space framework capture similar long run trends, only the state-space framework is able to incorporate persistent fluctuations in factor-augmenting technical change during transition periods.

The recent and growing interest in the global decline of the labor share has renewed the interest of the estimation of the substitution elasticity. Two popular explanations have typically been suggested as causing the decline in the labor share observed in recent decades: Changes in the relative price and technical change. Which channel is the most important is inherently linked to the value of the elasticity. Karabarbounis and Neiman (2014) argue that the decreasing price of investments relative to the wage is the main determinant behind the global decline in the labor share. However, this interpretation depends crucially on the elasticity being above unity. Gechert et al. (2021) show that the decline in the labor share following a transitory labor-augmenting technology shock is five times as large when the elasticity is equal to 0.9 compared to 0.5. Oberfield and Raval (2021) find that the decline in the US labor share since 1980 is mainly driven by biased technical change when the baseline elasticity is around 0.5 and the decline is more than four times as large as when the elasticity is equal to two. Consequently, the value of the elasticity has major implications for policy and its proper estimation is therefore of utmost importance.

We contribute by implementing an estimation framework that allows for several persistent transition periods in factor-augmenting technical change. This is in line with several papers that argue that technical change is not only the main determinant of the long run labor share, but also important in periods of transition (Beaudry, 2005; Comin and Gertler, 2006; Growiec et al., 2018; Leon-Ledesma and Satchi, 2019; Oberfield and Raval, 2021). As we find in this paper, incorporating these shifts is important to obtain unbiased elasticity estimates on the median. Thus, we provide a less parameteric framework for the estimation of the elasticity that incorporates several shifts in factor-augmenting technical change as compared to earlier studies that incorporate a linear or Box-Cox trend assumption (Antras, 2004; Klump et al., 2007, 2008; McAdam and Willman, 2013; Leon-Ledesma et al., 2015; Stewart and Li, 2018). Lastly, we apply a filter on technical change, whereas Chirinko and Mallick (2017) apply a low-pass filter to the time series prior to estimation.

The paper is organized as follows: In Section 2 we show how imposing assumptions on either the elasticity or technical change affect each other. The state-space framework is presented in Section 3. Section 4 presents the results of the simulation study and Section 5 the estimated elasticities and processes of technical change estimated in the PWT data. Lastly, Section 6 concludes.

2 The bias of technical change and the identification problem of the elasticity of substitution

As the capital-labor elasticity of substitution and technical change are both unobserved parameters, restrictions on one of the parameters are needed to obtain identification. In Section 2.1 we first present the firms optimization problem and argue that the effect of factor-augmenting technical change on the labor and capital share is determined by the elasticity of substitution. Section 2.2 illustrates the identification problem when imposing incorrect assumptions on either the elasticity or the process of technical change.

2.1 The elasticity of substitution and biased technical change

A representative firm in a given country produces in accordance with a CES production function using capital and labor as input factors in production. Technical change is allowed to influence the efficiency of the two factors differently allowing for so-called factor-augmenting technical change:

$$Y_t = \left[\pi\left(\Gamma_t^K K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi)\left(\Gamma_t^L L_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

Y_t is value-added, K_t is capital, and L_t is labor. Γ_t^K and Γ_t^L are capital- and labor-augmenting technologies, respectively. In the following we denote the growth rates of the factor-augmenting technologies as technical change. σ is the constant elasticity of substitution, and π is the capital share in total income. Assuming cost minimizing firms, the first order conditions (FOC) of capital and labor are:

$$r_t \equiv \frac{\partial Y_t}{\partial K_t} = \pi \left(\Gamma_t^K\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}}, \tag{2}$$

$$w_t \equiv \frac{\partial Y_t}{\partial L_t} = (1-\pi) \left(\Gamma_t^L\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}}. \tag{3}$$

r_t and w_t are the factor payments (i.e. the user cost of capital and the wage, respectively). To elaborate on how the factor-augmenting technologies and the elasticity interact, we derive the inverse relative FOC:

$$log\left(\frac{r_t K_t}{w_t L_t}\right) = \sigma log\left(\frac{\pi}{1-\pi}\right) + (1-\sigma)\left(log\left(\frac{\Gamma_t^L}{\Gamma_t^K}\right) + log\left(\frac{r_t}{w_t}\right)\right). \tag{4}$$

Equation (4) illustrates how relative factor-augmenting technical change and changes in the relative price interact with the elasticity of substitution: As an example, consider the case of relative labor-augmenting technical change, that is $\Delta log\left(\Gamma_t^L/\Gamma_t^K\right) > 0$. When $\sigma < 1$, technical change is biased towards capital, i.e. increasing the share of capital, whereas technical change is biased towards labor when $\sigma > 1$. A decrease in the relative factor payments, $\Delta log\left(r_t/w_t\right)$, will increase the share of labor when $\sigma < 1$ and decrease it when $\sigma > 1$. Two special cases emerge when $\sigma \to 1$ where production is Cobb-Douglas, resulting in constant relative factor shares, and $\sigma = 0$ where production is Leontief and there is no substitution between the production factors due to factor payment changes (perfect complements).

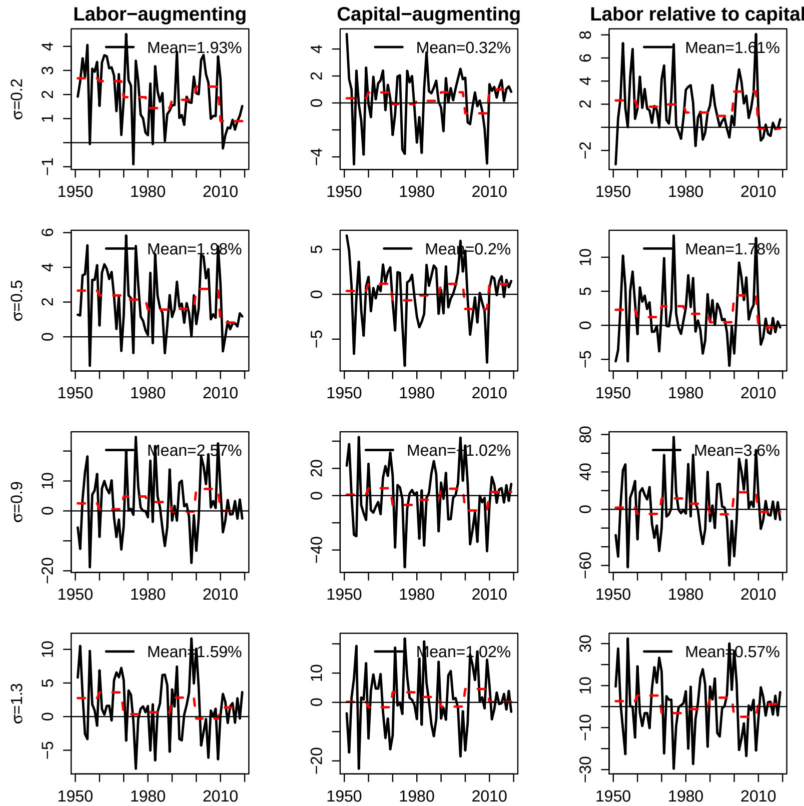
2.2 The identification problem of the elasticity and technical change

As both the elasticity, σ, and the relative factor-augmenting technologies, Γ_t^L/Γ_t^K, are unobserved variables in equation (4), we cannot identify both without further assumptions on either the elasticity or the process of the technologies. In this Section, we illustrate the non-identification problem with a hypothetical example based on US data for the time period 1950-2019.[4] The idea is to first impose different restrictions on the elasticity and illustrate how this affects the residually derived technologies. Next, and most relevant for our paper, we illustrate how imposing restrictions on the process of technical change affects the elasticity. In particular, we highlight how restrictive trend assumptions, such as a linear or Box-Cox time trend, cannot capture several shifts in technical change and that this affects the elasticity.

As we derive Γ_t^L/Γ_t^K residually in this example, it should be emphasized that Γ_t^L/Γ_t^K captures all factors that affect the relative factor shares (other than factor payments). Thus, the residually derived Γ_t^L/Γ_t^K reflects long run factors such as technical change and institutional quality (Acemoglu, 2009; Acemoglu and Autor, 2011; Cantore et al., 2014) but also factors at a business cycle frequency such

[4] Yearly data from the US is applied and all variables are from the PWT and described in more detail in Section 5.

Figure 1: Factor-augmenting technical change in the US data.



Notes: The Figure displays the residually derived growth rates in percentage of labor- and capital-augmenting technologies, Γ_t^L and Γ_t^K, respectively, for different values of σ. The processes are derived from the first order conditions of capital and labor. The dashed red lines show 10-year averages, one for each decade.
Source: Data is obtained from the PWT.

as markup variations, labor hoarding, factor utilization, as well as product-, search-, and labor market frictions (Bertola et al., 2005; Schneider, 2011). Yet, this is no different from other studies in the literature estimating production functions and is still informative on the time-variation in factor efficiency.

Imposing restrictions on σ

First, we impose restrictions on $\sigma \in (0.2, 0.5, 0.9, 1.3)$ and derive the yearly growth rates of Γ_t^L, Γ_t^K, and Γ_t^L/Γ_t^K residually from equations (2)-(4). The resulting technical changes are shown in Figure 1. Two main observations can be made: i) The imposed value of the elasticity has large implications for the volatility as well as the level of technical change. ii) Technical change is directed at improving the efficiency of labor relative to capital on the average. However, several persistent shifts in the direction of technical change, represented by the shifts in the 10-year average growth rates, are observed, in particular after the financial crisis.

Imposing restrictions on Γ_t^L/Γ_t^K

To illustrate how the elasticity of substitution is affected by the assumptions made about technical change, we next perform the opposite exercise where we impose restrictions on the process of technical change and derive the elasticity residually. The procedure consists of three steps: i) Assume $\sigma = 0.5$ and measure $log\left(\Gamma_t^L/\Gamma_t^K\right)$ residually based on equation (4). ii) Next, we estimate a technology process of the residually derived $log\left(\Gamma_t^L/\Gamma_t^K\right)$. iii) Lastly, we impose the fitted values of this estimation in equation (4) and derive σ residually. This hypothetical exercise implies that when σ deviates from 0.5 it reflects mismeasurement of technical change.

The widely cited paper by Berndt (1976) assumed Hicks-neutral technical change and estimated the US elasticity to unity, thus providing support for the Cobb-Douglas production function widely applied in economic models. The assumption of Hicks-neutral technical change implies that capital and labor are equally affected by technical change, i.e. the ratio Γ_t^L/Γ_t^K remains constant. The resulting residually derived value of the elasticity when imposing Hicks-neutrality is shown in the upper left graph of Figure 2. As we use the initial values of Γ_t^L/Γ_t^K derived above, the elasticity starts at $\sigma = 0.5$, but converges to 1 with a trend of 0.56%-points per year. The mean squared error (MSE) is 0.079 reflecting the poor performance of the Hicks-neutral assumption. As pointed out by Antras (2004) and Leon-Ledesma et al. (2015), this bias towards 1 shows that a trend in the relative technologies is needed to match a constant capital-to-labor ratio in the long run when the relative factor payments is trending.

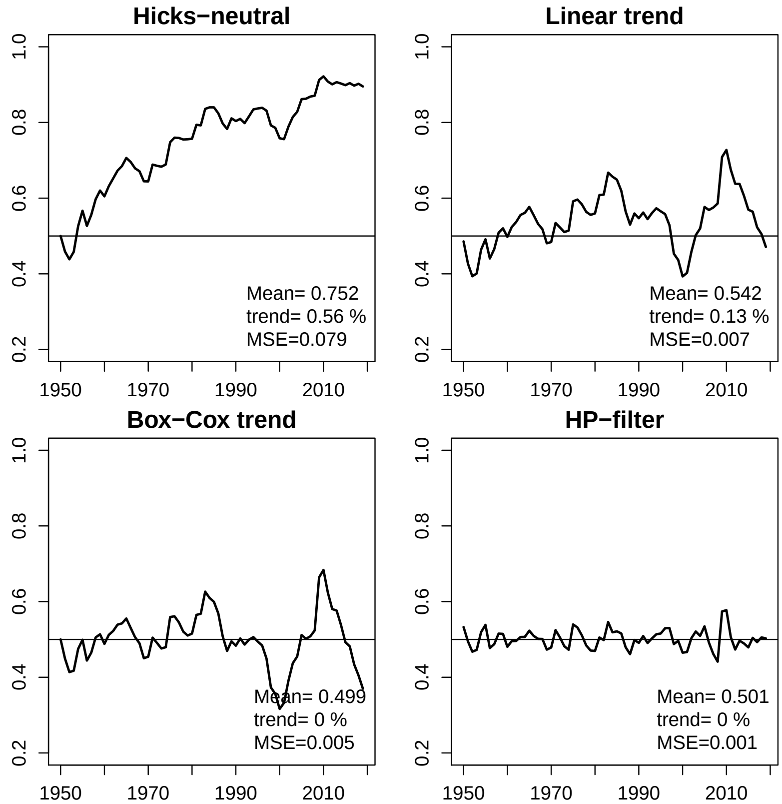
To account for the trend in the relative factor payments, we next assume that Γ_t^L/Γ_t^K follows a linear trend given by $(\gamma_L - \gamma_K)(t - t_0)$.[5] Compared to the Hicks-neutral technical change, using the linear trend assumption considerably improves the average value of the elasticity (upper right graph in Figure 2), which is only slightly upward biased (0.542 vs. imposed value of 0.5). The MSE is 0.007 and thus more than 10 times smaller than the Hicks-neutral assumption. However, considerable deviations from $\sigma = 0.5$ throughout the sample are present which is a mirrow of the fact that the linear trend is unable to reproduce the fluctuations of technical change during several persistent periods of transition.

To incorporate fluctuations of technical change during persistent periods of transition, Klump et al. (2007) suggested to apply a Box-Cox transformation to measure technical change. This assumption allows for accelerating, constant or decelerating growth rates and has been widely applied in later studies (Klump et al., 2008; McAdam and Willman, 2013; Stewart and Li, 2018). The residually derived elasticity when applying a Box-Cox transformation is shown in the lower left graph in Figure 2.[6] The Box-Cox transformation reproduces the average elasticity with high precision and reduces the MSE marginally to 0.005. However, considerable deviations from $\sigma = 0.5$ are present, in particular in the late half of the sample. This illustrates that the (incorrect) identification of technical change being accelerating or

[5]We measure γ_K as the average growth rate of the residually derived Γ_t^K when $\sigma = 0.5$ and γ_N as the average growth rate of Γ_t^N.

[6]The parameters of the Box-Cox transformation are estimated by a non-linear estimator where we minimize the sum of squared residuals. We apply parameter values estimated by Klump et al. (2007) as initial values. The estimated values imply average growth rates of Γ_t^L around 2% and average growth rates of Γ_t^K around 0.4%, close to the average growth rates observed in Figure 1.

Figure 2: Elasticity of substitution for different trend assumptions of technical change.



Notes: The Figure shows the residually derived elasticity, σ, for different trend assumptions of technical change. The elasticity is derived from the relative first order condition of capital and labor. $\sigma = 0.5$ implies correct modeling of technical change.
Source: Data is obtained from the PWT.

decelerating will adversely affect the estimate of the elasticity, in particular in periods of transition.

A process of technical change that allows for several persistent shifts in technical change, but still nests the methods most commonly applied in the literature, is the I(2) process known from the Hodrick and Prescott (1997) filter. In the lower right graph in Figure 2 we see that this method performs superior in terms of reproducing the true estimate on the mean and in particular in terms of minimizing the mismeasurement of technical change during persistent transition periods reflected by the low MSE on 0.001. This result is what motivated our estimation framework where we define technical change as an I(2) process as described in the next Section.

3 The state-space framework

In this Section, we present our new state-space estimation framework for the estimation of the elasticity of substitution. Section 3.1 describes the state-space representation and Section 3.2 presents how the parameters of the model are estimated.

3.1 State-space representation

The state-space representation starts from the relative inverse FOC of capital and labor (equation 4). As the equation is static, it is implicitly assumed that the economy is in a long run equilibrium. However, in macroeconomic modeling adjustment costs are often imposed to ensure lags in the response of quantities to relative price changes (Christiano et al., 2005; Smets and Wouters, 2007). Consequently, using equation (4) may result in a small sample bias due to the relatively short sample applied in the empirical analysis for several of the 16 OECD countries.[7] In this paper, we employ an error-correction model to allow for short run dynamics in factor shares. Specifically, we estimate the equation:

$$\Delta s_t = \alpha \left(s_{t-1} - (1-\sigma)\, p_{t-1} - \mu_{t-1} \right) + \sum_{i=0}^{k} \kappa_i \Delta p_{t-i} + \sum_{i=1}^{k} \omega_i \Delta s_{t-i} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Sigma^{\varepsilon}). \tag{5}$$

$s_t = log\left(\frac{r_t K_t}{w_t L_t}\right)$ is the relative factor share in national income, $p_t = log\left(\frac{r_t}{w_t}\right)$ is the relative factor payments, and $\mu_t = C + (1-\sigma)\, log\left(\frac{\Gamma_t^L}{\Gamma_t^K}\right)$ maps into the relative factor-augmenting technology.[8] Time-invariant factors, C, will affect the level of μ_t but not the dynamics, which are of main interest.[9] The speed of adjustment to the long run equilibrium is determined by α, and σ is the long run elasticity. κ_i and ω_i are the short run elasticities with respect to factor payments and factor shares, respectively. The optimal lag length (k) is determined such that no autocorrelation is present in the measurement errors, ε_t.

As shown in Knoblach et al. (2020) and Section 2, the estimation of σ is highly affected by the assumptions made about the process of the relative factor-augmenting technologies, μ_t. We specify the process of μ_t to satisfy three requirements motivated by the literature: i) As the relative factor payments contains a trend in many of the estimations, a trend should be allowed if the relative factor shares are approximately constant in the long run. ii) We deviate from the linear trend assumption to account for the persistent deviations from the long run trend in periods of transition. iii) μ_t is a slow moving process, meaning it should not reflect short run factors such as labor market frictions, adjustment costs, and risk premium fluctuations. The following I(2)-process for μ_t satisfies these three requirements:

$$\Delta \mu_t = \Delta \mu_{t-1} + \eta_t, \qquad \eta_t \sim N(0, \Sigma^{\eta}). \tag{6}$$

With this model setup, consisting of the observation equation (5) and the state equation (6), ε_t captures temporary deviations from the long run equilibrium, whereas η_t captures persistent changes in the relative

[7] To focus on the long run, researchers have typically relied on error-correction models (Caballero, 1994), applying a low-pass filter to the data (Chirinko and Mallick, 2017), or estimate in long differences (Karabarbounis and Neiman, 2014).

[8] Hicks neutral technical change, affecting both technology factors in the same way, will not affect the *relative* technology level and the *relative* factor shares. Thus, we only identify factor-augmenting technical change.

[9] Based on the non-normalized FOC, equation (4), $C = \sigma log\left(\frac{\bar{\pi}}{1-\bar{\pi}}\right)$ and $C = \sigma log\left(\frac{\bar{\pi}}{1-\bar{\pi}}\right) + (1-\sigma)\, log\left(\frac{\bar{K}}{\bar{N}}\right)$ is obtained from the normalized equation system (14)-(16) presented in Appendix A. As $\bar{\pi}$, $\bar{K}$ and $\bar{N}$ (sample averages) are based on the observed data series, it is possible to determine the value of C and thereby also the level of $log\left(\Gamma_t^L/\Gamma_t^K\right)$.

factor-augmenting technical change. The trend specification in equation (6) is a special case of the Local Trend Model where the variance of the drift term has been set to zero and the result is a model with a smooth trend, which is how we identify technical change. The degree of smoothness of technical change is determined by the noise-to-signal ratio, $\lambda \equiv \Sigma^{\varepsilon}/\alpha^2\Sigma^{\eta}$. A very low degree of smoothing (low values of λ) would ascribe almost the entirety of unexplained year-to-year fluctuations in factor shares to changes in technical change, whereas a very high degree of smoothing imposes a linear trend assumption on the model.

3.2 Estimation of the state-space model

The state parameters of the model $(\mu_t, \sigma, \alpha, \kappa_i, \omega_i)$ are estimated with the Kalman filter and the measurement error variance, Σ^{ε}, is estimated with a recursive application of the maximum likelihood estimator. As a starting point, λ is also estimated with maximum likelihood. However, in some estimations the resulting model is not well-specified, based on a filter consistency test (the Normalized Innovations Squared test (NIS) - see Appendix B for a description) and a Breusch-Godfrey test for autocorrelation. As shown in Section 5.2, a low degree of smoothing often leads to filter inconsistency (overfitting), whereas a high degree of smoothing results in autocorrelated residuals (trend is too restrictive). Therefore, the maximum likelihood estimate is combined with a grid searching procedure in the range 20 to 500 with increment 10. The value of λ that maximizes the likelihood conditional on being well specified at the 10% significance level is chosen.[10] By applying the grid search procedure, we also address the issue of having potentially multiple local minima of the likelihood function.

Technically, the elasticity and adjustment parameter are specified as unobservables with a zero variance in the state-space representation. Therefore, no standard errors are obtained when using the Kalman filter. Instead, we apply a standard recursive residual-based bootstrapping procedure with 1,000 iterations to obtain standard errors, which are valid given that the model's measurement errors are neither autocorrelated nor heteroscedastic.

To summarize, the estimation procedure is as follows:

1. Estimate the state-space representation with the Kalman filter for different initializations of σ and α. Pick the combination that maximizes the likelihood.
   (a) For every initialization of σ and α we estimate Σ^{ε} with maximum likelihood for different initializations. Choose the initial value of Σ^{ε} that maximizes the likelihood. If λ is freely estimated, it will be estimated in this step also.
2. If autocorrelation in ε_t or a violation of the consistency test is detected, lags are included and the procedure returns to step 1.

[10] A 10% significance level is used throughout the paper to acknowledge that we are working with a relatively small sample.

3. Perform steps 1-2 for a grid of different values of λ. The first is a free estimation and the proceeding values are calibrated from the interval 20 to 500 with increment 10. The preferred value of λ is the value that maximizes the likelihood conditional on being well-specified based on the autocorrelation and filter inconsistency test.

4. Obtain standard errors from the recursive residual-based bootstrapping procedure.

This procedure is relatively easy to implement, and codes are publicly available at Github. We refer the reader to Kastrup et al. (2021) for a detailed description of how to apply the package.

4 Simulation evidence

4.1 The simulation setup

Data is simulated according to the simulation studies by Leon-Ledesma et al. (2010, 2015). They simulate stochastic processes of labor (L_t), capital (K_t), labor-augmenting technologies $\left(\Gamma_t^L\right)$, and capital-augmenting technologies (Γ_t^K) and match the variance of these to US data for the time period 1950-2005. These processes are used to derive equilibrium value-added ($Y_t^\star$), observed value-added (Y_t) (i.e. equilibrium value-added plus measurement errors), and real factor payments (r_t and w_t). Measurement errors are added in terms of an interest rate and a wage shock. We deviate in two aspects: i) We update the standard deviations of the growth rate of L_t, K_t, and Y_t according to the US data applied in the empirical analysis. ii) Leon-Ledesma et al. (2010, 2015) assume that Γ_t^L and Γ_t^K consist of a linear trend and an i.i.d. error term. We test the performance of the estimators for different deterministic trend specifications, by first simulating Γ_t^L and Γ_t^K as Random Walks with a drift and a stochastic trend. Second, we simulate data according to a Box-Cox transformation of the growth rates. Lastly, we simulate data with several persistent shifts in the average growth rate of technical change, consistent with the 10-year averages shown in Figure 1 when $\sigma = 0.5$. In general we find that ii) (i.e. how technical change is specified) has the largest impact on the results and is the most important change relative to Leon-Ledesma et al. (2010).

As in Leon-Ledesma et al. (2010, 2015), the stochastic processes of capital and labor are simulated according to a Random Walk with a drift:

$$log\left(K_t\right) = \eta + log\left(K_{t-1}\right) + \varepsilon_t^K, \tag{7}$$

$$log\left(L_t\right) = \kappa + log\left(L_{t-1}\right) + \varepsilon_t^L. \tag{8}$$

η and κ are the average growth rate of capital and labor, respectively. Both error terms are assumed normally distributed with mean zero and standard deviations, $sd\left(\varepsilon_t^K\right)$ and $sd\left(\varepsilon_t^L\right)$. We set the initial values $K_0 = L_0 = 1$.

We deviate from Leon-Ledesma et al. (2010, 2015) by simulating the factor-augmenting technologies of capital and labor according to the following processes:

$$log\left(\Gamma_t^K\right) = g_K\left(t,\bar{t}\right) + log\left(\Gamma_{t-1}^K\right) + \varepsilon_t^{\Gamma^K}, \tag{9}$$

$$log\left(\Gamma_t^L\right) = g_L\left(t,\bar{t}\right) + log\left(\Gamma_{t-1}^L\right) + \varepsilon_t^{\Gamma^L}. \tag{10}$$

$g_K\left(t,\bar{t}\right)$ and $g_L\left(t,\bar{t}\right)$ are the deterministic trends and $\varepsilon_t^{\Gamma^K}$ and $\varepsilon_t^{\Gamma^L}$ the stochastic i.i.d. normally distributed error terms with standard deviations $sd\left(\varepsilon_t^{\Gamma^K}\right)$ and $sd\left(\varepsilon_t^{\Gamma^L}\right)$. We set $\Gamma_0^K = \Gamma_0^L = 1$ and apply three different specifications of the deterministic trends: i) A linear trend assumption, $g_K\left(t,\bar{t}\right) = \gamma_K$ and $g_L\left(t,\bar{t}\right) = \gamma_L$. ii) A Box-Cox transformation of the growth rates (see Appendix C for details). iii) Lastly, to analyze the performance of the estimators when several persistent deviations from the long run trend are present, we apply the 10-year averages for $\sigma = 0.5$ shown in Figure 1 as measures of $g_K\left(t,\bar{t}\right) = \gamma_{K,\tau}$ and $g_L\left(t,\bar{t}\right) = \gamma_{L,\tau}$ with τ indicating the decade.

Several papers highlight the importance of normalizing the production function when applying system estimators to estimate the substitution elasticity (Klump et al., 2007; Leon-Ledesma et al., 2010; Klump et al., 2012). As argued by Leon-Ledesma et al. (2010), the sample average of the distribution parameter, $\bar{\pi}$, provides plausible starting values for π when the system is normalized and the efficiency parameter, $\xi = Y_0/\bar{Y}$, should be close to unity without casting serious doubts on the estimation results. Oppositely, a non-normalized system provides no clear guidelines to the starting values of π and ξ which may bias the estimate of σ towards unity. To test the sensitivity of our state-space framework to normalization, we follow Leon-Ledesma et al. (2010) and simulate data according to a normalized production function. The normalized production function is given by:

$$Y_t^\star = Y_0^\star \left[\pi_0 \left(\frac{K_t}{K_0}\frac{\Gamma_t^K}{\Gamma_0^K}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{L_t}{L_0}\frac{\Gamma_t^L}{\Gamma_0^L}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{11}$$

$Y_t^\star$ is used to derive the first order conditions of capital and labor. $Y_0^\star = \frac{r_0}{\pi_0}K_0$ is the initial value. We initialize the capital share, π_0, and interest rate, r_0, to 0.4, which implies that $Y_0^\star = 1$. With this particular initialization and $K_0 = 1$, the estimate of σ is invariant to normalization. Therefore, we adjust the size of K_0 and r_0 in Appendix C to analyze how our framework performs when normalization matters.

The first order conditions (i.e. the factor payments) are obtained by differentiating (11) with respect to capital and labor, respectively:

$$r_t \equiv \frac{\partial Y_t^\star}{\partial K_t} = \pi_0 \left(\frac{\Gamma_t^K}{\Gamma_0^K}\frac{Y_0^\star}{K_0}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t^\star}{K_t}\right)^{\frac{1}{\sigma}} e^{\varepsilon_t^r}, \tag{12}$$

$$w_t \equiv \frac{\partial Y_t^\star}{\partial L_t} = (1-\pi_0) \left(\frac{\Gamma_t^L}{\Gamma_0^L}\frac{Y_0^\star}{L_0}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t^\star}{L_t}\right)^{\frac{1}{\sigma}} e^{\varepsilon_t^w}. \tag{13}$$

The terms ε_t^r and ε_t^w are an interest rate and a wage shock, respectively, and assumed normally distributed with zero mean. To remain consistent with national accounts we adjust value-added so that $Y_t = r_t K_t + w_t L_t$. It can be shown that $Y_t / Y_t^\star = \eta_t e^{\varepsilon_t^r} + (1-\eta_t) e^{\varepsilon_t^w}$.[11] Thus, to match the volatility of value-added observed in the US data, it is necessary to calibrate the standard deviations of ε_t^r and ε_t^w. However, as the volatility of $Y_t^\star$ and η_t depends on the trend assumptions of technical change, the standard deviations of ε_t^r and ε_t^w need to be adjusted accordingly. As in Leon-Ledesma et al. (2015) we impose the assumption that $sd(\varepsilon_t^r)/sd(\varepsilon_t^w) = 2$. By imposing this assumption, we calibrate the standard deviations of ε_t^r and ε_t^w such that the growth rate of Y_t matches the growth rate of GDP in the US data.

Parameterization

The parameterization is mainly based on the literature (see Table 1). In addition, we update the standard deviations based on the US data applied in the empirical analysis.[12] The initial capital share in total income, π_0, is 0.4, which corresponds well with an average value of 0.38 in the US data applied in the empirical analysis. The performance of the estimators are tested for four different values of the elasticity of substitution, σ. The first value is a near-Leontief elasticity of 0.2. The second elasticity is set to 0.5 and similar to the preferred US estimate from Klump et al. (2007) and Knoblach et al. (2020). The last two values of the elasticity are close to unity from either sides: 0.9 and 1.3, respectively.

γ_K and γ_L are set to the same values as in Leon-Ledesma et al. (2010) and the curvature parameters of the Box-Cox transformation, λ_K and λ_L, are taken directly from Klump et al. (2007). The average growth rates of labor (η) and capital (κ) are from Leon-Ledesma et al. (2010), corresponding to average growth rates of 1.5% and 3%, respectively, and similar to the US data applied in the empirical analysis (1.2% and 2.7%, respectively).

The standard deviation of the growth rate in the capital stock, $sd\left(\varepsilon_t^K\right)$, and labor, $sd\left(\varepsilon_t^L\right)$, and value-added, $sd\left(\Delta log\left(Y_t\right)\right)$, are calibrated based on the US data applied in the empirical analysis to 0.008, 0.02, and 0.021, respectively. As the residually derived processes shown in Figure 1 reflect technical change and other factors including measurement errors, the standard errors of the factor-augmenting technologies, $sd\left(\varepsilon_t^{\Gamma^K}\right)$ and $sd\left(\varepsilon_t^{\Gamma^L}\right)$, are unobserved. Both values are set equal to 0.01 which is close to the variance of a TFP shock in papers such as Backus et al. (1994) and Kose and Yi (2006) (0.009 and 0.018, respectively). As a robustness, we have also tried values of 0.02 and 0.005 and the quantitative results still holds true.[13] At the baseline, we set the number of time periods equal to 50. We test the

[11] $\eta_t = \dfrac{\pi_0\left(\frac{K_t}{K_0}\frac{\Gamma_t^K}{\Gamma_0^K}\right)^{\frac{\sigma-1}{\sigma}}}{\pi_0\left(\frac{K_t}{K_0}\frac{\Gamma_t^K}{\Gamma_0^K}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{L_t}{L_0}\frac{\Gamma_t^L}{\Gamma_0^L}\right)^{\frac{\sigma-1}{\sigma}}}.$

[12] We update the standard deviations of capital, labor, and GDP growth according to the PWT data and the time period 1950-2019. Using a 50 year period (1970-2019) leads to almost identical standard deviations.

[13] As an example, when the determinstic trend is the 10-year averages and $\sigma = 0.5$, the median estimate is almost unchanged when applying the state-space framework. Oppositely, the median estimates with the system estimators are increasing in the value of the standard errors. This is of no surprise as when the stochastic trend becomes increasingly important, technical change becomes increasingly non-linear and thus a worse fit of the linear or Box-Cox trend assumptions.

Table 1: Parameter values.

Parameter	Description	Value	Source
π_0	Capital share in income	0.4	Leon-Ledesma et al. (2010)
σ	Elasticity of substitution	0.2,0.5,0.9,1.3	Leon-Ledesma et al. (2010)
γ_K	Average growth rate of capital-augmenting technology	0.005	Leon-Ledesma et al. (2010)
γ_L	Average growth rate of labor-augmenting technology	0.015	Leon-Ledesma et al. (2010)
λ_K	Curvature parameter of capital-augmenting technology	-0.118	Klump et al. (2007)
λ_L	Curvature parameter of labor-augmenting technology	0.439	Klump et al. (2007)
η	Average growth rate of labor	0.015	Leon-Ledesma et al. (2010)
κ	Average growth rate of capital	$\gamma_L+\eta$	Leon-Ledesma et al. (2010)
$sd(\varepsilon_t^K)$	Standard deviation of the growth rate in the capital stock	0.008	US data
$sd(\varepsilon_t^L)$	Standard deviation of the growth rate in the labor	0.02	US data
$sd(\Delta log(Y_t))$	Standard deviation of the growth rate in value-added	0.021	US data
$sd(\Delta log(\varepsilon_t^{\Gamma^{K,L}}))$	Standard deviation of factor-augmenting technical change	0.01	
T	Number of yearly observations	20-70	
M	Number of monte carlo draws	1000	

sensitivity of the results to the number of time periods by applying values in the range 20-70 in Appendix C. Lastly, the number of simulations is set to 1,000.[14]

Table C1 in Appendix C reports the mean and standard deviations of the growth rates of the simulated variables K_t, L_t, Y_t, r_t, and w_t as well as the moments in the US data applied in the empirical analysis. Importantly, across the trend assumptions and elasticities, our simulation study performs well in terms of replicating the moments of K_t, L_t, and Y_t. The moments of r_t and w_t match the US data to a lesser extent and factors such as adjustment costs of capital and labor may be important for solving this issue. Including such factors in the simulation study is beyond the scope of this paper.[15]

4.2 Simulation results

In this Section, we present the simulation results. The performance of the state-space framework is compared to two system estimators, presented in detail in Appendix A.[16] The first system estimator imposes a linear trend assumption and the second system estimator a Box-Cox transformation of the growth rates.

[14]We find that our results are stable when the number of draws is above 200.

[15]In general, we find that adjusting the variance of r_t and w_t through changes in the measurement errors only have minor implications for the estimation results.

[16]When applying the state-space framework we do not include the tests for autocorrelation or NIS when determining the optimal value of λ. This adjustment only affects the distance between the percentiles in some cases and the median estimate is unchanged. As the measurement errors are added to the factor payments we estimate the inverse version of equation (4). If only estimating equation (4), we observe a downward bias in the estimates of the elasticity. By reversing the equation we obtain precise estimates which suggest, maybe not surprisingly, that an attenuation bias is present when the measurement errors are added to the explanatory variables instead of the dependent variable. In addition, estimating the relative first order conditions instead of equation (4) is more comparable to the system estimator that also estimate first order conditions. Thus, with these adjustments, the estimators are as comparable as possible.

Table 2: Estimated median elasticities in the simulated data.

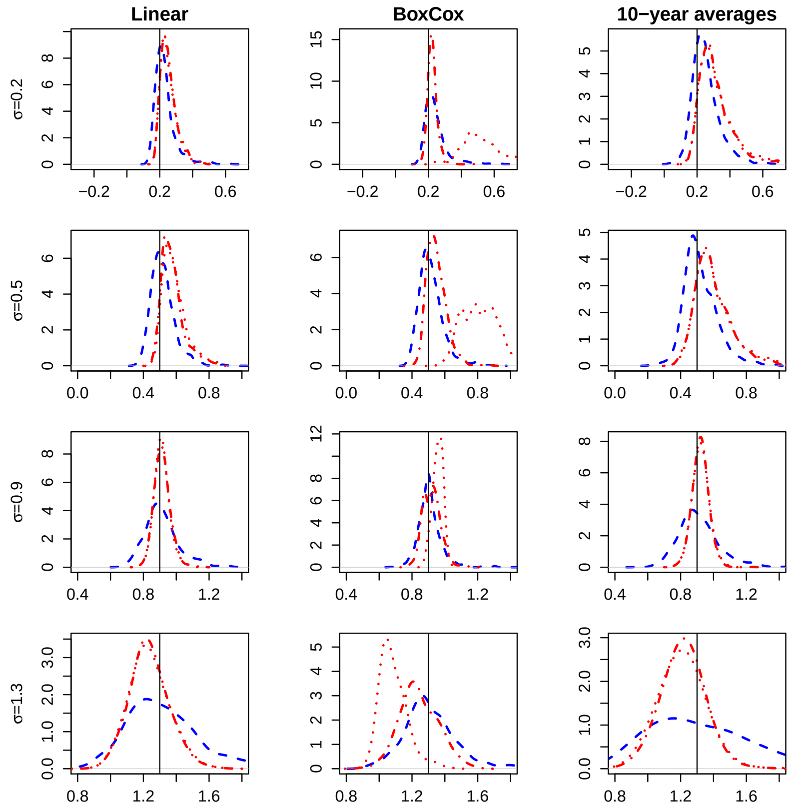
	σ= 0.2	σ= 0.5	σ= 0.9	σ= 1.3
		Linear		
State-space framework	0.22 (0.17;0.30)	0.51 (0.44;0.61)	0.90 (0.79;1.06)	1.30 (1.07;1.67)
System, Linear trend	0.24 (0.20;0.33)	0.58 (0.51;0.68)	0.91 (0.86;0.97)	1.22 (1.09;1.39)
System, Box-Cox	0.24 (0.20;0.31)	0.56 (0.51;0.67)	0.91 (0.86;0.98)	1.23 (1.09;1.39)
		BoxCox		
State-space framework	0.23 (0.18;0.33)	0.51 (0.43;0.61)	0.90 (0.83;0.99)	1.29 (1.13;1.53)
System, Linear trend	0.51 (0.39;0.73)	0.81 (0.67;0.95)	0.96 (0.91;0.99)	1.07 (1.00;1.18)
System, Box-Cox	0.22 (0.19;0.26)	0.54 (0.48;0.62)	0.92 (0.85;0.99)	1.23 (1.10;1.40)
		10-year averages		
State-space framework	0.24 (0.17;0.38)	0.50 (0.41;0.65)	0.90 (0.77;1.08)	1.31 (0.95;1.97)
System, Linear trend	0.29 (0.21;0.46)	0.58 (0.48;0.80)	0.92 (0.86;0.98)	1.21 (1.05;1.36)
System, Box-Cox	0.29 (0.21;0.45)	0.58 (0.48;0.77)	0.92 (0.87;0.98)	1.22 (1.07;1.34)

Notes: The table reports the estimated median elasticities in the simulated data. The results are displayed for four different values of the elasticity, σ, and three different trend assumptions in the data generating process. 10 and 90 percentiles are reported in paranthesis.
Source: Own simulations.

Table 2 reports the estimated median elasticities when varying the value of σ and the trend assumption of Γ_t^L and Γ_t^K in the data generating process. Importantly, across values of σ and trend assumptions in the data generating process, the state-space framework is very accurate on the median: The state-space framework is able at replicating the true elasticity for all three types of deterministic trends in the data generating process. Oppositely, the two system estimators are both biased towards unity on the median. This bias is consistent with papers such as Luoma and Luoto (2011) and Stewart and Li (2018) who argue that the system estimator does not sufficiently account for the cross-equation correlation of measurement errors. However, as the bias of the system estimators increases when technical change becomes increasingly non-linear, cross-equation correlation of measurement errors cannot be the sole explanation for the bias towards unity. Consequently, misspecification of technical change also biases the median estimate and a flexible specification of technical change, such as the state-space framework, is important for obtaining unbiased median estimates.

Along with the 10 and 90 percentiles reported in Table 2, density plots are used to shed further light

Figure 3: Density plots of the estimated elasticities in the simulated data.



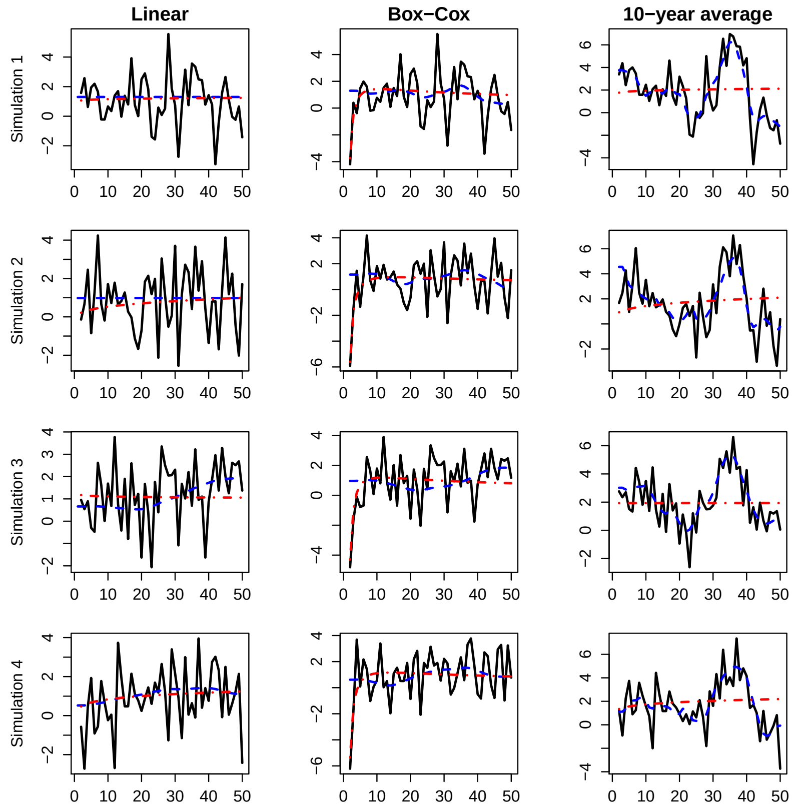
Notes: The Figure shows the density plots of the simulations for different values of σ and the three different trend assumptions in the data generating process. Dashed blue lines are the state-space framework, dotted red lines the system with linear trend, and dotdashed red lines the system with Box-Cox trend.
Source: Own simulations.

on the distribution of the estimators (Figure 3). The estimates obtained with the state-space framework are centered around the true value for all trend assumptions and values of the elasticity. In addition, the estimates with the state-space framework are determined with a high precision in the empirically plausible range of $\sigma \approx 0.5$. While the system estimators are in general more narrowly distributed than the state-space framework, the distributions are biased towards unity. This holds in particular true for the system estimator with a linear trend when the deterministic trend in the data generating process is a Box-Cox transformation.

Applying a flexible trend assumption is important not only for the precision of the estimates but also for the estimation of technical change.[17] When the deterministic trend in the data generating process is linear (Figure 4, linear trend simulation 1-2) both the state-space framework and system estimator with Box-Cox trend estimate processes that are close to linear. However, when the stochastic trend deviates systematically from the deterministic trend (linear trend simulation 3 and 4), the estimated processes with the state-space framework is a smooth trend and deviates from the deterministic trend, whereas this to a

[17] In Figure 4, we show the first four simulations ($k \in 1,2,3,4$) with seed number $1234+k$ with k being the draw number. Estimation results shown for $\sigma = 0.5$.

Figure 4: Estimated relative factor-augmenting technical change in the simulated data.



Notes: The graphs show the simulated (solid black line) and estimated processes (dashed blue: state-space framework, dotdashed red: system estimator with Box-Cox trend) of $\Delta log(\Gamma_t^L/\Gamma_t^K)$ in percentage for different trend assumptions in the data generating process. The first four simulations are displayed.
Source: Own simulations.

lesser extent is captured by the system estimator with Box-Cox trend. When the deterministic trend in the data generating process is Box-Cox (Box-Cox trend simulation 1-4), the state-space framework to a high extent captures the dynamics of technical change during the sample and outperforms the system estimator with Box-Cox trend, except for the initial shift in technical change. When the deterministic trend is the 10-year averages (10-year average growth rate simulation 1-4), technical change is time-varying with several persistent shifts in technical change, in agreement with the US data shown in Figure 1. Importantly, the state-space framework reproduces the dynamics of technical change with high precision and outperforms the Box-Cox trend, which is unable to capture several persistent shifts in technical change.

In Appendix C we test the sensitivity of the state-space framework to normalization (i.e. different initializations of K_0 and r_0). We find that the state-space framework is close to unaffected by normalization. This is a major advantage of our framework relative to the system estimators where proper normalization is necessary to obtain unbiased estimates. In addition, we also test the sensitivity to the number of observations and find that when the number of observations is above 30, increasing the number of observations has primarily gains for the precision of the estimates reflected by narrower confidence bands.

5 Empirical application

The simulation exercise highlights that considerable gains in the estimate of the elasticity of substitution and factor-augmenting technologies are obtained by applying the state-space framework compared to estimators that assume a linear or Box-Cox trend. This in particular holds true in the empirically realistic case where the trend process is time-varying with several persistent shifts in the growth rate of technical change.

In this Section we apply the state-space framework to estimate the capital-labor substitution elasticity on actual data from 16 OECD countries. The applied data is described in Section 5.1. In Section 5.2 we report the main estimates of the elasticity of substitution and the estimated processes of technical change are shown in Section 5.3.

5.1 Data

Four data series are necessary to estimate the elasticity: The capital stock, labor in hours, the user cost of capital, and the hourly wage. The main data source is the PWT version 10 (Feenstra et al., 2015). The capital stock and labor in hours are obtained directly from the PWT. For most countries, the PWT also includes data on the labor share needed to obtain the hourly wage. However, for some countries the labor share data is limited prior to 1995. In these cases, we apply data from the Structural Analysis Database (STAN) from OECD, which enables us to extend the labor share data backwards to at least 1976.[18]

The main data issue in any study estimating the elasticity of substitution is how to measure the unknown user cost of capital. From national accounts it follows that aggregate value-added is divided into the share of labor, capital, and profits (Π_t), $Y_t = r_t K_t + w_t N_t + \Pi_t$. As mentioned in Hulten (2010), two approaches have typically been taken to separate the two unknowns, r_t and Π_t. The first imposes assumptions on r_t, such as the Hall and Jorgenson (1967) user cost, and derive Π_t residually. The main advantage of this approach is that time variation in markups, e.g. the global increase since 1980 documented by De Loecker and Eeckhout (2018), will not affect the user cost. However, we find this approach problematic for at least two reasons: i) As mentioned by Karabarbounis and Neiman (2019), the Hall and Jorgenson (1967) user cost does not take factors such as investment risk, adjustment costs, financial constraints, and risk premiums into account. ii) Caballero et al. (2017) find that the growing risk premiums have generated an increasing wedge between the treasury bond rate and corporate borrowing costs in recent decades. As a consequence, the user cost will not reflect the actual cost of borrowing.

The second approach instead imposes assumptions on Π_t and derive r_t residually. While this approach requires assumptions on the aggregate profits, it is able to overcome many of the challenges with the Hall and Jorgenson (1967) user cost and generates a more stable outcome (Karabarbounis and Neiman, 2019).

[18] The data series on the labor compensation from OECD do not include self-employment. This is normally corrected for by assuming that the self-employed obtain the same wage as the employed (Blanchard, 1997; Klump et al., 2008; McAdam and Willman, 2013; Leon-Ledesma et al., 2015). We have tried to measure labor compensation with and without this correction. The resulting two labor shares were highly correlated (e.g. a correlation of 0.81 in France, 0.97 in Canada, and 0.99 in the US). As data on self-employment is limited in the time dimension, we prefer not to correct for self-employment.

Importantly, Karabarbounis and Neiman (2019) find that by adjusting sales to not only include the cost of goods sold but also selling, general, and administrative expenses, the markup has been almost constant over time. Based on these considerations, we follow Leon-Ledesma et al. (2015) and Cantore et al. (2017) by assuming a 10% markup and derive the implied user cost of capital residually. It is important to note that this assumption only affects the data applied in the empirical analysis and not the estimation framework. As such, it is possible to implement different levels of the markup or time variability (e.g. increase in recent decades) without changing the estimation framework. The resulting data series consist of at least 44 observations for each country, fulfilling the minimum number of observations necessary to obtain reliable estimates in the simulation study.

5.2 The elasticity of substitution

In Table 3 we report the estimated elasticities of substitution between capital and labor for the 16 OECD countries in our sample. We apply the state-space framework as presented in Section 3 and compare to the Box-Cox system estimator described in Appendix A.[19] The shortest time period is 1976-2019 (Austria, column 1) and the longest 1950-2019 (US, Sweden, and France). The unweighted average elasticity is 0.35 and the average weighted with GDP is 0.42 (column 2).[20] This difference is primarily driven by a relatively high estimate in US data on 0.54. The estimates range from 0.11 (Norway) to 0.65 (Korea).[21] Even though some of the estimates are close to zero, all are significantly different from zero as well as unity. Therefore, we reject the cases of Leontief and Cobb-Douglas production function in all countries. Based on the meta regression studies (Knoblach et al., 2020; Gechert et al., 2021) and the literature review in Chirinko (2008), the consensus US value in aggregate time series is in the range 0.4-0.6 when allowing for factor-augmenting technical change and controlling for different sources of bias. Consequently, our US estimate is consistent with the literature. Significant error-correction is present in all countries (column 3). The unweighted average yearly error-correction is 33% and the weighted 39%, ranging from 16% (Japan) to 50% (US). The fact that the optimal allocation of input factors do not respond immediately to factor payments and technical change highlights the importance of frictions such as adjustment costs.

In roughly two thirds of the estimations, the maximum likelihood estimate of the noise-to-signal ratio, λ, is well specified and preferred relative to the grid searching procedure (column 4, see Section 3 for details). The majority of the maximum likelihood estimates are in the range specified by the grid searching procedure, except for Denmark and Australia (3765 and 7458, respectively). The remaining noise-to-signal ratios are in the range 10-95, similar to values traditionally applied in the business cycle

[19] As we construct the user cost by assuming a 10% markup, we estimate the system estimator by imposing a markup consistent with Leon-Ledesma et al. (2015).

[20] As only well-specified estimations are kept, the methodology secures automatically that the estimates are well specified based on the autocorrelation and NIS test. Therefore, we do not report these test statistics.

[21] The low estimate in Norway is likely driven by the fact that an extra lag of first-difference of relative prices is added to the observation equation. When not including this lag, the estimate is around 0.3. However, not including this lag comes with the cost of autocorrelated residuals.

literature (Backus et al., 1994; Ravn and Uhlig, 2002). Lemoine et al. (2010) also apply the Kalman filter, but find that a maximum likelihood estimation of λ has a tendency to result in $\lambda \to 0$, thereby attributing most of the variation not explained by the factor prices to technical change, i.e. close to a dynamic calibration of technical change. Importantly, our finding that most of the estimates of λ obtained with the state-space framework are far from zero highlights that our framework is not subject to this over-fitting issue. As we shall see in the next Section, smoothing in the range 10-95 allows for several persistent shifts in technical change and the high values for Denmark and Australia imply an approximately linear trend of technical change.

To elaborate on the high noise-to-signal ratio in Denmark and Australia, we have decomposed the relative factor shares net of long run price effects $(s_t - (1-\sigma)\, p_t)$ in a trend component (μ_t) and a cyclical component $(\kappa \Delta p_t + \varepsilon_t)$ (available upon request). The cyclical component is trending in Denmark and have a level break in Australia due to a high wage inflation in the 70'es and 80'es in both countries, similar to many European countries (Blanchard, 1997). When removing short run price effects from the cyclical component, the remaining term is stationary and appear as an i.i.d. error term. Thus, when controlling for short run price effects, a linear trend fits well with the long run trend of factor shares net of prices. Oppositely, the cyclical component is stationary in the US implying that the medium term deviations from the long run level of factor shares net of prices are instead captured by the trend component (hence a lower noise-to-signal ratio).

Column 6-8 in Table 3 report the main parameter estimates when applying the system estimator with a Box-Cox trend and the full set of the parameters are displayed in Table D1 in Appendix D. The average elasticity is above the average obtained when applying the state-space framework (0.45 compared to 0.35). The estimates deviate significantly from the state-space framework in more than half of the estimations, highlighting that the choice of estimation framework and in particular process of technical change have major implications for the estimated elasticity. In all countries $\gamma_L > \gamma_K$, implying that technical change is predominantly labor-augmenting (the average of γ_L is approximately 2% and the average of γ_K is 0%). Whereas λ_L is estimated on the unit interval, implying a decelerating growth rate of labor-augmenting technical change and estimated with a high precision, λ_K ranges from -0.92 (Sweden) to 4.12 (Great Britain) and is in general imprecisely estimated.

In Appendix D we report the estimated elasticities with the state-space framework for different noise-to-signal ratios. We find that i) in line with Chirinko and Mallick (2017), the estimated elasticity is tightly bound by the filtering assumptions made. This highlights that in line with the literature (Klump et al., 2007; McAdam and Willman, 2013; Knoblach et al., 2020), our findings reiterate the tight link between identifying restrictions on technical change and the elasticity of substitution. Although our model specification is less parametric and seemingly less a priori restrictive we do not escape this fact. ii) Some (but not too high) degree of smoothing is needed to obtain a well specified model. This indicates that persistent deviations from the long run trend of technical change are not only important from an economic perspective but also an econometric perspective. iii) For most countries, a certain range of moderate degrees of smoothing lead to almost the same estimate, implying that the framework is robust

Table 3: Main estimates of the elasticity of substitution.

		State-space framework				Box-Cox system estimator		
	Time period	σ	α	λ	lags	σ	γ_K	γ_L
Australia	1959-2019	0.28	−0.23	7458	0	0.44	−0.001	0.016
		(0.07)	(0.07)			(0.00)	(0.000)	(0.000)
Austria	1976-2019	0.39	−0.4	10	0	0.29	−0.002	0.017
		(0.09)	(0.09)			(0.00)	(0.000)	(0.000)
Belgium	1970-2019	0.34	−0.45	20	0	0.27	0.002	0.015
		(0.03)	(0.07)			(0.00)	(0.000)	(0.000)
Canada	1970-2019	0.27	−0.37	20	0	0.34	−0.007	0.011
		(0.04)	(0.10)			(0.00)	(0.001)	(0.001)
Denmark	1970-2019	0.43	−0.3	3765	0	0.28	−0.001	0.016
		(0.05)	(0.05)			(0.00)	(0.000)	(0.000)
Finland	1970-2019	0.5	−0.27	34	0	0.45	−0.004	0.023
		(0.07)	(0.05)			(0.00)	(0.001)	(0.001)
France	1950-2019	0.12	−0.31	20	0	0.52	−0.002	0.027
		(0.05)	(0.05)			(0.00)	(0.000)	(0.001)
Great Britain	1970-2019	0.36	−0.35	30	0	0.44	0.002	0.017
		(0.05)	(0.09)			(0.00)	(0.001)	(0.001)
Italy	1970-2019	0.48	−0.25	59	0	0.41	−0.009	0.012
		(0.07)	(0.05)			(0.00)	(0.000)	(0.000)
Japan	1970-2019	0.13	−0.16	60	0	0.4	−0.005	0.02
		(0.06)	(0.05)			(0.00)	(0.000)	(0.000)
Korea	1970-2019	0.65	−0.37	95	0	0.78	−0.023	0.064
		(0.08)	(0.06)			(0.02)	(0.002)	(0.002)
Netherlands	1970-2019	0.25	−0.28	20	0	0.29	−0.001	0.014
		(0.03)	(0.06)			(0.01)	(0.000)	(0.000)
Norway	1970-2019	0.11	−0.34	45	1	0.6	−0.001	0.021
		(0.05)	(0.07)			(0.01)	(0.001)	(0.001)
New Zealand	1971-2019	0.29	−0.5	11	0	0.75	−0.008	0.019
		(0.05)	(0.09)			(0.02)	(0.003)	(0.002)
Sweden	1950-2019	0.47	−0.23	95	0	0.6	0	0.022
		(0.06)	(0.04)			(0.01)	(0.001)	(0.000)
USA	1950-2019	0.54	−0.5	16	0	0.33	0.003	0.017
		(0.09)	(0.08)			(0.00)	(0.000)	(0.000)
Mean		0.35	−0.33	735	0	0.45	−0.004	0.021
		(0.06)	(0.07)			(0.00)	(0.001)	(0.001)
Weighted mean		0.42	−0.39	298	0	0.39	−0.001	0.019
		(0.07)	(0.07)			(0.00)	(0.000)	(0.000)

Notes: The Table reports the main estimation results of the elasticity of substitution for the 16 OECD countries. Results are reported for the state-space framework and the system estimator with Box-Cox trend. From the state-space framework, the estimated elasticity, σ, adjustment parameter, α, noise-to-signal ratio, λ, and number of lags included in the estimation are reported. From the system estimator, the estimated elasticity, the average growth rates of labor-augmenting technical change, γ_L, capital-augmenting technical change, γ_K are reported. Standard errors reported in paranthesis.
Source: Data is obtained from PWT and the OECD STAN database.

to changes in λ in a certain range. iv) The level of smoothness has not only implications for the elasticity of substitution, but also for the estimated process of technical change. We refer the interested reader to Appendix D for a further discussion of these points.

5.3 Factor-augmenting technical change

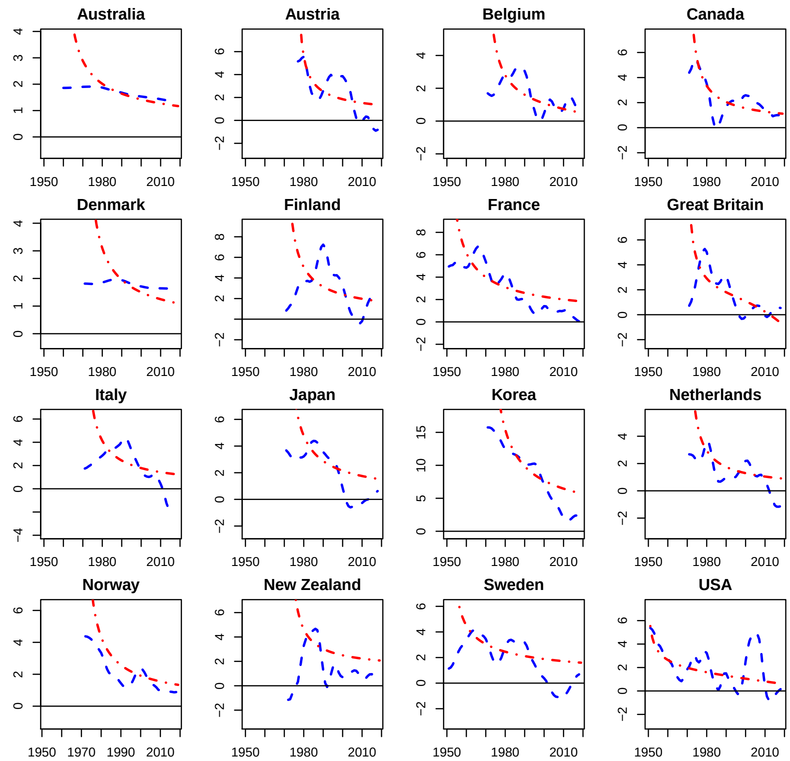
The relative factor-augmenting technical change, $\Delta log\left(\Gamma_t^L/\Gamma_t^K\right)$, estimated with the state-space framework is displayed in Figure 5 for all the 16 OECD countries (dashed blue line). Technical change has been directed at improving the relative efficiency of labor during the entire sample period, except for the years after the financial crisis in Austria, Netherlands, Sweden, and the US. However, the relative growth rate has decreased over time, i.e the speed of labor-augmenting technical change has declined (or capital-augmenting technical change increased). Several persistent fluctuations are observed in the growth rates, indicating an important role for capital-augmenting technical change during periods of transition. In particular a tendency of capital-augmenting technical change in the 90'es and after the financial crisis is observed, e.g. the US and Great Britain. These findings are consistent with previous papers that have tried to rationalize shifts in the direction of technical change. Blanchard (1997) and Acemoglu (2002) argue that labor abundance, e.g. increasing unemployment rates in the European countries in the 70'es to 90'es, lead technical change to become capital-augmenting. Acemoglu (2003) and Klump et al. (2008) argue that profit-maximizing incentives drive technical change, e.g. the IT-boom in the 90'es.

The process of technical change in Australia and Denmark with high estimated noise-to-signal ratios are approximately linear trends. Oppositely, the processes of technical change in the countries with low estimated ratios such as New Zealand, Austria, and the US are smooth trends with several persistent shifts in technical change. This emphasizes that the state-space framework nests many different types of processes of technical change. In addition, Figure 5 also includes the relative factor-augmenting technical change estimated with the system estimator with Box-Cox trend (dotdashed red line). Even though the long run trend of technical change estimated with the state-space framework is similar to the process estimated with the system estimator with a Box-Cox trend, they differ in two important points: i) The Box-Cox transformation implies initial growth rates in the range of 5-15%, which seems empirically implausible, but is needed to match the curvature in later periods. ii) Most important, the state-space framework incorporates several persistent shifts in technical change in contrast to the Box-Cox transformation. This is the major difference between the two methods and underscores the ability of the state-space framework to capture several persistent shifts in technical change as observed empirically.

6 Concluding remarks

How to specify the unknown process of technical change has long been an issue when estimating the elasticity of substitution between capital and labor. In this paper we present a new state-space framework to simultaneously provide an estimate of the elasticity of substitution and identify time-varying and

Figure 5: Estimated relative technical change, $\Delta log\left(\Gamma_t^L/\Gamma_t^K\right)$.



Notes: The Figure display the estimated growth rates of relative augmenting technical change in percentage with the state-space framework (dashed blue) and system estimator with Box-Cox trend (dotdashed red).
Source: Data is obtained from PWT and the OECD STAN database.

potentially factor-augmenting technical change. By exploiting the natural state-space representation of the problem, we avoid a full parametric specification of the structural changes in the economy. Instead, persistent shifts in the factor-augmenting technical change during transition periods as well as a long run trend are identified by a smoothness restriction, the noise-to-signal ratio. We show in a simulation study with non-linear technical change that our approach performs superior in terms of reproducing the true elasticities on the median and outperforms the system estimators widely applied in the literature. Based on an empirical analysis including data for 16 OECD countries from the PWT and the time period 1950-2019 we conclude the following: i) All estimates are significantly above zero, but below unity. Consequently, our results suggest that the business cycle literature should abandon the Cobb-Douglas assumption. ii) Long run technical change has been directed at improving the efficiency of labor relative to capital, but several periods of persistent shifts in technical change during transition periods are observed. Importantly, the state-space framework is able at incorporating these shifts, opposite to the existing Box-Cox or linear trend assumptions. Therefore, the state-space framework is recommended as an improvement to models with parametric assumptions on factor-augmenting technical change and is made publicly available through the statistical software program R.

Several extensions are manageable to implement in our state-space framework: i) Several papers have recently shown that the US elasticity has changed during the last decades (Cantore et al., 2017; Chirinko and Mallick, 2017; Oberfield and Raval, 2021). Incorporating time-variation in the elasticity is manageable in our framework by including a stochastic trend in the Random Walk process of the elasticity. ii) In the business cycle literature, spectral analysis is often applied to decompose a time series in a long run, a medium run, and an idiosyncratic error term component (Comin and Gertler, 2006; DeJong and Dave, 2011). This is indeed also possible in our framework but is likely to require variance restrictions as in Lemoine et al. (2010). iii) Whereas our paper apply aggregate data, Chirinko and Mallick (2017) estimate the US elasticity in industry-level data and Oberfield and Raval (2021) apply US firm-level data. A panel data extension of the state-space framework is already under development and codes are available upon request.

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A Estimators in the literature

In previously published papers, most often different versions of the first order conditions or a system of equations are estimated. Of the studies investigated in the meta regression study by Knoblach et al. (2020), 78% estimate first order conditions and 14% estimate equation systems.

The equation system estimators estimate the two FOC and the production function simultaneously.[22] Recently, it has been acknowledged that normalization of the system is necessary to obtain consistent estimates.[23] Therefore, the equation system consisting of the first order conditions of capital and labor, respectively, and the production function is normalized:

$$log\left(r_t\right) = log\left(\bar{\pi}\frac{\bar{Y}}{\bar{K}}\right) + \frac{1}{\sigma}log\left(\frac{Y_t/\bar{Y}}{K_t/\bar{K}}\right) + \frac{\sigma-1}{\sigma}\left(log\left(\xi\right) + g_K\left(t,\bar{t}\right)\right), \tag{14}$$

$$log\left(w_t\right) = log\left(\left(1-\bar{\pi}\right)\frac{\bar{Y}}{\bar{L}}\right) + \frac{1}{\sigma}log\left(\frac{Y_t/\bar{Y}}{L_t/\bar{L}}\right) + \frac{\sigma-1}{\sigma}\left(log\left(\xi\right) + g_L\left(t,\bar{t}\right)\right), \tag{15}$$

$$log\left(Y_t/\bar{Y}\right) = log\left(\xi\right) + \frac{\sigma}{\sigma-1}log\left[\bar{\pi}\left(e^{g_K\left(t,\bar{t}\right)}\frac{K_t}{\bar{K}}\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\bar{\pi}\right)\left(e^{g_L\left(t,\bar{t}\right)}\frac{L_t}{\bar{L}}\right)^{\frac{\sigma-1}{\sigma}}\right]. \tag{16}$$

The normalized variables are $\bar{Y},\bar{K},\bar{L},\bar{\pi}$ and $\bar{t}$ with a bar referring to the sample average, geometric for the first three and arithmetic for the last two. $\xi\bar{Y} = Y_0$ is a normalizing constant expected to be close to unity. A linear trend assumption implies that $g_K\left(t,\bar{t}\right) = \gamma_K\left(t-\bar{t}\right)$, $g_L\left(t,\bar{t}\right) = \gamma_L\left(t-\bar{t}\right)$ and a Box-Cox transformation of the growth rates implies that $g_K\left(t,\bar{t}\right) = \bar{t}\frac{\gamma_K}{\lambda_K}\left(\left(t/\bar{t}\right)^{\lambda_K}-1\right)$ and $g_L\left(t,\bar{t}\right) = \bar{t}\frac{\gamma_L}{\lambda_L}\left(\left(t/\bar{t}\right)^{\lambda_L}-1\right)$. With both trend assumptions, γ_K and γ_L are the determinstic trend terms. λ_K and λ_L are the curvature parameters. When this parameter is equal to one, it corresponds to a linear trend, below one is a decelerating growth rate and above one is an accelerating growth rate. Thus, the linear trend assumption is nested in the Box-Cox transformation.

B Filter inconsistency test

To evaluate filter performance we would like to know if the filtered state is a reasonably prediction of the true value. However, as the true state is unknown filter consistency is usually based on information on the innovations in the observation equation. If the filter is consistent, the standardized forecast errors will be a zero-mean and homoskedastic white noise process. This can be evaluated either by graphically inspecting the standardized innovations or (more formally) by considering the Normalized Innovations Squared test (NIS). The NIS test has the following test statistic:

[22] Equation systems are most often estimated with non-linear least squares, such as the SUR estimator applied in papers such as Klump et al. (2007, 2008).

[23] Leon-Ledesma et al. (2010) find that failure to normalize the equation system leads to a bias of the elasticity towards one. We refer to Klump et al. (2012) for a review of the literature.

$$m_t = \varepsilon_t^T F_t^{-1} \varepsilon_t, \tag{17}$$

where F_t is the co-variance matrix of the innovations. If the assumptions are correct, m_t will be $\chi^2(1)$ distributed, implying that the T period moving average, $\bar{m}_T$, has a $T\chi^2(T)$ distribution (applying the ergodic property of the innovations). Hence, the null hypothesis is $E[m] = 1$ and can be tested by computing the moving average of (17) recursively for an increasing sample size and compare the test statistics to the critical values.

C Simulation evidence

Simulating Box-Cox transformation

As the processes of Γ_t^K and Γ_t^L are Random Walks, we adjust the Box-Cox transformations such that $g_K(t,\bar{t}) = \bar{t}^{1-\lambda_K} \frac{\gamma_K}{\lambda_K} \left(t^{\lambda_K} - (t-1)^{\lambda_K} \right)$ and $g_L(t,\bar{t}) = \bar{t}^{1-\lambda_L} \frac{\gamma_L}{\lambda_L} \left(t^{\lambda_L} - (t-1)^{\lambda_L} \right)$, where $\bar{t}$ is the sample average. γ_K and γ_L determine the average growth rates and λ_K and λ_L the curvature of the growth rates: Values of the curvature parameters below one correspond to a decelerating growth rate, above one an accelerating growth rate, and equal to one a constant growth rate. By initializing the first observation to $g_K(1,\bar{t}) = \bar{t}^{1-\lambda_K} \frac{\gamma_K}{\lambda_K} \left(1 - \bar{t}^{\lambda_K} \right)$ and $g_L(1,\bar{t}) = \bar{t}^{1-\lambda_L} \frac{\gamma_L}{\lambda_L} \left(1 - \bar{t}^{\lambda_L} \right)$ it can be shown by iterating backwards that $\sum_{k=1}^{t} g_K(k,\bar{t}) = \bar{t} \frac{\gamma_K}{\lambda_K} \left((t/\bar{t})^{\lambda_K} - 1 \right)$ and $\sum_{k=1}^{t} g_L(k,\bar{t}) = \bar{t} \frac{\gamma_L}{\lambda_L} \left((t/\bar{t})^{\lambda_L} - 1 \right)$, i.e. resembling the Box-Cox transformations described in Appendix A.

Table C1: Mean and variance of the simulated variables compared to US data.

	US data		σ=0.2		σ=0.5		σ=0.9		σ=1.3	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
					Linear					
K_t	2.72	0.83	3.00	0.79	3.01	0.79	3.01	0.79	3.01	0.79
L_t	1.17	2.02	1.48	1.99	1.48	1.99	1.48	1.99	1.48	1.99
Y_t	3.07	2.14	3.12	2.09	3.16	2.05	3.18	2.05	3.19	2.06
r_t	0.51	2.88	-1.32	9.20	-0.16	4.19	0.15	3.35	0.25	3.16
w_t	1.83	1.25	2.21	3.80	1.88	2.28	1.73	1.89	1.66	1.77
					BoxCox					
K_t	2.72	0.83	2.98	0.80	3.00	0.79	3.01	0.79	3.01	0.79
L_t	1.17	2.02	1.63	1.81	1.54	1.89	1.49	1.92	1.48	1.93
Y_t	3.07	2.14	3.69	2.13	3.70	2.12	3.71	2.11	3.73	2.11
r_t	0.51	2.88	-0.95	10.16	0.26	3.65	0.66	2.61	0.83	2.47
w_t	1.83	1.25	2.40	3.07	2.38	2.57	2.25	2.15	2.18	1.98
					10-year averages					
K_t	2.72	0.83	3.01	0.79	3.01	0.79	3.01	0.79	3.01	0.79
L_t	1.17	2.02	1.48	1.99	1.48	1.99	1.48	1.99	1.48	1.99
Y_t	3.07	2.14	3.10	2.11	3.13	2.06	3.14	2.06	3.14	2.05
r_t	0.51	2.88	0.67	6.90	0.29	4.05	0.15	3.31	0.11	3.16
w_t	1.83	1.25	1.14	7.96	1.55	2.68	1.65	1.90	1.69	1.77

Notes: The Table shows the mean and variance of growth rate in simulated observed value-added, $\Delta log(Y_t)$, the real wage, $\Delta log(w_t)$, and the real interest rate, $\Delta log(r_t)$. Median values are reported and compared to the values in US data.
Source: Data is obtained from PWT and own simulations.

Sensitivity to normalization

Leon-Ledesma et al. (2010) show that the estimated elasticity is biased towards unity if the system estimator is not appropriately normalized. A legitimate question to raise is therefore if the state-space framework is robust to normalization.

The initialization of this simulation study implies that $K_0 = L_0 = Y_0 = \Gamma_0^L = \Gamma_0^K = 1$. As argued by Leon-Ledesma et al. (2010) the estimates are invariant to normalization with this particular initialization. In Table C2 we report the median elasticities for different initializations of K_0 and r_0 for the three different trend assumptions and $\sigma = 0.5$ and $\sigma = 1.3$. For all trend assumptions and values of σ, the state-space framework replicates the median elasticities with high precision and varying initialization only affects the distance between the 10 and 90 percentiles marginally. This illustrates that the state-space framework is not sensitive to normalization, which is an important advantage of the state-space framework compared

Table C2: Estimated median elasticities with the state-space framework for different initializations.

	Linear		Box-Cox		10-year average	
	σ= 0.5	σ= 1.3	σ= 0.5	σ= 1.3	σ= 0.5	σ= 1.3
$K_0 = 1$, $r_0 = 0.4$ and $Y_0^\star = 1$	0.51 (0.44;0.61)	1.30 (1.07;1.67)	0.51 (0.43;0.61)	1.29 (1.13;1.53)	0.50 (0.41;0.65)	1.31 (0.95;1.97)
$K_0 = 5$, $r_0 = 0.05$ and $Y_0^\star = 0.625$	0.52 (0.44;0.62)	1.29 (1.06;1.64)	0.51 (0.44;0.62)	1.30 (1.13;1.54)	0.53 (0.43;0.68)	1.29 (0.95;1.96)
$K_0 = 8$, $r_0 = 0.05$ and $Y_0^\star = 1$	0.52 (0.45;0.63)	1.30 (1.06;1.65)	0.52 (0.44;0.62)	1.29 (1.12;1.53)	0.54 (0.44;0.70)	1.28 (0.95;1.90)

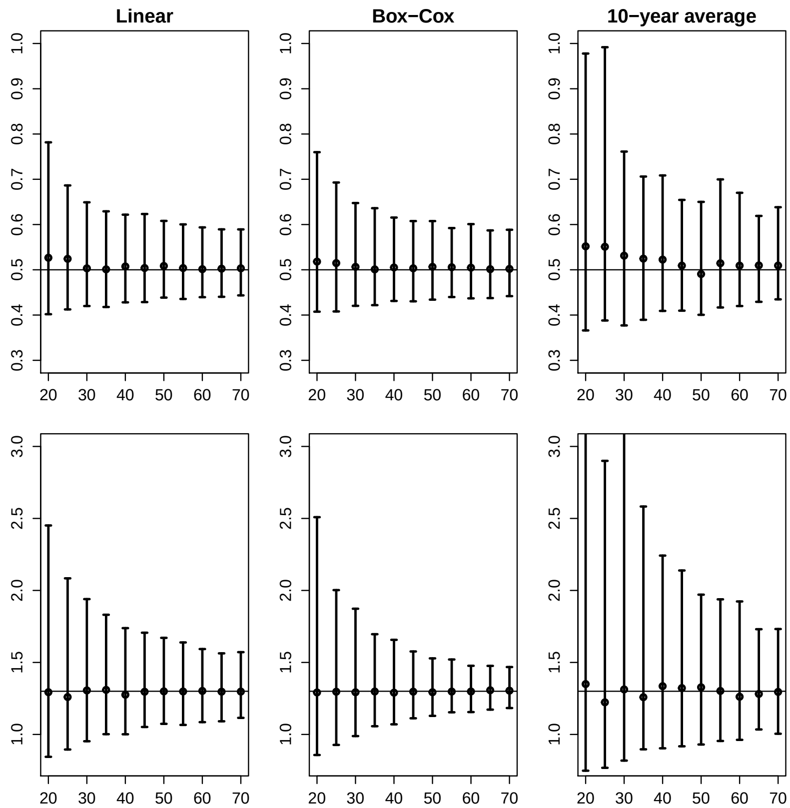
Notes: The Table reports the estimated median elasticities in the simulated data by applying the state-space framework and with different initializations of K_0 and r_0. The results are shown for σ=0.5,1.3 and the three different trend assumptions. 10 and 90 percentiles are reported in paranthesis.
Source: Own simulations.

to the system estimators.

Sensitivity to the number of observations

To test the sensitivity of the state-space framework to the number of observations, we simulate the data for time periods between 20 and 70 observations and display the median elasticities and the percentiles of the distribution (Figure C1). When the trend assumption in the simulated data is the linear or Box-Cox trend, increasing the number of observations above 30 has primarily gains for the precision of the estimates (narrower confidence bands). When the trend in the simulated data is the 10-year averages, it takes around 40 observations before unbiased estimates on the median are obtained. Thus, across the three trend assumptions, at least 40 observations are necessary to obtain reliable estimates.

Figure C1: Median estimated elasticities for different number of observations.



Notes: The graphs show the median estimated elasticitites (y-axis), σ, for different trend assumptions and values of σ in the data generating process and increasing number of observations, T (x-axis). The state-space framework is applied and 10 and 90 percentiles lines are reported with a dot indicating the median.
Source: Own simulations.

D Empirical analysis

How different degrees of smoothness affect the elasticity and technical change

Table D2 contains estimates for all the 16 countries with the following noise-to-signal ratios: $\lambda \in (1, 10, 50, 100, 200, 500, 10000)$. On the average, low values of λ imply a relatively low estimate of σ and high values a high estimate: The unweighted average ranges from 0.24 with $\lambda = 1$ to 0.79 with $\lambda = 10,000$. For some countries, the estimate even exceed unity with high degrees of smoothing, e.g. 2.51 in Austria, 2.09 in Japan, 1.49 in Korea, and 1.06 in the US when $\lambda = 10,000$. This highlights that in line with the literature (Klump et al., 2007; McAdam and Willman, 2013; Knoblach et al., 2020), our findings reiterate the tight link between identifying restrictions on technical change and the elasticity of substitution. Although our model specification is less parametric and seemingly less a priori restrictive we do not escape this fact.

For most countries, the estimates are reasonably stable in the range $\lambda \in 50-500$ with the average only increasing by 0.07. Consequently, even though the ratio is increased by a factor up to 10, the estimates are still stable in this certain range, to a lesser extend when $\lambda = 10$ also. However, the largest increase is

Table D1: System estimator estimates of the elasticity of substitution.

		Box-Cox system estimator					
	Time period	σ	γ_K	γ_L	λ_K	λ_L	ξ
Australia	1959-2019	0.44 (0.00)	−0.001 (0.000)	0.016 (0.000)	−0.63 (0.35)	0.51 (0.04)	1.05 (0.01)
Austria	1976-2019	0.29 (0.00)	−0.002 (0.000)	0.017 (0.000)	−0.29 (0.08)	0.52 (0.03)	1.03 (0.00)
Belgium	1970-2019	0.27 (0.00)	0.002 (0.000)	0.015 (0.000)	2.25 (0.68)	0.26 (0.02)	1.04 (0.00)
Canada	1970-2019	0.34 (0.00)	−0.007 (0.001)	0.011 (0.001)	0 (0.06)	0.47 (0.05)	1.03 (0.00)
Denmark	1970-2019	0.28 (0.00)	−0.001 (0.000)	0.016 (0.000)	−0.56 (0.16)	0.39 (0.02)	1.04 (0.00)
Finland	1970-2019	0.45 (0.00)	−0.004 (0.001)	0.023 (0.001)	−0.11 (0.12)	0.39 (0.04)	1.06 (0.01)
France	1950-2019	0.52 (0.00)	−0.002 (0.000)	0.027 (0.001)	0.73 (0.47)	0.34 (0.02)	1.14 (0.01)
Great Britain	1970-2019	0.44 (0.00)	0.002 (0.001)	0.017 (0.001)	4.12 (1.61)	0.37 (0.06)	1.02 (0.01)
Italy	1970-2019	0.41 (0.00)	−0.009 (0.000)	0.012 (0.000)	0.4 (0.05)	0.08 (0.04)	1.04 (0.00)
Japan	1970-2019	0.4 (0.00)	−0.005 (0.000)	0.02 (0.000)	−0.1 (0.05)	0.37 (0.02)	1.04 (0.00)
Korea	1970-2019	0.78 (0.02)	−0.023 (0.002)	0.064 (0.002)	−0.07 (0.07)	0.53 (0.03)	1.03 (0.01)
Netherlands	1970-2019	0.29 (0.01)	−0.001 (0.000)	0.014 (0.000)	−0.39 (0.26)	0.29 (0.03)	1.04 (0.00)
Norway	1970-2019	0.6 (0.01)	−0.001 (0.001)	0.021 (0.001)	−0.81 (0.52)	0.28 (0.06)	1.06 (0.01)
New Zealand	1971-2019	0.75 (0.02)	−0.008 (0.003)	0.019 (0.002)	−0.16 (0.17)	0.84 (0.17)	0.99 (0.01)
Sweden	1950-2019	0.6 (0.01)	0 (0.001)	0.022 (0.000)	−0.92 (0.55)	0.5 (0.04)	1.05 (0.01)
USA	1950-2019	0.33 (0.00)	0.003 (0.000)	0.017 (0.000)	2.46 (0.35)	0.63 (0.02)	1.02 (0.00)
Mean		0.45 (0.00)	−0.004 (0.001)	0.021 (0.001)	0.37 (0.35)	0.42 (0.04)	1.04 (0.00)
Weighted mean		0.39 (0.00)	−0.001 (0.000)	0.019 (0.000)	1.53 (0.37)	0.5 (0.03)	1.03 (0.00)

Notes: The Table reports the main estimation results of the elasticity of substitution for the 16 OECD countries. Results are reported for the system estimator with Box-Cox trend. The Table reports the estimated elasticity, the average growth rates of labor-augmenting technical change, γ_L, capital-augmenting technical change, γ_K, the curvature parameters, λ_L and λ_K, and normalization parameter, ξ. Standard errors reported in paranthesis.
Source: Data is obtained from PWT and the OECD STAN database.

Table D2: Estimation with different noise-to-signal ratios.

	λ						
	1	10	50	100	200	500	10000
Australia	0.09	0.17*·	0.18*·	0.2*·	0.21*·	0.23*·	**0.29***·
	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.06)	(**0.07**)
Austria	0.27	**0.39***·	0.74*·	0.91*·	1.05*·	1.14*·	2.51*·
	(0.04)	(**0.08**)	(0.38)	(0.26)	(0.63)	(1.82)	(93.69)
Belgium	0.3	**0.34***·	0.33*·	0.33*·	0.33·	0.36·	0.52·
	(0.03)	(**0.03**)	(0.03)	(0.03)	(0.04)	(0.05)	(0.12)
Canada	0.19	**0.26***·	0.27*·	0.25*·	0.23·	0.2·	0.16·
	(0.02)	(**0.04**)	(0.04)	(0.04)	(0.04)	(0.04)	(0.06)
Denmark	0.21	0.33·	0.44*·	0.46*·	0.46*·	0.44*·	**0.44***·
	(0.03)	(0.06)	(0.06)	(0.06)	(0.06)	(0.05)	(**0.05**)
Finland	0.27	0.46*·	**0.51***·	0.51*·	0.54*·	0.55·	0.68·
	(0.05)	(0.06)	(**0.07**)	(0.08)	(0.08)	(0.08)	(0.11)
France	0.13	**0.12***·	0.11*·	0.11*·	0.11*·	0.14*·	0.74·
	(0.03)	(**0.05**)	(0.06)	(0.07)	(0.08)	(0.11)	(0.42)
Great Britain	**0.26**	0.34*	0.36·	0.35·	0.35·	0.36·	1.06·
	(**0.03**)	(0.05)	(0.05)	(0.06)	(0.06)	(0.09)	(13.88)
Italy	0.2*	0.35*·	**0.48***·	0.49*·	0.47·	0.44·	0.25·
	(0.05)	(0.07)	(**0.06**)	(0.07)	(0.07)	(0.08)	(0.22)
Japan	0.18	0.16*·	**0.13***·	0.1*·	0.04*·	0*·	2.09·
	(0.03)	(0.04)	(**0.06**)	(0.07)	(0.11)	(0.44)	(23.09)
Korea	0.41*	0.52*·	0.63*·	**0.65***·	0.68*·	0.76*·	1.49·
	(0.06)	(0.06)	(0.07)	(**0.08**)	(0.09)	(0.10)	(1.68)
Netherlands	**0.17**	0.23*·	0.27*·	0.29·	0.3·	0.31·	0.26·
	(**0.02**)	(0.03)	(0.04)	(0.04)	(0.05)	(0.05)	(0.07)
Norway	**0.09***	0.15*·	0.11*·	0.13*·	0.17*·	0.19*·	0.21*·
	(**0.02**)	(0.04)	(0.05)	(0.06)	(0.08)	(0.09)	(0.15)
New Zealand	0.14	**0.28***·	0.28*·	0.26*·	0.25·	0.23·	0.28·
	(0.04)	(**0.05**)	(0.06)	(0.07)	(0.07)	(0.09)	(0.20)
Sweden	0.26	0.32*·	0.44*·	**0.47***·	0.51*·	0.54·	0.68·
	(0.04)	(0.04)	(0.06)	(**0.06**)	(0.06)	(0.05)	(0.06)
USA	0.5*	**0.53***·	0.64*·	0.71*·	0.81*·	0.94·	1.06·
	(0.06)	(**0.08**)	(0.12)	(0.14)	(0.16)	(0.18)	(0.19)
Mean	0.23	0.31	0.37	0.39	0.41	0.43	0.8
	(0.04)	(0.05)	(0.08)	(0.08)	(0.11)	(0.21)	(8.38)
Weighted mean	0.24	0.32	0.37	0.39	0.41	0.44	0.83
	(0.04)	(0.05)	(0.07)	(0.08)	(0.10)	(0.19)	(6.38)

Notes: The Table reports for every country the estimated elasticities with different values of the noise-to-signal ratios, λ. Bold is the estimate with the highest likelihood value. A "*" indicates that the autocorrelation test is satisfied at the 10 percent level. A "." indicates that the NIS test is within the confidence bands at the 10 percent level. Zero lags are included in all estimations except for Norway.
Source: Data is obtained from PWT and the OECD STAN database.

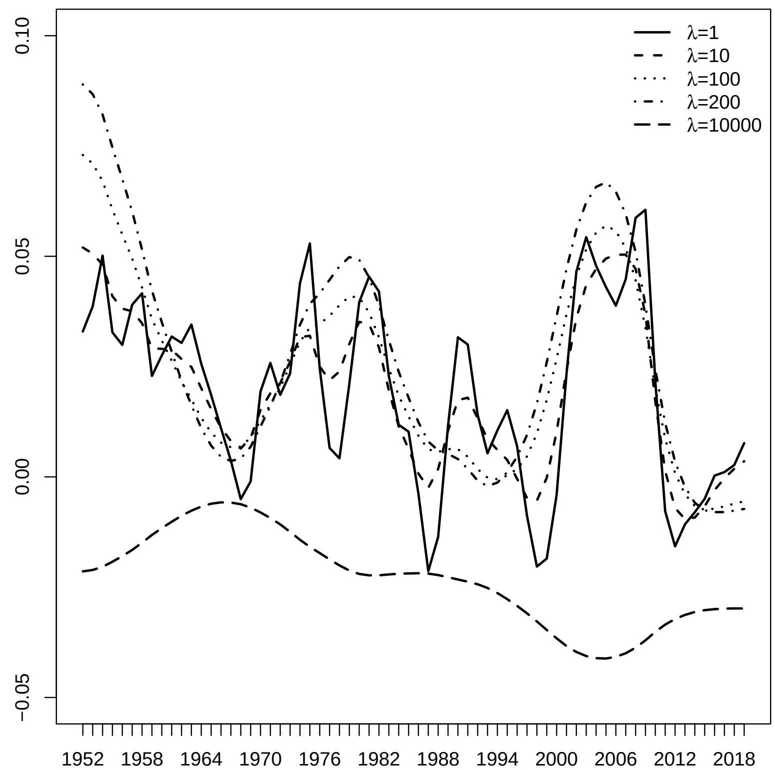
seen when moving from $\lambda = 500$ to $\lambda = 10,000$ where the average increases with 0.39.

In Table D2, the estimate that maximizes the likelihood is reported in bold. In most cases, these are in the range 10-100, corresponding well with the estimates in Table 3. In three cases (Great Britain, Netherlands, and Norway), the estimate with $\lambda = 1$ has the highest likelihood. However, all of these are misspecified, based on the autocorrelation and the NIS test (a "*" indicates that the autocorrelation test is satisfied, a "." that the NIS test is satisfied). In two cases, the estimate with $\lambda = 10,000$ are preferred, and well specified. These are Australia and Denmark that also have high values of λ in Table 3.

As the Kalman filter tends to produce serially correlated innovations if the noise-to-signal ratio becomes too high, we would expect autocorrelation to show up if the model imply excessive smoothing. This is also what is observed in Table D2 where the autocorrelation test fails in many cases when λ exceeds 100. This suggests that an approximately linear trend in the factor-augmenting technical change is generally too restrictive to describe the persistent variations of technical change. On the other hand, if our model imply too little smoothing of the relative technical change (λ small) one might expect to reject the NIS test as the filter can be sensitive to tuning of the measurement noise in particular. From Table D2, we do find the test for well calibrated innovations to be violated for $\lambda = 1$, based on a too low NIS for all countries. In addition, the autocorrelation test also fails in most cases when $\lambda = 1$ indicating that some degree of smoothing of factor augmenting technical change is necessary to obtain a well specified model.

In Figure D1, we show the estimated processes of US technical change for different degrees of smoothing. When $\lambda = 1$, the resulting process of relative augmenting technical change is a flexible process with many year-to-year fluctuations. As λ increases, the process becomes increasingly smooth, but still feature several persistent shifts in the direction on technical change. When $\lambda = 10,000$, the estimated process converges to a process that is closely related to the linear trend, but with several persistent changes in the average growth rate. The fact that the elasticity estimate is above 1 when $\lambda = 10,000$ implies that the estimated growth rate of relative augmenting technical change becomes capital-augmenting during the entire sample, which seems empirically implausible.

Figure D1: US relative factor-augmenting technical change, $\Delta log\left(\Gamma_t^L/\Gamma_t^K\right)$, for different levels of smoothness.



Source: Data is obtained from PWT and the OECD STAN database.