DREAM

Danish Research Institute for
Economic Analysis and Modelling

MAKRO

Estimating foreign shocks in a VAR model

Anders F. Kronborg

Working paper
7 December 2021

www.dreamgruppen.dk

Preface

This note describes how foreign shocks are estimated in the work with MAKRO. The foreign block consists of the total export market, foreign prices, the interest rate, and the oil price. Thus, a shock to either foreign variable is now part of an endogenous system - akin to a number of related papers using VAR models, including the literature on spillovers. The problem of identifying multiple shocks is addressed by combining different types of restrictions, including an assumption of Denmark as a small open economy as well as sign restrictions to separately identify foreign demand- and supply-type shocks as well as controlling for generic domestic shocks. This approach leads to results, which are broadly in line with theory and related empirical literature, albeit they are associated with non-negligible uncertainty for some shocks and variables.

1 Introduction

Since Sims [1980], vector autoregressive (VAR) models have been one of the main tools in empirical macroeconomics. Specifically, the resulting impulse response functions have been a popular way to examine the monetary policy transmission mechanism (Eichenbaum and Evans, 1995), the effects of fiscal policy (Blanchard and Perotti, 2002), and later supply shocks such as changes in technology (Dedola and Neri, 2007) and labor supply (Foroni et al., 2018). For smaller economies, the framework has been applied to examine the effects of monetary and economic activity spillovers, e.g. from the US or Euro Area (Vasishtha and Maier, 2013). Further, VAR models have been used to estimate the structural parameters of DSGE models by minimizing the distance between the model's impulse responses and those found in the data (for example Christiano et al. [2005], Christiano et al. [2016], and Aursland et al. [2019]). In the work with MAKRO, a similar approach has been taken to parameterize a key set of parameters that govern the frictions in the model system, i.e. the estimated impulse responses are used to calibrate the model's adjustment to shocks.

This note describes how a VAR model is used to find the effects on the Danish economy to foreign shocks. As a small open economy, the Danish business cycle is naturally highly dependent on the state of the foreign economy, not least via the effect on exports. Further, the foreign interest rate is particularly relevant for Denmark due to the fixed exchange rate policy against the Euro: This implies that monetary policy shocks are de facto foreign shocks as well.[1] The benchmark specification shows the impulse response of GDP, private consumption, exports, the GDP deflator and wages. Subsequently, the response of additional domestic variables are examined.

The foreign economy itself is modeled as a simple 4 variable system with output, prices, the interest rate, and the oil price. A similar setup is often found at the core of VAR analyses of the Euro Area, for example in a number of ECB-papers (e.g. Bonci, 2011) and similarly in empirical papers on real and financial spillovers (see for example Jensen et al. [2017] for a Danish example). This approach has at least two advantages: First, since the primary objective is not to model the foreign economy itself in great

[1] Equivalently, the interest rate, exchange rate, foreign demand, and foreign prices are taken as exogenous variables in many Danish applied macroeconomic models.

detail, but instead to focus on a set of core variables and their transmission to domestic Danish variables, this justifies a more »reduced-form« modeling of the rest of the world while simultaneously saving some degrees of freedom. Second, it is consistent with the rather simple model of the foreign economy applied in MAKRO and other models (where the foreign economy enters only as a few exogenous variables). Hence, for later matching purposes a simple foreign economy suffices.

Identification of structural shocks in multivariate time series models such as VAR models is not possible without imposing further restrictions on the estimated reduced-form of the VAR model. In the benchmark model, foreign shocks are identified by combining two types of identifying restrictions, often used in the literature, namely sign and zero (short-term) restrictions, which lend themselves naturally to the small open economy setup. Specifically, short-term restrictions are implemented by assuming that domestic shocks can have no impact on the foreign economy. As a consequence, the foreign economy is modeled as a separate block, a standard assumption in the spillover literature (e.g. Cushman and Zha, 1997) and - one might add - an assumpion made in MAKRO as well. The consequence is of course that it implies a causality structure from the foreign to the domestic economy from which foreign shocks can be identified from the contemporaneous correlation of the variables included in the empirical model. Further, the model identification »controls« for generic domestic shocks, using the idea in Mountford and Uhlig [2009] by applying sign restrictions which are universal across different classes of macroeconomic models (so-called robust sign restrictions). This approach avoids having to specify a full recursive ordering of the domestic variables as in the -decomposition (sometimes without strong theoretical justification). To summarize, the impulse responses below are found as follows: First, a candidate identification matrix is drawn which satisfies the short-run restrictions. Second, the signs of the resulting impulses are compared to the imposed restrictions. If they satisfy the identification restriction, the draw is kept, otherwise it is discarded. Third, the estimation is continued until 1,000 accepted draws are obtained. The impulse response function of this SVAR model is the median response of all accepted draws across the chosen horizon of the impulse response function. The specific implementation follows the algortihm in Arias et al. [2018] which allows the combination of sign and zero (or short-term) restrictions (this was originally implemented in R in a DREAM master's thesis by by Lund-Thomsen, 2016). The sensitivity to the identification is subsequently examined in the robustness section.

As the number of parameters to be estimated in a VAR model increases rapidly as more variables are included, the variables included in the estimated model will necessarily be a subset of the total variables in a large-scale macroeconomic model such as MAKRO. This naturally raises the question whether the model includes sufficient information or whether relevant and crucial outside information is available to the real world agents that is not used by the econometrician (i.e. that is included in the VAR model's information set). This is adressed specifically by formal testing of the orthogonality condition of the structural shocks, using principal components of a large macroeconomic dataset in the spirit of Forni and Gambetti [2014]. In the benchmark specification, additional information is needed for this condition to be satisfied. Consequently, the model is augmented by factors a la Bernanke et al. [2005].

The rest of this note is organized as follows: Section 2 describes the model as well as the data used in the estimation. Section 3 shows the results of the benchmark model. Section 4 concludes.

2 The VAR model and the data

The analysis is based on the estimation of the VAR model in the so-called reduced form:

$$y_t = \Gamma_0 + \Gamma_1 t + \Gamma_2 Z_t + \tilde{y}_t, \quad \tilde{y}_t = \Pi_1 \tilde{y}_{t-1} + ... + \Pi_p \tilde{y}_{t-p} + u_t, \quad u_t \sim N(0, \Sigma) \qquad (1)$$

where y_t is a $K \times 1$-vector of endogenous variables at time t while Z_t is a $n_Z \times 1$-vector of exogenous variables (such as dummy-variables). The model includes p lags of its own endogenous variables. The matrices and vectors (Γ_0,$\Gamma_1, \Gamma_2, \Pi_1, ..., \Pi_p$,$\Sigma$) contain the model coefficients that we want to estimate. Alternatively, the model in (1) can be written as:

$$\Pi(L)(y_t - \Gamma d_t) = u_t, \tag{2}$$

where $\Pi(L) = I_K - \Pi_1 L - \Pi_2 L^2 - ... - \Pi_p L^p$ (L is the lag-operator), $\Gamma = (\Gamma_0, \Gamma_1, \Gamma_2)$ and $y_t - \Gamma d_t = \tilde{y}_t$ (where d_t is appropriately defined to contain the deterministic components of the model). Writing the VAR model as in (2) follows Villani [2009] which allows for explicit priors on the steady state. The parameters can then be estimated by a Bayesian approach using a so-called Gibbs-sampler by iterating over the conditional distributions of the parameter matrices (see for example Del Negro and Schorfheide [2011] for further details). Villani [2009] shows that the convergence of the Markov-chain in the specification in (1) happens relatively quickly as long as the priors for Γ are not too diffuse. All results presented below are based on 1,000 accepted draws.

The data series used are quarterly series where the benchmark specification uses data from 1994Q1 to 2020Q2. The choice of estimation sample reflects a compromise of conflicting wishes to include as many observations as possible and to obtain a model based on stable relationsships without severe breaks. Even though data is available for earlier periods, several related VAR studies on Danish data have preferred to start in 1994 when estimating the effects of interest rates on macroeconomic variables, citing for example the much more stable monetary policy regime (e.g. seen from the interest spread to Germany) after the ERM crisis in the early 1990s (Beier and Storgaard, 2006 and Jensen and Pedersen, 2019). Weber et al. [2009] even argue that there are breaks in the transmission of monetary policy of the Euro Area countries as late as 1999 due to the implementation of the common currency (relevant for Denmark due to the fixed exchange rate policy of the Krone against the Euro). Estimating the model with a much earlier starting point than used in the benchmark specification would make it hard to argue that the relations considered are stable. Similarly, when considering oil price shocks, the empirical litterature prefers to use data no earlier than the mid 1980s. This is due to a normalization problem in identifying oil price shocks: If a supply shock to the oil production is defined in terms of a specific price change, the effects are found to be diminishing over time due to a lower oil demand elasticity in recent years, thus making pre and post 1986 periods incomparable ([Peersman and van Robays, 2012]). To summarize, the benchmark model is estimated using data from 1994. The one-step ahead residuals in the bechmark specification indicate that the model parameters are stable when choosing this sample periode.

All variables except for the foreign interest rate have been log-transformed and have been appropriately seasonally adjusted when relevant prior to estimation. To avoid modeling different long-run trends in the domestic versus the foreign economy, the foreign variables have been filtered prior to estimation, using the approach suggested in Hamilton [2018]. All variables except for the foreign interest rate come from the database of the macroeconometric model used by the Danish central bank (see Danmarks Nationalbank [2003] (in Danish)). The interest rate was collected from the webpage of the Deutche Bundesbank (https://www.bundesbank.de/en/statistics).

The domestic block of the economy in the benchmark specification includes the following 5 variables: Real GDP, real private consumption, real (non-energy) exports, domestic prices (measured as the GDP deflator) relative to foreign prices, and a measure of the real wage (measured as the average nominal wage relative to domestic prices). The choice of domestic variables in the benchmark specification is meant to capture the main effects of the propagation mechanism of most common macroeconomic shocks, including open economy aspects and foreign shocks. For example GDP is included as a measure of total economic activity whereas private consumption is the most important component of domestic aggregate demand. Exports are included to obtain the most direct estimate of the effect on the Danish economy through the trade channel on foreign shocks. Including, say only GDP would make it harder to distinguish the direct effects via exports and those due to the resulting increase in domestic demand following the subsequent change in the business cycle. Prices and wages are included both to infer aggregate supply effects of demand shocks as well as to give an indication of the level of nominal rigidity (»sticky prices«). Further, including a measure of domestic prices and wages will to some extend indicate the effects of competitiveness and the effects on exports through the terms of trade.

The foreign block consists of four variables: The foreign interest rate, foreign prices, foreign output, and the oil price. The interest rate is the quarterly average of the interbank market in the euro area after 1999 and the corresponding interest rate in Germany before that. As argued above, this is the relevant interest rate to capture monetary policy effects due to Denmark's fixed exchange rate policy. The fixed exchange rate of the Krone vis-a-vis the Euro (and before that the D-mark) implies that monetary policy is exogenously determined relative to the domestic variables. To ensure the best mapping to MAKRO, the foreign prices and output variables are included as their relevant representations in the model. Hence, foreign prices are included as the price index for Danish exporters and foreign output is an index of total market, relevant for Danish exporters. This is chosen to ensure the best possible mapping to MAKRO at the cost of a more »reduced-form«/less stringent modeling of the foreign economy (e.g. of the policy decision in the Euro Area). Naturally, such a 4-variable foreign block represents a further simplified view of the economy and the links to the Danish economy. However, a similar setup with 3 variables (interest rate, prices, and output) has been used in other studies of the effects of foreign variables on the Danish economy, for example in Pedersen and Ravn [2013][2] and Jensen et al. [2017]. Further, a number of papers, published by the ECB are based on some extension of a VAR model which includes output, prices and the monetary policy rate at its core (for example, see Sousa and Zaghini [2008], Bonci [2011], and Peersman [2011]). Similarly, the literature on spillover effects of US shocks to emerging markets, often include US GDP, inflation and an interest rate measure as the foreign economy (for example see Buckle et al. [2007], Mumtaz and Surico [2009], Vasishtha and Maier [2013], and Almansour et al. [2015]). Finally, it is consistent with the the way the foreign economy enters in MAKRO and hence, it is consistent with the subsequent matching procedure.

The fact that the model is estimated in levels merits a comment: While in theory it is clear which type of specification to use (i.e. a VAR in levels or differences and whether to use a VECM-specification, based on the number of unit roots and cointegrating relations), in practice this is less clear. One reason for this is that pre-testing the data before specifying the model type has the problem that the associated tests have notoriously low power. Further, structural breaks in (trend-) stationary series might make the test falsely conclude that there is a unit root (Lai, 2004). Since the true data generating process is unknown, one concern is how model misspecification affects the estimated impulses. Gospodinov et al. [2013] examine the robustness of the impulse responses

[2] In Pedersen and Ravn [2013] the foreign economy is modeled as two sets of three equation economies which are assumed not to affect each other.

from estimated VAR models and find that the level specification is generally more robust than the VECM and VAR in differences in terms of impulse response estimation when the true data generation process is unknown. This echoes Ashley and Verbrugge [2009] who find that overdifferencing of the model yields poor estimation of the impulse response functions, including confidence interval coverage.[3] Perhaps as a result, most studies that match impulse responses to theoretical models include real variables in levels instead of differences (some of the more well-known and recent examples include Rotemberg and Woodford [1997], Iacoviello [2005], Altig et al. [2011], Christiano et al. [2016] and Castelnuovo and Pellegrino [2018]). Further, modeling the real variables in levels with a deterministic trend corresponds to the constant growth corrections made in the equilibrium conditions in MAKRO, both in the baseline forecast and when conducting shock analysis on the model. Finally, since the motivation for estimating the impulse responses is to have a set of data-driven moments to match against the model's short-run properties (given its long-run structure and parametrization) in face of shocks this motivates the choice to focus on short-run in stead of long-run identification. Hence, this contributes to consistency between the empirical impulse responses and those of the model as well as the intended use of the empirical application.

Determining the lag order p is based on information criteria. Data clearly favorises a model with a limited lag-length and as a result is estimated with one lag. The model includes a constant and a linear trend term. Finally, the benchmark specification includes a dummy for the financial crisis, which takes on the value 1 during 2008Q4-2010Q4 and 0 otherwise as well as a few impulse dummies to account for extreme outliers.

2.1 Identification of structural shocks

The residuals, u_t, in (2) can be interpreted as one-period-ahead forecast errors and do not lend themselves to any economic interpretation per se. Instead, they might be seen as linear combinations of the structural shocks that hit the economy simultaneously. Hence, identification of a particular structural shock (which can then be compared with

[3]As noted in Kilian and Lütkepohl [2017], the consequences of imposing a unit root are asymmetric: Incorrectly imposing an I(1)-assumption implies overdifferencing while failing to impose a unit root preserves consistency, albeit with less precise parameter estimates.

a theoretical model) cannot be obtained by the VAR model's reduced form, i.e. without imposing further identifying restrictions. Such restrictions lead to the structural VAR (SVAR) representation of the model:

$$B_0 y_t = B_1 y_{t-1} + ... + B_p y_{t-p} + \varepsilon_t, \quad , \varepsilon_t \sim N(0, I_K) \tag{3}$$

where B_0 is a non-singular $K \times K$-matrix, $B_1 = B_0 \Pi_1$, $B_2 = B_0 \Pi_2$, etc. and $\varepsilon_t = B_0 u_t$. The key difference between (2) and (3) is that the covariance-matrix of the error term in (3) is now diagonal: Since the shocks in ε_t are uncorrelated at time t they are said to be »structural«. While it is potentially possible to assign an economic interpretation to all elements in ε_t, this need not be the case. Hence, the error term can simultaneously contain the structural shocks considered while remaining elememts can be measurement errors or unidentified shocks (see for example Kilian and Lütkepohl [2017] for a discussion hereof). Since Sims [1980], the most widely used identification strategy has been to obtain the structural parameters in B_0 from a Cholesky decomposition of the covariance matrix of the reduced form residuals, Σ. Since the estimate of $\mathbb{E}(u_t u_t') = B_0^{-1} I_K (B_0^{-1})' = B_0^{-1} (B_0^{-1})'$ has $K(K+1)/2$ free parameters but B_0 contains K^2 parameters, $K^2 - K(K+1)/2$ further restrictions on B_0 are necessary for exact identification (which is exactly what is obtained by the Cholesky decomposition). This identification strategy has often been used for example to identify a monetary policy shock in the seminal paper by Christiano et al. [2005]. The disadvantage of this approach is that it implies a full recursive ordering of the variables in terms of weak exogeneity which might have a weak theoretical basis.

In the following, identification is obtained by combining short-term restrictions with sign restrictions. The former is imposed on the foreign economy while the latter is used to »control for« a number of domestic shocks, following the idea in Mountford and Uhlig [2009].[4] The specific algorithm used is the one proposed in Arias et al. [2018]

[4] We abstain from imposing long-run restrictions a' la' Blanchard and Quah [1989] (for example, Souki [2008] identifies a foreign demand shock as the only shock allowed to have a permanent effect on the Canadian economy). The reason for this is that identifying assumptions on the long-run structure of the data are highly sensitive to the trend specification of the empirical model (and the true data generating process is of course unknown as discussed above). A prominent example is in the estimation of hours to technology shocks: While Gali [1999] finds that hours worked decline in face of a positive technology shock (thus questioning RBC-type models) when using a difference specification, Christi-

which allows the combination of sign and zero- (or short-term) restrictions (this was originally implemented in a DREAM master's thesis by Lund-Thomsen, 2016).

Since the foreign economy included in (2) consists of 4 variables this allows for up to 4 structural shocks. With sign restrictions it is logical to try an estimate foreign demand, foreign supply, an intereste rate, and an oil price shock respectively. In the matching of shocks against MAKRO, we settle for the 3 shocks, excluding foreign supply shocks for which we found the assumptions necessary to map the empirical impulse reponses to those of the model too uncertain.[5]

Hence in the following, the effects of a shock to foreign demand, the interest rate, and the oil price will be presented, respectively. The foreign shocks are partly identified via a recursive ordering. This implies: Shocks to the foreign interest rate are identified through the assumption that the monetary policy authorities are assumed to incorporate contemporaneously (intra-quarter) changes in output and prices in their interest rate decision. On the contrary, foreign output is assumed to be able to react to monetary policy only with a lag. This is reasonable since the policy lags are typically thought to be short in monetary policy and because output is well-documented to react with some lag to changes in the interest rate (Belongia and Ireland, 2016). This identification of monetary policy is reminiscent to much of the comparable literature (examples include Christiano et al. [2005], Bjoernland [2008], and Jensen et al. [2017]). Similarly, a shock to foreign output can affect foreign prices contemporanously but not vice versa. The positive contemporaneous correlation between foreign price and output implies that »shocks to foreign output« within a given quarter predominantly reflects movement along an aggregate supply curve and thus might be interpreted as aggregate demand shocks. A shock to foreign prices on the other hand can be interpreted as an aggregate supply shock, and hence is required to have a negative effect on foreign output (although not shown in the following). A shock to the oil price is interpreted as an increase in the price of oil that is not driven by movements in foreign output, i.e. by the Danish export market. This shock could come from a specific demand sho-

ano et al. [2003] come to the opposite conclusion using the same identification scheme but using a model estimated in levels. The severe lack of robustness of long run restrictions combined with model misspecification is also highlighted in Ravenna [2007] and Gospodinov et al. [2013].

[5]For example, a shock to foreign prices contain effects of both productivity movements, which may correlate across countries, as well as identify the endogenous effetcs on relative prices on exports.

ck to the oil market or through an exogenous change in the production of oil and we do not discriminate between the two type in the following. In both cases it is assumed that no domestic Danish shock affects the oil price - again consistent with the assumptions made in MAKRO. To separate oil shocks from price movements due to foreign aggregate demand shocks, opposing signs are imposed the effect on the Danish export market which is required to respond negatively to any of the two shock specific to the oil market (similar to [Peersman and van Robays, 2012]). Further, the other foreign variables (the Danish export market, foreign prices and the interest rate) are assumed to affect the oil price only with a lag. Finally, following the idea in Mountford and Uhlig [2009], the model »controls« for some more generic domestic shocks using sign restrictions. These restrictions are based on »robust« sign restrictions (this term was first introduced Peersman and Straub [2009]) in the sense that they are applicable across different classes of macroeconomic models. This includes the following set of domestic shocks (which do not affect the foreign economy): First, a domestic demand shock affects GDP, consumption and domestic prices positively. Second, a supply shock affects GDP positively but prices negatively. Such a response is consistent with both an increase in the labor supply as well as an increase in productivity. Finally, no domestic shocks can affect the Danish exports in the same quarter as the shock. This amounts to an assumption that the effects on exports from competitiveness are initially negligible which is probably a reasonable assumption. The two domestic shocks are well defined due to their opposite effect on prices as well as the open economy assumption. Table 1 summarizes the identification used in the benchmark specification. A »0« indicates that this variable cannot move contemporaneously in response to the particular shock. A »+« (»-«) indicates that this variable must respond positively (negatively) to the particular shock. All sign restrictions are imposed for a total of 4 quarters in total. The impulse responses are based on the median impulse of 1,000 accepted draws, i.e. impulses that satisfy the sign and zero restrictions.

2.2 Are the identified shocks structural?

An inherent disadvantage of VAR models is that the number of parameters increase rapidly as more variables are included. Further, due to structural breaks, the model has to be estimated on a limited number of observations. This necessitates that the empirical model contains fewer variables than those used by consumers and firms in their decision-making process. Omitting important variables can introduce slack in the estimation, since the true state of the economy is inaccurately observed. That agents and policy makers may have more information than the econometrician has been considered in terms of VAR models at least since Sims [1992] who argue that part of the »price puzzle« he observes in terms of monetary policy may be due to the fact that the central bank incorporates inflation expectations in their decision making proces.[6] Later, this has been discussed in terms of news shocks (Sims [2012]) and fiscal foresight (Leeper et al., 2013).

To investigate whether the identified shocks are in fact structural we follow Forni and Gambetti [2014]. The approach has two steps: First, we summarize the information from more than 70 macroeconomic and financial variables at quarterly frequency from the database described in Section 2 using principal components. As suggested in Stock and Watson [2002], in this way one can pool the information in all possible predictor variables in a large macroeconomic data set while discarding idiosyncratic variation in one particular series. Second, we test the orthogonality condition that implicitly underpins the SVAR-representation of the empirical model: All relevant information used to identify the shock of interest must be contained in the information set of the empirical model. In other words, the structural shock must be orthogonal to the lagged values of the principal components. In the benchmark specification, the data suggests that the first principal component should be added in the spirit of Factor-Augmented VAR (FAVAR) models (Bernanke et al., 2005) and it is subsequently included in the final estimation.

[6] This insight led Fernandez-Villaverde et al. [2007] to propose the "poor man's invertibility condition" that must hold for all the residuals in the VAR model to have a structural mapping. However, as noted in Forni et al. [2016], one or more structural shock may well be identified even though the condition in Fernandez-Villaverde et al. [2007] is not satisfied.

3 Results

The endogenous response to the three foreign shocks are shown in Figure 1. All graphs have been annualized for comparison with MAKRO. They show both the response to the shocks of the foreign economy itself as well as the spillovers to Danish prices and quantities. In the most broad sense, the impulse responses are reasonable, in that the signs and magnitudes are for the most parts as expected and in line with theory and comparable empirical literature. The confidence bands indicate that the impulse responses for some of the shocks and impulses are associated with a non-negligible uncertainty, although seemingly not to a larger degree than other VAR studies.

One thing should perhaps be noted here: The foreign shocks have general equilibrium effects in the foreign economy as well as in the domestic economy. For example, at least qualitatively we will expect increasing foreign demand to affect prices in the foreign economy, equivalently to those of the Danish economy. Hence, a particular foreign shock will affect all endogenous foreign variables as is seen from the graphs, as opposed to more stylized (partial) shocks which you need a model for. For example, this means that the crowding out mechanism in Denmark to expansionary foreign shocks can happen from either a return of foreign demand to a normal state, a worsening of competitiveness - or both.

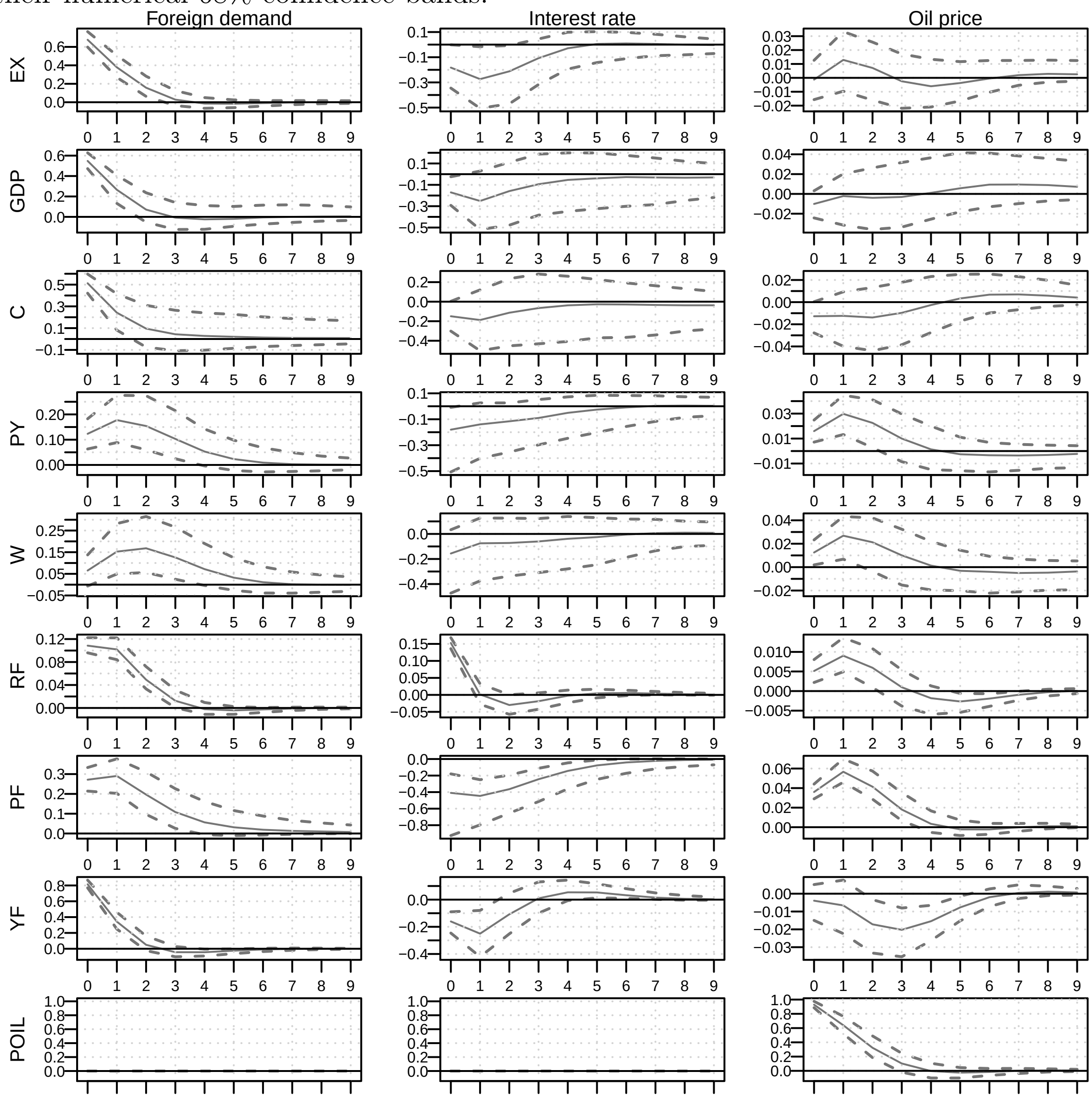
The first column of Figure 1 shows the impulse responses to a shock to foreign demand. As a response to the increase in economic activity, prices in the foreign economy go up as we would expect, since aggregate supply will only to some extend accomodate the increase in demand. However, while foreign prices increase they do so sluggishly with inflationary preassure for the first 4 quarters even though the increase in demand is largest on impact. As expected, the increase in output and prices makes the European Central Bank raise its monetary policy rate to stabilize prices in line with its mandate. For the Danish variables, the immediate effect of increased foreign demand is higher exports. However, it can be seen that exports react less that one-to-one against increased foreign demand (the short-run output elasticity of exports is less that 1). One explanation for this could be that the composition of Danish exports implies that it is less cyclical than the export of other countries (Andersen et al., 2012). Such a composition effect is hard to capture in a model in a completely microfounded way.

Instead, at the aggregate level it might be captured by introducing a rigidity in export demand for Danish goods. Similar to foreign prices, Danish export prices increase as aggregate demand goes up as well. The increase in GDP increases the factor demand of firms, all else equal. A higher demand for labor input means that wages increase as well. This leads to positive wage inflation for slightly longer than two years, around the point after which output is no longer above trend. Subsequently, wages slowly decline back towards the baseline level. Further, prices and (especially) wages seem to be »sticky« in Denmark as well. Both export prices and wages peak at a later point and are considerably more persistent than for example the GDP response. Further, it seems that wages are more sticky that prices in Denmark as they peak slightly later (a similar conclusion is drawn in Abildgren [2010]). This motivates the implementation of nominal wage rigidity in wage-changes. Hence, the results indicate that real (average) wages may be midly counter cyclical in the initial quarters following a foreign demand shock (the effect later reverses). This echoes the findings in Messina et al. [2009] who find Danish real wages to be counter cyclical in general, a feature which tends to be more prominent for more open economies.[7]

As higher foreign demand stimulates the domestic economy, income increases as well which in turn has a positive effect on private consumption. In face of a foreign demand shock, it is seen from Figure 1 that the magnitude and shape of the impulse response of private consumption follows that of GDP fairly closely in terms of percentage deviation from steady state (with some degree of uncertainty). Since private consumption consitutes nearly half of Danish GDP (and if GDP is taken as a proxy for disposable), this indicates a fairly high marginal propensity to consume conditional on foreign demand shocks. This finding is consistent with several recent microeconometric studies on Danish consumers (for example Crawley and Kuchler, 2018 and Kreiner et al., 2019) as well as the short-run income elasticity of aggregate consumption of 0.4 to 0.5 in large-scale Danish macroeconometric models (Grinderslev and Smidt, 2006 and Borge and Knudsen, 2019).

[7]Of course, a counter cyclical average wage does not imply that the wage for a giver worker is counter cyclical. It has been known at least since Solon et al. [1994] that this result might be due to a composition effect in aggregate time series. For example, the employment of high wage-earners may be less cyclical than that of low wage-earners, which makes the average real wage less procyclical.

Figur 1: Annualized impulse responses to a shock to foreign demand. The identification scheme is the benchmark specification (Table 1). The impulses are shown including their numerical 68% confidence bands.



The second column of Figure 1 shows the impulse responses to a shock to the foreign interest rate. Since the shock itself is set identified in the benchmark specification, the shock is normalized to a shock of 25 basis points to the interest rate. Due to the linearity of the VAR model, the initial size of the shock itself can be scaled without changing the interpretation of the shock or its endogenous effects. Thus, this scaling keeps the general parameter uncertainty associated with the estimation but not that which is associated with the size of the shock itself. As the shock is meant to be compared (and matched) to a model shock where a specific value of the shock must be used as an input, this merely removes an uncertainty that will not be taken into account at a later point anyways.[8] One way to interpret this shock is as a contraction in the monetary policy stance that is not in accordance with the average reaction to economic activity and prices, i.e. the contractionary policy is unexpected. By definition, the shock to the foreign interest rate increases the interest rate which is elevated for between 1 and 2 years (after which there seems to be a slight overshoot of policy - also found in similar VAR studies of the Euro Area, e.g. Weber et al. [2009]). Overall, contractionary monetary policy leads to a slowdown in real economic activity and lower prices in the foreign and the domestic economy as one would expect. However, the effects from changes in the interest rate happens with a lag, which is typically found in related studies as well (the lags associated with the monetary policy transmission mechanism is often used as an argument that monetary policy must be forward-looking, see for example ECB [2000]). As earlier, foreign prices are longer at adjusting to the baseline level than output. The fall in foreign output reduces the demand for Danish exports which fall in similar fashion. Although the size of the drop in exports is associated with non-negligible uncertainty, the central estimate suggest that it is smaller than the drop in the total export market, thus reiterating the finding for foreign demand shocks that the short-run output elasticity for Danish exports may be less than 1. The higher nominal interest rate combined with sticky prices means that the real interest rate increases following the shock. This effect reduces consumption and GDP besides the drop in exports. The delayed effect on quantities from foreign output is found for Danish variables similar to the foreign economy. As a result, the impulses of output and consumption are »hump-shaped«, with GDP decreasing for approximately 1 year before returning to the baseline level (similar dynamics are found in for example Abildgren, 2010). In general, the hump-shaped responses are a standard result in VAR model and typically used as the motivation in the DSGE literature for including certain frictions,

[8] See Peersman and van Robays [2012] for a discussion of an example of a similar normalization when looking at the effects of oil prices, identified by sign restrictions only.

for example habit formation in consumption (Christiano et al., 2005). Further, it is worth noting that the effects of the monetary policy shock on the real economic variables are fairly persistent, even though the shock in itself is not as persistent (this result, that interest rate shocks have persistent effects is also found in Pedersen and Ravn [2013] and Jensen and Pedersen [2019]). With respect to the magnitude, the negative effect on GDP is found to be between 0.2-0.3% below the baseline level when the effect is highest. Comparing the effects to the two other Scandinavian countries, this is substantially larger than what Bjoernland [2008] finds for Norway (a 1 p.p. increase in the monetary policy rate decreases GDP by only 0.25% 1-2 years later) but fairly close to the effect in Aursland et al. [2019] (1 p.p. increase has a peak effect of 1.4%). In Ramses II, the DSGE model of Sweden, the peak effect on GDP is 0.1-0.2% relative to baseline for a 0.25 p.p. shock to the monetary policy rate (Adolfson et al., 2013). Both Norway and Sweden however have a flexible exchange rate as opposed to Denmark, which makes the effects - not least the direct effects from exports - different. In the estimated DSGE model of the Euro Area, Ratto et al. [2008] report a 1 p.p. monetary policy shock to have a peak effect on GDP of slightly below 2% which is a substantially larger effect than found in this paper. Finally, the estimated magnitude of the peak GDP effect is somewhat higher than estimated for Denmark in the DGSE model in Pedersen and Ravn [2013] (a 0.25 p.p. increase in the ECB policy rate have a peak effect of 0.1-0.2% of GDP relative to baseline) but of the same magnitude to the effects found in Abildgren [2010] (where the peak effect on GDP (in %) is approximately the same magnitude as the peak effect on the interest rate (in p.p.), depending on the model specification).

The thirds column of Figure 1 shows the impulse responses to a shock of one percent to the oil price. The increase in the oil price reduces foreign output, increases foreign price and induces an endogenous contraction of monetary policy. This rise in the monetary policy rate however is smaller in magnitude and shorter in duration than in the case of a foreign demand shock. The reason for this is probably the opposing effects that the two shocks have on output in the foreign economy (as opposed to foreign demand shocks for which foreign prices are procyclical). It is seen that - while foreign output decreases - Danish (non-energy) exports increase in the short run, although this effect is insignificant. The reason is the asymmetric effects on foreign relative to domestic prices following the oil price shock. This finding is consistent with the results in Peersman and van Robays [2012] who find that, while oil price changes from aggregate global demand shocks are similar across countries, oil specific shocks have highly asymmetric effects, including on inflation. On reason might be that Danish firms are more energy efficient than their foreign counterparts, which reduces the pass-through to output prices. As a result of this asymmetric effect on prices, export competitiveness improves.

Finally, Figures 2-4 makes the same impulses as the benchmark specification with the following changes: Estimating the model from 1999, estimating the model with the shadow short rate of Wu and Xia [2017], an alternative identification based on Tabel 2, and a model including the Danish GDP deflator instead of export prices.

4 Conclusion

In this working paper, the effects of foreign shocks to the Danish economy are assessed and analyzed. This is done through an estimated VAR model with domestic and foreign variables. The foreign shocks are those to foreign demand, the foreign interest rate and the oil price. As an identification scheme, a combination of sign and zero restrictions is used, applying a small-open economy assumption as well as controlling for generic business cycle shocks. This approach leads to results which are broadly in line with theory and related empirical literature, albeit they are associated with non-negligible uncertainty for some shocks and impulses.

Appendix A: Overview of identifying restrictions

Tabel 1: Identification, benchmark model

	Domestic demand	Domestic supply	Foreign interest rate	Foreign supply	Foreign demand	Oil price
GDP	+	+				
Consumption	+					
Domestic prices / foreign prices	+	-				
Real wage						
Exports	0	0				
Foreign interest rate	0	0	+			
Foreign prices	0	0	-	+	+	
Foreign output	0	0	0	0 (-)	+	
Oil price	0	0	0	0	0	+

Note: A "0"indicates that this variable cannot move contemporaneously in response to the particular shock. A "+"(-") indicates that this variable must respond positively (negatively) to the particular shock. Signs in () indicate that this is not imposed contemporaneously. All sign restrictions are imposed for a total of 4 quarters in total. The impulse responses are based on 1,000 accepted draws.

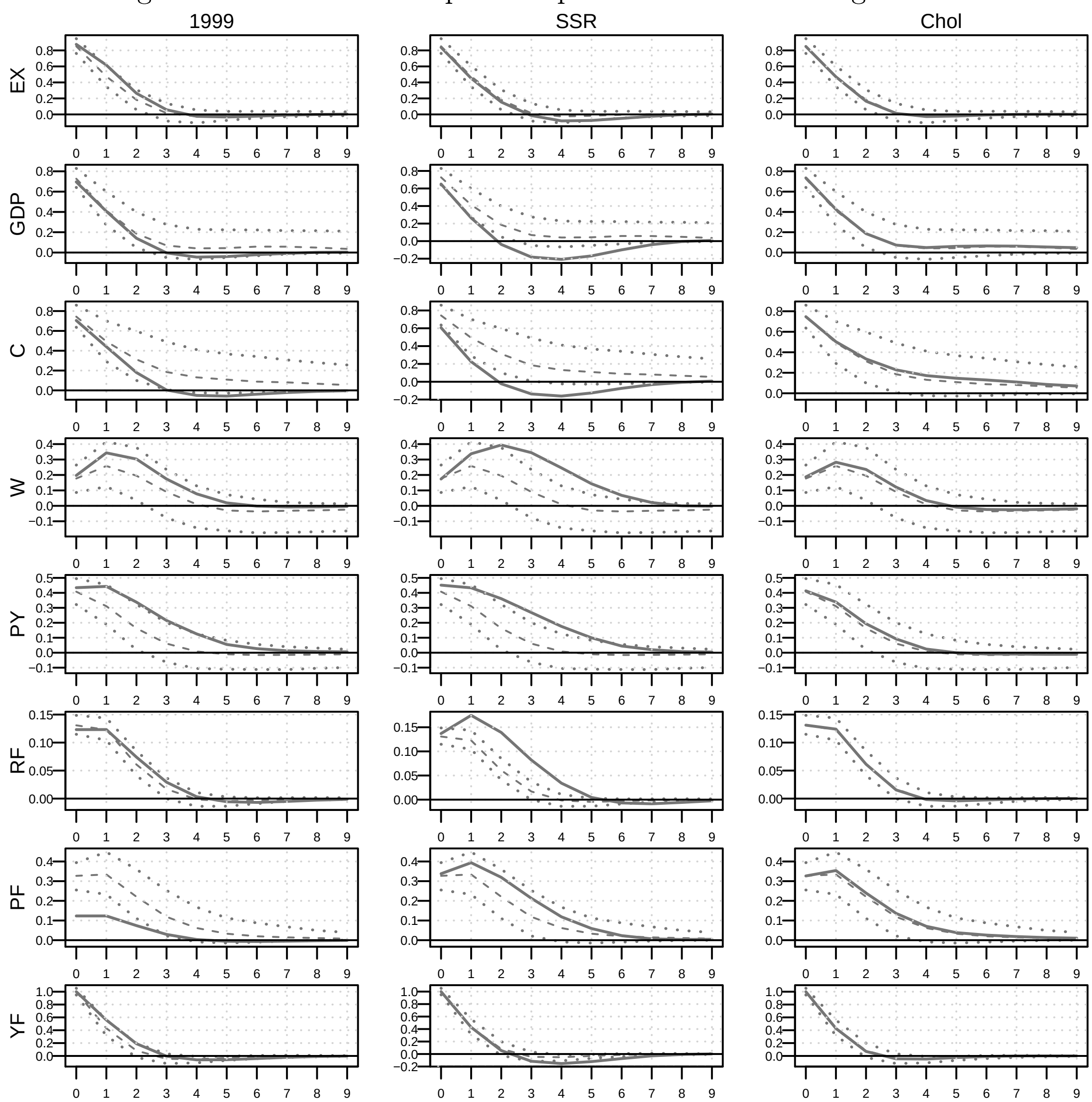
Tabel 2: Robustness check: Alternative identification

	Domestic demand	Domestic supply	Foreign interest rate	Foreign supply	Foreign demand	Oil price
GDP	+	+				
Consumption	+					
Domestic prices / foreign prices	+	-				
Real wage						
Exports	0	0				
Foreign interest rate	0	0	+			
Foreign prices	0	0	0	+		
Foreign output	0	0	0	0	+	
Oil price	0	0	0	0	0	+

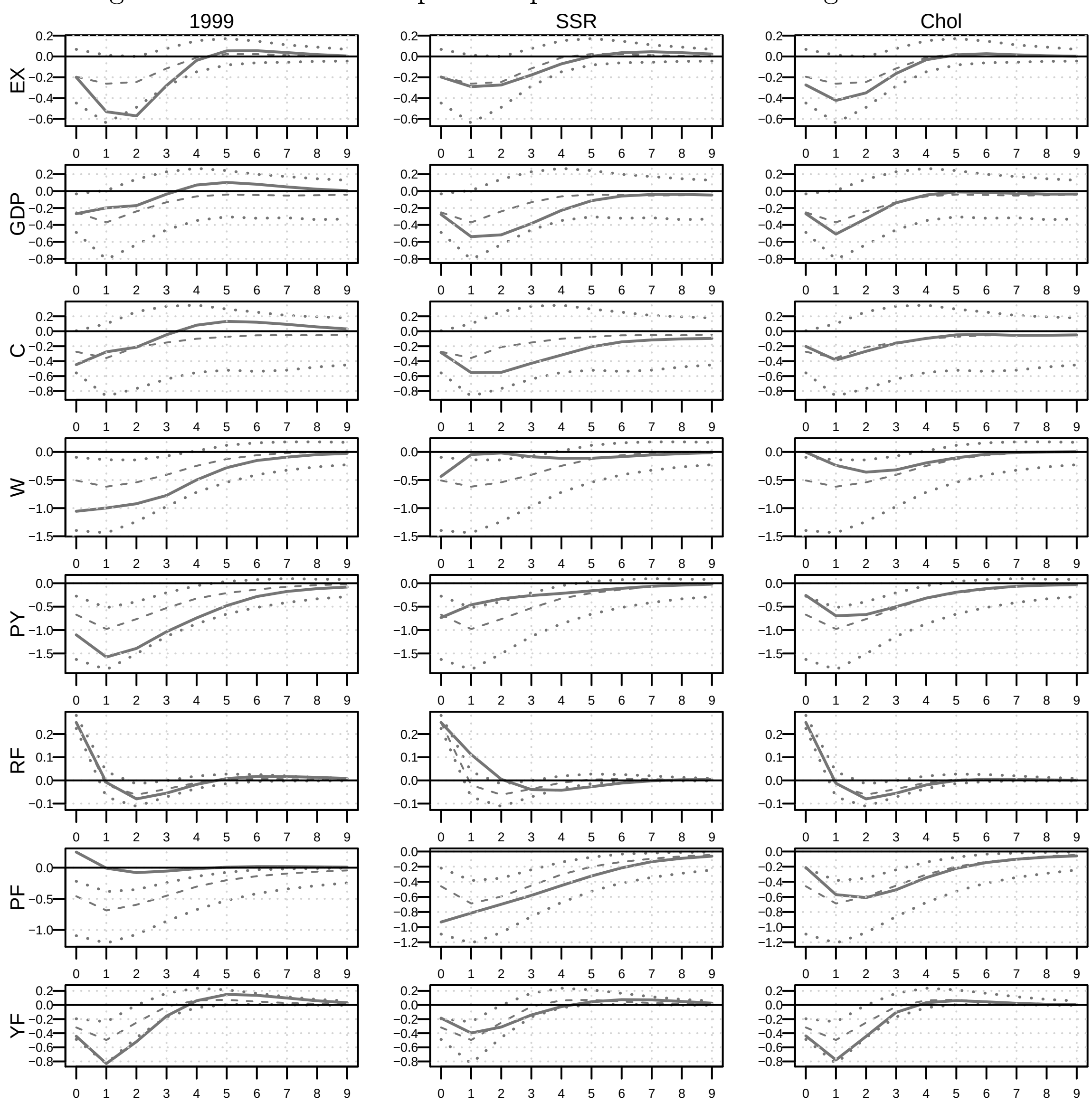
Note: A "0"indicates that this variable cannot move contemporaneously in response to the particular shock. A "+"(-") indicates that this variable must respond positively (negatively) to the particular shock. All sign restrictions are imposed for a total of 4 quarters in total. The impulse responses are based on 1,000 accepted draws.

Appendix B: Additional figures and tables

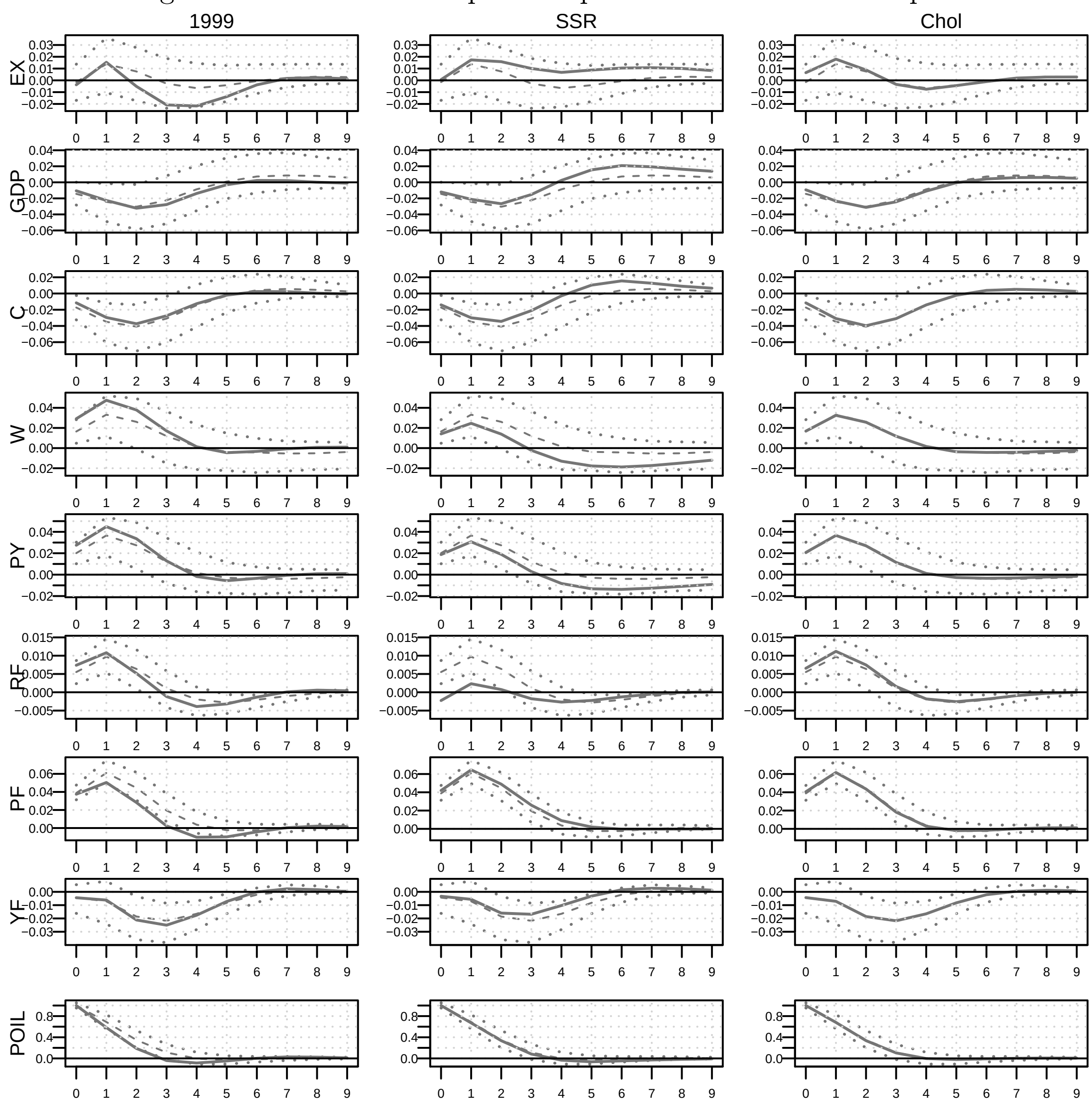
Figur 2: Robustness of impulse responses: Shocks to foreign demand



Figur 3: Robustness of impulse responses: Shocks to foreign interest rate



Figur 4: Robustness of impulse responses: Shocks to the oil price



Litteratur

K. Abildgren. Business cycles, monetary transmission and shocks to financial stability - empirical evidence from a new set of danish quarterly national accounts. *Danmarks Nationalbank working paper*, 2010.

M. Adolfson, S. Laseen, L. Christiano, M. Trabandt, and K. Walentin. Ramses ii - model description. *Sveriges Riksbank Occasional Paper Series*, 2013.

A. Almansour, A. Aslam, J. Bluedorn, and R. Duttagupta. How vunerable are emerging markets to external shocks? *Journal of Policy Modeling*, 2015.

D. Altig, L. J. Christiano, M. Eichenbaum, and J. Linde. Firm-specific capital, nominal rigidities and the business cycle. *Review of Economic Dynamics*, 2011.

C. H. Andersen, J. Isaksen, and M. Spange. Denmark's competitiveness and export performance. *Danmarks Nationalbank - Monetary Review 2012Q2*, 2012.

J. E. Arias, J. F. Rubio-Ramirez, and D. F. Waggoner. Inference based on svars identified with sign and zero restrictions: Theory and applications. *Econometrica*, 2018.

R. A. Ashley and R. J. Verbrugge. To difference or not to difference: a monte carlo investigation of inference in vector autoregression models. *International Journal of Data Analysis Techniques and Strategies*, 2009.

T. A. Aursland, I. Frankovic, B. Kanik, and M. Saxegaard. Nora - a microfounded model for fiscal policy analysis in norway. *NORA Documentation*, 2019.

N. C. Beier and P. E. Storgaard. Indentifying monetary policy in a small open economy under fixed exchange rates. *Danmarks Nationalbank working paper*, 2006.

M. T. Belongia and P. N. Ireland. Money and output: Friedman and schwartz revisited. *Journal of Money, Credit and Banking*, 2016.

B. S. Bernanke, J. Boivin, and P. Eliasz. Measuring the effects of monetary policy: A factor-augmented vector autoregressive (favar) approach. *Quarterly Journal of Economics*, 2005.

H. C. Bjoernland. Monetary policy and exchange rate interactions in a small open economy. *The Scandinavian Journal of Economics*, 2008.

O. Blanchard and R. Perotti. An empirical characterization of the dynamic effects of changes in government spending and taxes on output. *The Quarterly Journal of Economics*, 2002.

O. J. Blanchard and D. Quah. The dynamic effects of aggregate demand and supply disturbances. *American Economic Review*, 1989.

R. Bonci. Monetary policy and the flow of funds in the euro area. *ECB Working Paper*, 2011.

A. Borge and D. Knudsen. Reestimation af makroforbrugsrelationen til adam model-version jun19. *Danmarks statistik arbejdspapir*, 2019.

R. A. Buckle, K. Kim, H. Kirkham, N. McLellan, and J. Sharma. A structural var business cycle model for a volatile small open economy. *Economic Modeling*, 2007.

E. Castelnuovo and G. Pellegrino. Uncertainty-dependent effects of monetary policy shocks: A new-keynesian interpretation. *Journal of Economic Dynamics & Control*, 2018.

L. J. Christiano, M. Eichenbaum, and R. Vigfusson. What happens after a technology shock? *NBER Working Papers*, 2003.

L. J. Christiano, M. Eichenbaum, and C. L. Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 2005.

L. J. Christiano, M. S. Eichenbaum, and M. Trabandt. Unemployment and business cycles. *Econometrica*, 2016.

E. Crawley and A. Kuchler. Consumption heterogeneity: Micro drivers and macro implications. *Danmarks Nationalbank working paper*, 2018.

D. O. Cushman and T. Zha. Identifying monetary policy in a small open economy under flexible exchange rates. *Journal of Monetary Economics*, 1997.

Danmarks Nationalbank. Mona - en kvartalsmodel af dansk økonomi. *Danmarks Nationalbank publikation*, 2003.

L. Dedola and S. Neri. What does a technology shock do? a var analysis with model-based sign restrictions. *Journal of Monetary Economics*, 2007.

M. Del Negro and F. Schorfheide. Bayesian macroeconometrics. *Handbook of Bayesian Econometrics*, 2011.

ECB. Monetary policy transmission in the euro area. *ECB Monthly Bulletin*, 2000.

M. Eichenbaum and C. L. Evans. Some empirical evidence on the effects of shocks to monetary policy on exchange rates. *Quarterly Journal of Economics*, 1995.

Jesus Fernandez-Villaverde, Juan F. Rubio-Ramirez, Thomas J. Sargent, and Mark W. Watson. Abcs (and ds) of understanding vars. *American Economic Review*, 97: 1021–1026, 2007.

M. Forni and L. Gambetti. Sufficient information in structural vars. *Journal of Monetary Economics*, 66:124–136, 2014.

Mario Forni, Luca Gambetti, and Luca Sala. Var information and the empirical validation of dsge models. *RECent Working Paper*, 2016.

C. Foroni, F. Furlanetto, and A. Lepetit. Labor supply factors and economic fluctuations. *International Economic Review*, 2018.

J. Gali. Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *American Economic Review*, 1999.

N. Gospodinov, A. M. Herrera, and E. Pesavento. Unit roots, cointegration, and pretesting in var models. *Advances in Econometrics*, 2013.

D. Grinderslev and J. Smidt. Smec - modelbeskrivelse og modelegenskaber, 2006. *Det Økonomiske Råd - sekretatiatet, dokumentation*, 2006.

J. D. Hamilton. Why you should never use the hodrick-prescott filter. *Review of Economics and Statistics*, 2018.

M. Iacoviello. House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review*, 2005.

J. R. Jensen and J. Pedersen. Macro financial linkages in a svar model with applications to denmark. *Danmarks Nationalbank working paper*, 2019.

J. R. Jensen, J. G. Mikkelsen, and M. Spange. The ecb's unconventional monetary policy and the role of exchange rate regimes in cross-country spillovers. *Danmarks Nationalbank working paper*, 2017.

L. Kilian and H. Lütkepohl. *Structural Vector Autoregressive Analysis.* Cambridge University Press, 2017.

C. T. Kreiner, D. D. Lassen, and S. Leth-Petersen. Liquidity constraint tightness and consumer responses to fiscal stimulus policy. *Americal Economic Journal: Economic Policy*, 2019.

K. S. Lai. On structural shifts and stationarity of the ex ante real interest rate. *International Review of Economics & Finance*, 2004.

Eric M. Leeper, Todd B. Walker, and Shu-Sung Susan Yang. Fiscal foresight and information flows. *Econometrica*, 81:1115–1145, 2013.

F. Lund-Thomsen. Matching macro theory with data. *Kandidatafhandling ved University of Copenhagen*, 2016.

J. Messina, C. Strozzi, and J. Turunen. Real wages over the business cycle: Oecd evidence from the time and frequency domains. *Journal of Economic Dynamics & Control*, 2009.

A. Mountford and H. Uhlig. Whare are the effects of fiscal policy shocks? *Journal of Applied Econometrics*, 2009.

H. Mumtaz and P. Surico. The transmission of international shocks: A factor-augmented var approach. *Journal of Money, Credit, and Banking*, 2009.

J. Pedersen and S. H. Ravn. What drives the business cycle in a small open economy? evidence from an estimated dsge model of the danish economy. *Danmarks Nationalbank Working Papers*, 2013.

G. Peersman. Macroeconomic effects of unconventional monetary policy in the euro area. *ECB working papers*, 2011.

G. Peersman and R. Straub. Technology shocks and robust sign restrictions in a euro area svar. *Internatonal Economic Review*, 2009.

G. Peersman and I. van Robays. Cross-country differences in the effects of oil shocks. *Energy Economics*, 2012.

M. Ratto, W. Roeger, and J. i. Weld. Quest iii: An estimated dsge model of the euro area with fiscal and monetary policy. *European Commission Economic Papers*, 2008.

Frederico Ravenna. Vector autoregressions and reduced form representations of dsge models. *Journal of Monetary Economics*, 54:2048, 2007.

J. Rotemberg and M. Woodford. An optimization-based econometric framework for the evaluation of monetary policy. *Chapter in NBER Macroeconomics Annual*, 1997.

C. A. Sims. Macroeconomics and reality. *Econometrica*, 1980.

C. A. Sims. Interpreting the macroeconomic time series facts: The effects of monetary policy. *European Economic Review*, 1992.

Eric R. Sims. News, non-invertibility and structural vars. *NBER Working Paper*, 2012.

G. Solon, R. Barsky, and J. A. Parker. Measuring the cyclicality of real wages: How important is composition bias. *The Quarterly Journal of Economics*, 1994.

K. Souki. Assessing the effects of u.s. shocks on the canadian economy using alternative identification methods. *The North American Journal of Economic and Finance*, 2008.

J. Sousa and Andrea Zaghini. Monetary policy shocks in the euro area and global liquidity spillovers. *International Journal of Finance and Economics*, 2008.

J. Stock and M. Watson. Macroeconomic forecasting using diffusion indexes. *Journal of Business Economics and Statistics*, 2002.

G. Vasishtha and P. Maier. The impact of the global business cycle on small open economies: A favar approach for canada. *North American Journal of Economics and Finance*, 2013.

M. Villani. Steady-state priors for vector autoregressions. *Journal of Applied Econometrics*, 24:630–650, 2009.

A. A. Weber, R. Gerke, and A. Worms. Has the monetary transmission process in the euro area changed? evidence based on var estimates. *BIS working papers*, 2009.

J. C. Wu and F. D. Xia. Time-varying lower bounds of interest rates in europe. *Chicago Booth Research Paper*, 2017.