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GREEN REFORM

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In this note, we propose a general methodology for incorporating exogenous marginal tax rates in input demand functions of firms, while still respecting national accounts statistics on total production costs, tax payments etc. of the firms in any given sector.

GreenREFORM is calibrated against very detailed input-output tables and various sattelite accounts from Statistics Denmark from a specific year (the data year), at present 2018. These statistics are build on strict accounting principles, which the model must also respect. Case in point, the value of production in a given sector is equal to total production costs including payments of a range of product specific taxes and net profits. In constructing the model, we typically connect the payment of these product specific taxes to the value of inputs in production from the most relevant sector, and calibrate an effective tax rate as the ratio between the tax payment and the value of those inputs. Say, if the payments of chocolate-tax (it is a real tax in Denmark) is 100 EUR, and the value of input from the food and beverage industry is 10,000 EUR, the effective chocolate tax rate would på 1 percent ad valorem.

Now, product specific taxes are not ad valorem but unit taxes. Since the chocolate tax is not important for the purpose of the GreenREFORM model, and since we therefore do not make account of the sales and purchases of chocolate (or other foods and beverages) in physical units, any discrepancies between the effective tax and actual tax rates (DDK per gram) will be obscured. The opposite is the case for energy and pollutants, which are obviously important for the purpose of GreenREFORM, and which we thus make account for in great detail and in physical units of energy content (joule) and mass (gram) respectively.

The effective tax rate based on national accounts are in practice never perfectly alligned with the objective marginal tax rates as per legislation. It may be due to discrepancies between data sources, lack of reliable data and subsequent errors of distribution of tax revenues, it may be because firms are not facing uniform tax rates, or that they are eligible to bottom deductions. The later is the case for some energy intensive industries with regard to the danish energy tax (energiafgift for virksomheder på proceslisten). The distribution of free allowances in the EU ETS system is another example.

Introducing explicit marginal tax rates in the model

As explained, the value of production is given as a markup over total costs. With a few simplifications, this is stated as follows for each sector j:

$$p_jY_j = (1+m_j)\left(wL_j + P^K_j K_j + \sum_i \left(p^x_i x_{ij} + \sum_n \tau_{ijn}\left(x_{ij} - \bar{x}_{ijn}\right)\right)\right), \tag{1}$$

where the left hand side is the value of production and m_j is the markup. The costs of production is the sum of returns to primary factors, labour L capital K respectively, and costs of material inputs x_{ij} in production incl. product specific taxes.

Tax payed of tax type n on input i by sector j is given by:

$$\tau_{ijn}\left(x_{ij} - \bar{x}_{ijn}\right) \tag{2}$$

where τ_{ijn} is the marginal tax rate and $\bar{x}_{ijn}$ is bottom deductions.

The average tax is given by:

$$\frac{\tau_{ijn}\left(x_{ij} - \bar{x}_{ijn}\right)}{x_{ij}}, \tag{3}$$

and the Government Revenue of tax type n:

$$TR_n = \sum_j \left(\sum_n \tau_{ijn}\left(x_{ij} - \bar{x}_{ijn}\right)\right) \tag{4}$$

The default approach in calibration of product taxes in CGE models is to assume no deductions $(\bar{x}_{ijn} = 0)$, and calculate the level of the tax rate τ_{ijn} that implies that the tax payed by sector j $(\tau_{ijn}x_{ijn})$ is correct in the base year according to national account statistics. This is called *effective tax rates*.

In the case of energy and pollutants, where quantities are expressed in physical untits, prices and taxes have an explicit interpretation, ie DKK per joule og per gram, and can hence be directly compared with observable prices and legal tax rates. When information is available, we want firms to internalize the correct legal tax rates (and not the effective tax rates). In those cases, we set the tax rates exogenously as equal to the legal rates, and instead calibrate the bottom deduction $\bar{x}_{ij}$ to match the amount of tax n payed by sector j according to the national accounts.

By using the corrext legal taxes we get the correct marginal effects in the model. This can be demonstrated by an example. Assume production to be described by a standard CES production function:

$$Y_j = \left[\left(\mu^L\right)^{\frac{1}{E}} L_j^{\frac{E-1}{E}} + \left(\mu^K\right)^{\frac{1}{E}} K_j^{\frac{E-1}{E}} + \sum_i \left(\mu^x_{ij}\right)^{\frac{1}{E}} x_{ij}^{\frac{E-1}{E}}\right]^{\frac{E}{E-1}}$$

Crucially, we will assume that firms $\bar{x}_{ijn}$ treat the bottom deduction as external to the choice of inputs. Firms demand for inputs in production can thus be shown to be given by:

$$x_{ij} = \mu_{ij}\left(\frac{p^x_i + \sum_n \tau_{ijn}}{P^O_j}\right)^{-E} Y_j \tag{5}$$

where the optimization price P_j^O is given by the CES-price index:

$$P_j^O = \left[\mu^L w^{1-E} + \left(P_j^K\right)^{1-E} + \sum_i \mu_{ij}^x \left(\left(p_i^x + \sum_n \tau_{ijn}\right)\right)^{1-E}\right]^{\frac{1}{1-E}}$$

It can be realised from (5), that relative inputs are given by:

$$\frac{x_{ij}}{x_{sj}} = \frac{\mu_{ij}}{\mu_{sj}} \left(\frac{p_i^x + \sum_n \tau_{ijn}}{p_s^x + \sum_n \tau_{sjn}}\right)^{-E}$$

Note in the expressions above, that only the marginal tax rates are important for the relative inputs, hence the bottom deduction$\bar{x}_{ijn}$ does not influence the distortion between inputs.

Further, it can be shown, that the output price p_j is given by:

$$p_j = (1 + m_j) P_j^O.$$

Modelling of the bottom deduction

As explained in the introduction, there may be various reasons as to why the marginal and effective tax rates are not alligned in the data year. This may influence how the bottom deduction is modelled in forecast years, once the model is calibrated to data as explained above.

If the discrepancy is caused by statistical noise in the national accounts data, with a calibrated mean value of $\bar{x}_{i,nj}$ close to zero and low variance across sectors, such that it does not have significant importance in calculating the government revenue or the costs of production, it may be tempting to let it gravitate towards zero in forecast years, for example as by

$$\bar{x}_{ijn,t} = a \cdot \bar{x}_{ijn,t-1}, 0 < a < 1$$

When this is not the case, adjusting the bottom deduction to zero will distort government revenue and possibly also the costs of production and output prices of firms. The bottom deduction can thus be forecasted exogenously, or it can be linked endogenously to some other variables as seems fit.

When there is no real reason for a non zero bottom deduction but for statistical error, another tempting approach is to forecast the bottom deduction in fixed proportion to the amount of inputs x_{ij} as by

$$\bar{x}_{ijn,t} = \bar{x}_{ijn,t-1} \frac{x_{ij,t}}{x_{ij,t-1}}$$

This has the tractable feature, that the tax payed by firms is proportional to the amount of inputs (2), and so is the government revenue (4). This could be important in scenarios with large changes in the amount of inputs. An example could be a tax on coal, which is expected to be phased out completely. It also solves the problem of non-symmetry mentioned at the bottom of the text. The caveat however, is that this aproach creates an apparant logic inconsistency with the assumption that the bottom deduction is perceived as external to the choice of inputs of the individual firms, cf. 5. From a sector perspective however, it seems a better first assumption, that firms are ellegible to a deduction in fixed proportion to the amount of input, rather than all firms sharing a fixed total bottom deduction.

In some cases, firms are in fact elligible to a bottom deduction, and we may even have information on the size theroff. The only such case at present is the EU ETS-system, where we have information on the allocation of free allowances $\hat{x}_{i,j,ETS,t}$.

In that specific case, we will introduce an error term α_t, to be calibrated in the data year, and adjusted towards zero in forecast years:

$$\bar{x}_{i,j,ETS,t} = \hat{x}_{i,j,ETS,t} + \alpha_t$$

The ETS system is symmetric in that unused allowances can be sold, and the marginal "tax" is thus the same above and below the level of free allowances. An example with non-symmetry is the bottom deduction in the danish CO2-tax (CO2-afgift for virksomheder på proceslisten), where firms do not receive a subsidy below the bottom deduction. Forecasting of the bottom deduction as exogenous carries with it an implicit assumption of symmetry. This could be an issue if the bottom deduction is large, or in cases where inputs are expected to be phased out like coal.