DREAM
Danish Research Institute for
Economic Analysis and Modelling

GREEN REFORM

Modelling abatement technologies: A comparison of approaches

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1. Introduction

This economic memo compares GreenREFORM's approach to integrate end-of-pipe abatement technologies into a computable general equilibrium model with the conventional method in the literature. The memo is based on the Bachelor thesis of the author. The Bachelor thesis was handed in at the Department of Economics at the University of Copenhagen on the 31st of April 2021

Technology catalogues contain technical and economic information about multiple technologies that businesses can bring into use in order to reduce their emissions. By using the information from such catalogues, one can construct a step-curve showing the marginal abatement cost (MAC), i.e. the firms' cost of reducing emissions by one tonne CO2e, as a function of total abatement. There exists different methods to incorporate the information from technology catalogues in a CGE-model. By imposing different level of taxation in the model, it is possible to construct the "model-based" MAC curve that shows how much the firms will abate at different level of taxation. This MAC curve will differ from the original step-curve consisting of the technologies from the catalogue. However, the size of the deviation depends on how the information from the catalogue is implemented in the CGE-model. The traditional way to do this is described in Kiuila and Rutherford (2013). The method suggested in this paper requires calibration of an abatement function outside the CGE-model of interest. Stephensen, P., Beck, U.R. and Berg, R.K suggest an alternative method. This method is presented in the economic memo of 25 August 2020, "End-of-pipe emissions abatement technologies in a CGE-model", which can be downloaded from https://dreamgruppen.dk/publikationer/2020/august/end-of-pipe-emissions-abatement-technologies-in-a-cge-model. The authors explain how the information from technology catalogues can be incorporated by introducing heterogeneity in the firms' costs of implementing the technologies. I find that the method suggested in Stephensen et al. (2020) has a number of advantages.

Firstly, applying the approach suggested in Stephensen et al. (2020) makes it possible to keep more of the information from a given technology catalogue in the model. Secondly, this approach makes it possible to obtain a "model-based" MAC curve that is infinitely close to the original step-curve curve. Thirdly, the Stephensen et al. approach fits the step-curve better in some cases, notably when the step-curve is "s-shaped". Moreover, this method makes inclusion of endogenous technology feasible. If the costs of the technologies are not stated explicitly in the technology catalogue, the costs can be determined endogenously in the CGE model. In such cases, the Stephensen et al. (2020) approach is applicable but the method suggested in Kiuila and Rutherford (2013) is not. Finally, it is far easier to include new technologies and adjust information about existing technologies using the Stephensen et al. (2020) approach.

There exists a range of Danish technology catalogues consisting of so-called end-of-pipe technologies, which means that the technologies do not affect the production process besides the impact on emissions. The Danish Energy Agency publishes most of these catalogues. There are catalogues describing industrial energy savings, industrial process heat, electric appliances, transport and - as considered in this memo - agricultural technologies. In the spirit of Kiuila and Rutherford (2013), this memo contains a calibration of the abatement function to include bottom-up information on agricultural abatement technologies in a top-down model. Subsequently, the same technological data is implemented in a top-down model by applying the method suggested in Stephensen et al. (2020). In section 2, I describe the agricultural data that enter the models and which is used to calibrate the abatement function. In section 3, I set up a general equilibrium framework and calibrate the parameters of the abatement function. Section 4 outlines a similar general equilibrium framework and describes how to impose heterogeneity using the approach suggested in Stephensen et al. (2020). Section 5 compares the effects of environmental tax policy using the presented models. Section 6 compares the two models in more detail and discusses the potential issues of the approach in this memo.

2. Description of the Data

The technology catalogue consists of agricultural technologies, each of which has an implementation potential, a reduction share and a unit cost

I consider a preliminary dataset of agricultural abatement technologies. This dataset consists of five different abatement technologies that can be implemented by farms to reduce emissions, cf. table 2.1. The final dataset used in the GreenREFORM model may differ from the data used here; however, the conclusions regarding the relative efficacy of the employed methods will carry through.

The technologies in the preliminary dataset are all "end-of-pipe"-technologies, which means that they do not affect the production process besides the impact on emissions. Each of the technologies has an implementation potential which measures how large a fraction of total emissions the technology is applicable to. For instance, acidification of swine manure can be applied to 26 pct. of the CH4-emissions from manure management. Moreover, a reduction share is attached to each of the technologies. The reduction share measures the abated fraction of the emissions that a given technology is applied to. For instance, if "acidification of swine manure" is implemented by firms, 60 pct. of the CH4-emissions that the technology is applied to are abated by the technology[1]. These reduction shares are calculated based on Denmark's Energy and Climate Outlook 2020 by The Danish Energy Agency. By combining the reduction share and the implementation potential it appears that acidification of swine manure has a potential of removing 0.26* 0.6 = 15.6 % of the total CH4-emissions from manure management according to the data. Finally, there is a cost of implementing each of the abatement technologies. The costs are measured in DKK per ton abated CO2e. The data for the costs and implementation potentials are based on the findings in Dubgaard and Ståhl (2018).

[1] Beck, U.R., Berg, A.K., Christiansen, S. and Jørgensen, C.L. (2020), p. 17

Table 2.1

Agricultural abatement technologies

Technology (t)	Reduction share	Implementation potential	Cost (c_t)
$\text{Acidification}_{\text{swine}}$	0.6	0.26	774
$\text{Biogasification}_{\text{swine}}$	0.17	0.66	1374
$\text{Biogasification}_{\text{cattle}}$	0.41	0.60	1374
$\text{Acidification}_{\text{beef cattle}}$	0.60	0.27	1827
$\text{Acidification}_{\text{cattle}}$	0.60	0.27	1827

Note: Cost is measured in DKK per ton CO2e

Source: Dubgaard and Ståhl (2018), Denmark's Energy and Climate Outlook 2020

By assuming a global warming potential of 25, the total amount of agricultural CH4-emissions related to manure management was 2219.3 kt CO2e[2] in 2018. The Danish Centre for Environment and Energy reports this number in Denmark's National Inventory Report published in 2020. The estimate is based on a bottom-up approach where the number of animals on Danish farms, excreted manure per animal per year and the estimated methane conversion factor is taken into consideration. I employ the estimated amount of CH4-emissions as the observed net emissions related to manure management in baseline in *model 1*, cf. section 3.

[2] Nielsen et al. (2020), p. 65

3. Kiuila-Rutherford

This section outlines a general equilibrium framework and explains the calibration of the parameters in the abatement function using the approach suggested in Kiuila-Rutherford (2013)

3.1 General Equilibrium Framework

In the following section, I set up a simple general equilibrium framework. The framework consists of equations (18)-(21). Since I only analyze the long run effects, the present modelling framework is static. The demand for the agricultural firms' composite good is given by a linear, downward-sloping curve

$$Y = 25000 - P \tag{1}$$

where Y is aggregate demand for the agricultural composite good, and P is the price of the good. The farms produce using Leontief technology such that the input factor demand is given by

$$Xi_i = \mu_i \cdot Y, \quad i = \text{manure, other inputs} \tag{2}$$

and

$$CH4^{gross}_{manure} = \mu_{CH4} \cdot Y \tag{3}$$

where Xi_i is the firms' demand for input *i*, μ_i is a distribution parameter for input *i* in production, and $\mathrm{CH4}^{\mathrm{gross}}_{manure}$ is the level of gross emissions. *Other inputs* is an aggregate input that contains all other inputs than manure and gross emissions. The components of *other inputs* are left out for the sake of simplicity. The weight of *other inputs* in production is, of course, large. Thus, the distribution parameter for *other inputs* is fixed at 0.8.[3] The two other distribution parameters are fixed at $\mu_{CH_4} = \mu_{manure} = 0.1$. The zero profit condition for the upper nest reads:

$$P \cdot Y = Pi_{manure} \cdot Xi_{manure} + Pi_{other\ inputs} \cdot Xi_{other\ inputs} + Pi_{CH4} \cdot CH4^{gross}_{manure} \tag{4}$$

where Pi_i is the price of input *i*.

[3] See section 6.4

Emissions can be divided into abatable and unabatable emissions, i.e.

$$CH4_{manure}^{abatable} = \psi_{manure} \cdot CH4_{manure}^{gross} \tag{5}$$

$$CH4_{manure}^{unabatable} = (1 - \psi_{manure}) \cdot CH4_{manure}^{gross} \tag{6}$$

where the share of emissions that are abatable ψ_{manure} equals the accumulated potential for the last technology $q_{Acidification_{cattle}} = \psi_{manure}$ = 0.83. The corresponding zero profit condition is

$$Pi_{CH4} \cdot CH4_{manure}^{gross} = P_A \cdot CH4_{manure}^{abatable} + \tau \cdot CH4_{manure}^{unabatable} \tag{7}$$

where P_A is the price of abatable emissions, and τ is an emissions tax. The firms can substitute towards abatement capital instead of emitting. Abatement capital is given by a CES demand equation:

$$K = \theta^{\sigma} \cdot (\frac{P_K}{P_A})^{-\sigma} \cdot CH4_{manure}^{abatable} \tag{8}$$

where θ is a distribution parameter that measures baseline capital's share of total baseline expenditures for the firms, σ is the elasticity of substitution between abatement capital and the part of abatable emissions that is left after abatement, and P_K is the price of abatement capital which I normalize to 1. The part of abatable emissions that is left after abatement has taken place is given by

$$CH4_{manure}^{after} = (1 - \theta)^{\sigma} \cdot (\frac{\tau}{P_A})^{-\sigma} \cdot CH4_{manure}^{abatable} \tag{9}$$

Finally, the zero profit condition for this part of the nest is given by

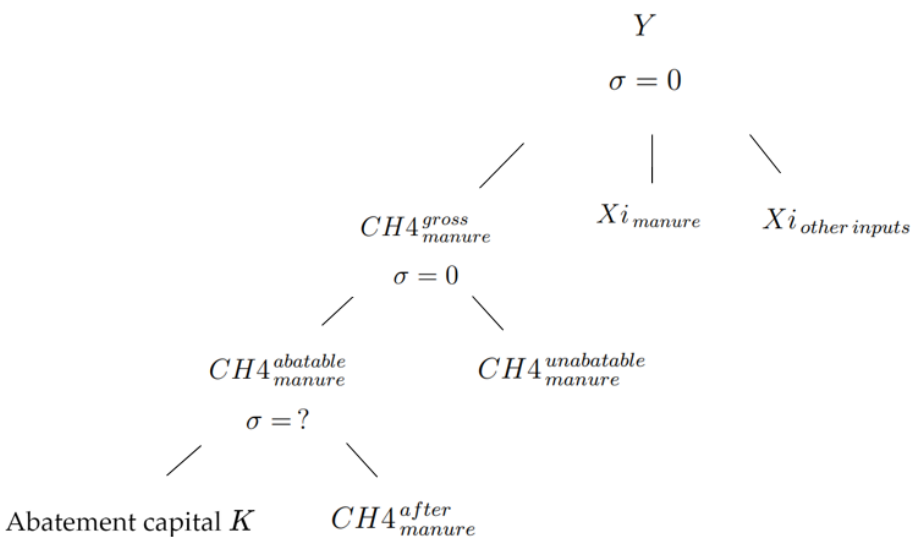
$$P_A \cdot CH4_{abatable}^{manure} = P_K \cdot K + \tau \cdot CH4_{manure}^{after} \tag{10}$$

3.2 Calibration Process

In order to include data for technological abatement opportunities in a top-down model, it is proposed in Kiuila and Rutherford (2013) to fit an abatement function to the abatement cost data. In order to implement this method, I assume that the abatement equipment are sequentially applicable such that technology $t-1$ has to be installed before technology t can be installed.

Figure 3.1

Production nest



The structure of the production nest in *model 1* is shown in figure 3.1. Similar to Kiuila and Rutherford (2013), all the elasticities of substitution except the one in the lowest nest are assumed to be zero. The elasticity of substitution between abatement capital and the part of abatable emissions that is left after abatement needs to be calibrated outside *model 1*.

I assume that the observed emissions from the data, cf. section 2, represent net emissions in baseline, i.e. $\overline{CH4^{unabatable}_{manure}} + \overline{CH4^{after}_{manure}} = 2219.3$ kt CO2e. In the present method, baseline abatement is required to have a non-zero level. Thus, one needs to assume a hypothetical baseline abatement level[4]. I assume that the level of baseline abatement is 200 kt CO2e. Similar to Kiuila and Rutherford (2013), the baseline tax is below the marginal cost of the cheapest technology, cf. table 2.1. This implies the following level of baseline gross emissions:

$$\overline{CH4^{gross}_{manure}} = 2219.3 + 200 = 2419.3 \text{ kt CO2e} \qquad (11)$$

The isoquant that implicitly defines the abatement function is given by

$$K = \overline{K}\left(\frac{1-(1-\theta)\left(1-(CH4^{abatable}_{manure} - CH4^{after}_{manure})/(\overline{CH4^{unabatable}_{manure}} + \overline{CH4^{after}_{manure}})\right)^{\frac{\sigma-1}{\sigma}}}{\theta}\right)^{\frac{\sigma}{\sigma-1}} \qquad (12)$$

[4] Kiuila, O. and Rutherford, T.F. (2013), p. 66

where $\bar{K}$ is baseline abatement capital, and θ is given by:

$$\theta = \frac{\bar{K}}{\bar{P}_K\bar{K}+\bar{\tau}\left(\overline{CH4^{unabatable}_{manure}}+\overline{CH4^{after}_{manure}}\right)} \tag{13}$$

The isoquant measures combinations of abatement capital and net emissions that result in the same quantity of gross emissions. The abatement cost when technology t is implemented, K_t, is the marginal cost of technology t times the abatement related to that technology. The data used for calibration, including the accumulated abatement costs, are shown in table 3.1.By normalizing the price of abatement capital $\bar{P}_K$ to 1 and isolating baseline capital $\bar{K}$ in (13), the calibrated value of baseline capital must be:

$$\widehat{K} = \frac{\bar{\tau}\cdot\left(\overline{CH4^{unabatable}_{manure}}+\overline{CH4^{after}_{manure}}\right)\cdot\theta}{(1-\theta)} \tag{14}$$

Table 3.1

Data used for calibration

Technology	Accumulated potential	Emissions	Abatement cost
Acidification$_{swine}$	0.156	2.219	0.318
Biogasification$_{swine}$	0.265	1.933	0.711
Biogasification$_{cattle}$	0.511	1.286	1.598
Acidification$_{beef}$	0.673	0.860	2.378
Acidification$_{cattle}$	0.835	0.434	3.156

Note: Emissions are measured in mt CO2e, and abatement costs are measured in bln. DKK

Source: Dubgaard and Ståhl (2018), Denmark's Energy and Climate Outlook 2020

In order to calibrate the parameters of the isoquant given by (12), I use an OLS-technique to minimize the sum of the squared deviations between the calibrated values and the data values. The baseline capital has to be given by (14) in order to keep the definition of θ in place.

Let $S = \{\sigma, \theta, \widehat{K}, \widehat{K}_t, \widehat{CH4^{unabatable}_{manure}}, \widehat{CH4^{after}_{t,manure}}\}$ be a set of variables. The entire non-linear optimization problem then reads:

$$\min_{x \in S} \sum_t \left[\left(\widehat{CH4_{manure}^{unabatable}} + \widehat{CH4_{t,manure}^{after}} - CH4_{manure}^{unabatable} - CH4_{t,manure}^{after} \right)^2 + \left(\widehat{K}_t - K_t \right)^2 \right] \quad (15)$$

s.t.

$$\widehat{K}_t = \widehat{K} \left(\frac{1 - (1-\theta)(1 - (CH4_{manure}^{abatable} - CH4_{t,manure}^{after}) / (\overline{CH4_{manure}^{unabatable}} + \overline{CH4_{manure}^{after}}))^{\frac{\sigma-1}{\sigma}}}{\theta} \right)^{\frac{\sigma}{\sigma-1}} \quad (16)$$

$\wedge$

$$\widehat{K} = \frac{\bar{\tau} \cdot \left(\overline{CH4_{manure}^{unabatable}} + \overline{CH4_{manure}^{after}} \right) \cdot \theta}{1-\theta} \quad (17)$$

Applying the data from table 2.1 in the problem given by (15)-(17) yields $\sigma = 5.67$ and $\theta = 0.00247$. These calibrated parameters are used in *model 1*.

4. Stephensen et al.

This section outlines a general equilibrium framework and imposes heterogeneity as proposed in Stephensen et al. (2020)

4.1 General Equilibrium Framework

The demand for the composite agricultural good is given by a linear, downward-sloping curve

$$Y = 25000 - P \tag{18}$$

where Y is aggregate demand for the composite agricultural good, and P is the price of the good. The farms produce using Leontief technology such that the input factor demand is

$$Xi_i = \mu_i \cdot Y, \quad i = \text{manure, otherinputs} \tag{19}$$

where Xi_i is the farms' demand for input i, and μ_i is a distribution parameter for input i. *Other inputs* is an aggregate input that contains all other inputs than manure. The components of *other inputs* are left out for the sake of simplicity. The parameters of the production function are $\mu_{manure} = 0.1$ and $\mu_{otherinputs} = 0.9$[5]. The following zero-profit-condition is applied:

$$\sum_i PiX_i \cdot Xi_i = P \cdot Y \tag{20}$$

where the left-hand-side are the expenses of the firms and the right-hand-side are the revenues of the firms. CH4-emissions are given by a constant share of the polluting input *manure*, that is

$$CH4_i^{gross} = \eta_i \cdot Xi_i \tag{21}$$

where $\eta_{manure} = 1$ and $\eta_{otherinputs} = 0$ such that only the use of the input *manure* pollutes. Only abatement technologies for manure management are considered. Thus, it is redundant to include other polluting inputs for the sake of this analysis.

[5] See section 6.4

4.2 Imposing Heterogeneity

By following the procedure proposed in Stephensen et al. (2020) I assume that firms are heterogeneous. Heterogeneity in this sense means that the costs of installing abatement technologies *on average* are given by the values in the column "Cost (c_t)" in table 2.1 but the costs of the individual firms can differ from these values. More specifically, I assume that the costs are log-normally distributed[6]. This assumption implies that the fraction of firms who install the abatement equipment t is given by:

$$Q_t = \Phi\left(\frac{\ln(\tau) - \ln(c_t) + \frac{\rho^2}{2}}{\rho}\right) \tag{22}$$

where Φ is the cumulative distribution function (cdf) for the standardized version of the normal distribution, and ρ is the degree of heterogeneity. The larger the ρ, the more the costs of the individual firms will differ from the average values given in the column "Cost (c_t)" in table 2.1. For $\rho \to 0$ the costs of the firms converge. The limit case is homogeneity. I choose to put ρ equal to 1, but the choice of a more reasonable value is a topic for further investigation, cf. section 6.2. The share of firms who chooses to install abatement equipment t, Q_t, times the potential of t, q_t, which is calculated as the reduction share times the implementation potential, cf. table 2.1, must be the abatement in percent for that technology. By adding the abatement percentages for each of the technologies, I obtain an expression for the total abatement in percent, i.e.:

$$\lambda_q = \sum_{t=1}^{T} q_t Q_t \tag{23}$$

Thus, the CH4-emissions after abatement are given by:

$$CH4_{manure}^{net} = CH4_{manure}^{gross}(1 - \lambda_q) \tag{24}$$

By using (22), integral calculus and probability theory, it can be shown that realized costs' share of potential costs from technology t is given by:

$$P_t = \Phi\left(\frac{\ln(\tau) - \ln(c_t) - \frac{\rho^2}{2}}{\rho}\right) \tag{25}$$

Multiplying the cost of technology t (measured in DKK per ton abated CO2e) with the potential of technology t yields the abatement cost per ton of gross emissions (measured in DKK per ton CO2e) related to technology t given that all firms implement technology t. By multiplying this product with the actual realized costs' share of potential costs regarding

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[6] For the specific distribution and the derivation of (22), see appendix 8

technology t and summing over all technologies, I find the total abatement cost per ton of gross emissions (measured in DKK per ton CO2e):

$$\lambda_c = \sum_{t=1}^{T} q_t c_t P_t \tag{26}$$

By multiplying λ_c with the level of gross emissions, I obtain an expression for the total abatement cost measured in DKK

$$C = CH4_{manure}^{gross} \cdot \lambda_c \tag{27}$$

By adding the total tax cost (measured in DKK per ton of gross CO2e) $\tau_{manure}(1-\lambda_q)$ to the total abatement cost per ton of gross CO2e, I obtain an expression for the total cost related to emissions (considering both tax expenditures and abatement expenditures) per ton of gross CO2e:

$$\lambda = \lambda_c + \tau(1-\lambda_q) \tag{28}$$

The "production price" of $manure$ consists of three elements: 1) the basic input price, 2) the abatement cost and 3) the tax cost. By adding the basic input price of input i to λ which measures the two latter elements, I obtain an expression for the "production price" related to input i[7]:

$$PiX_{manure} = Pi_{manure} + \eta_{manure} \cdot \lambda \tag{29}$$

Since the second (aggregate) input, $otherinputs$, does not pollute, the "production price" of this input is just given by the basic input price, i.e.

$$PiX_{otherinputs} = Pi_{otherinputs} \tag{30}$$

The input price of both inputs are normalized to 1. *Model 2* consists of (18)-(30).

[7] To change the unit of λ from $\frac{dkr.}{CH4_{manure}^{gross}}$ to $\frac{dkr.}{Xi_{manure}}$, I multiply λ with $\eta_{manure} = \frac{CH4_{manure}^{gross}}{Xi_{manure}}$

5. Results

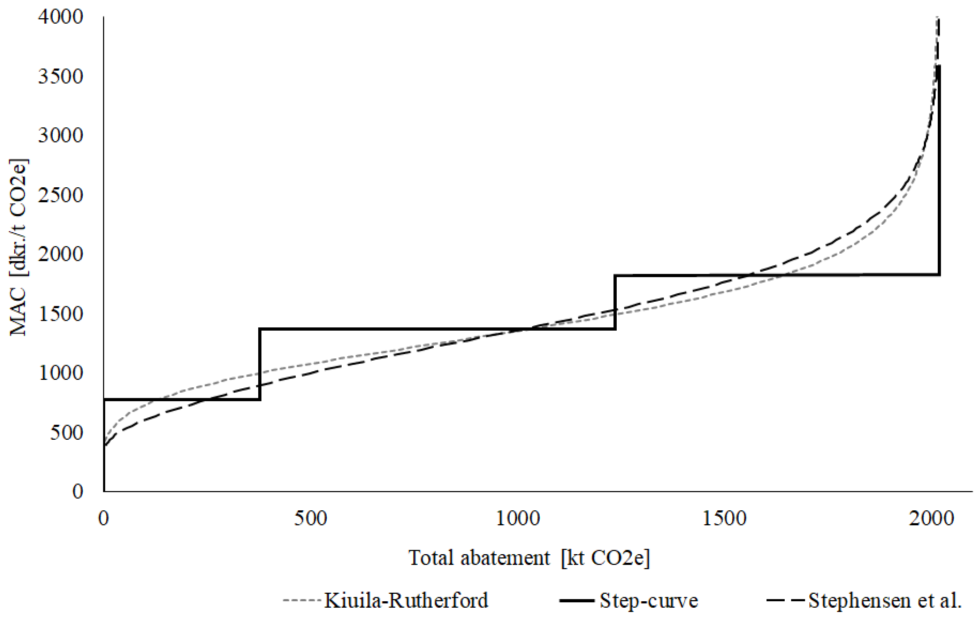
This section compares the resulting MAC curves using each of the two approaches. Moreover, an alternative case with an S-shaped step-curve is illustrated

5.1 MAC Curves

In the following section, 201 different levels of emissions taxation are imposed in *model 1* and *model 2*. By choosing $\rho = 0.3$, a similar curve to the one using the method proposed in Kiuila-Rutherford (2013) is found. The levels of taxation measured in DKK/ton CO2e are $\tau = [0{,}20{,}40, \ldots ,4000]$. The firms will abate, until their marginal abatement cost equals the level of taxation. By plotting the abatement levels corresponding to each level of taxation, I construct the smoothed MAC-curves of *model 1* and *model 2*. These curves are compared to the step-curve consisting of the marginal abatement costs of each technology and the corresponding abatement levels in figure 5.1.

Figure 5.1

Marginal abatement cost curves



Both curves fit the step-curve in a similar fashion. Below are the smoothed curves using the method proposed by Stephensen et al. for different values of the smoothing parameter.

Figure 5.2

Marginal abatement cost curves for different values of the smoothing parameter

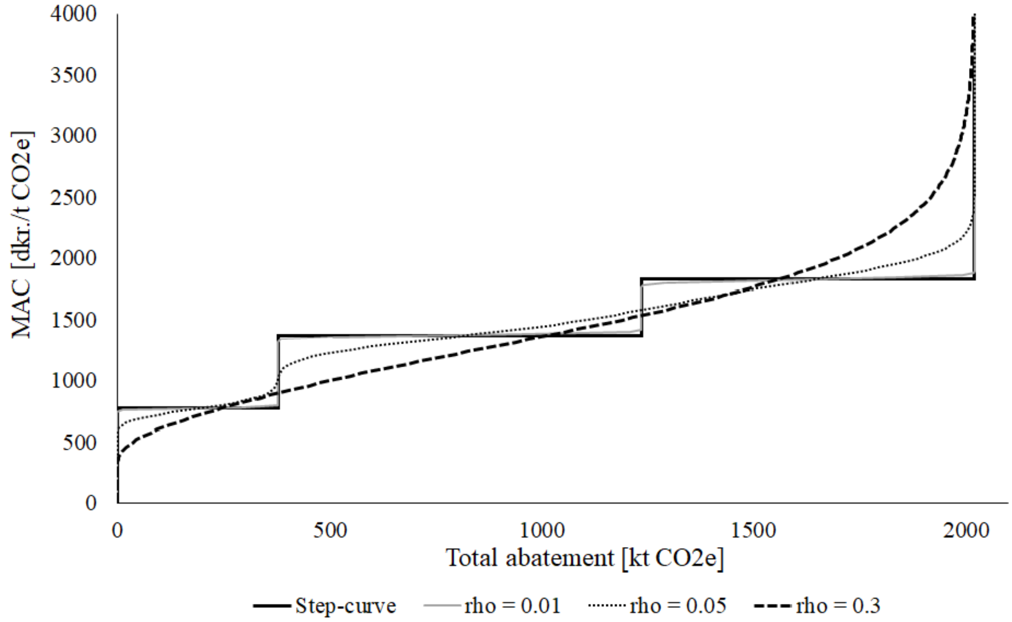


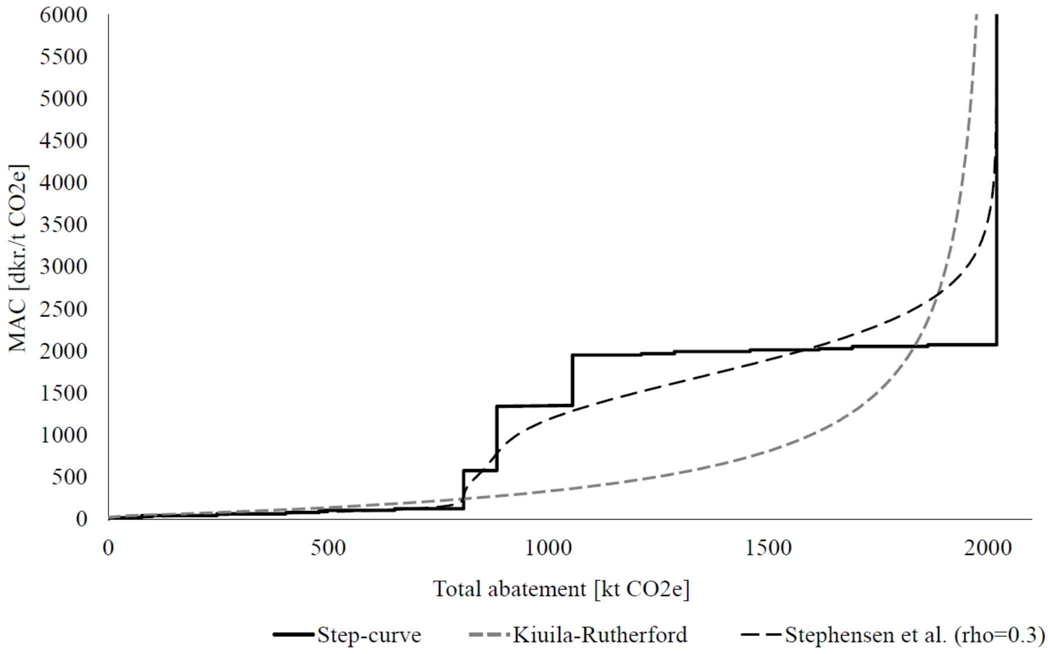
Figure 5.2 shows that it is possible to get arbitrarily close to the original step-curve by lowering the smoothing parameter.

5.2 Alternative Step-curve

When the shape of the step-curve is as in figure 2, both methods do a good job fitting the curve. However, if the step-curve is S-shaped, the method suggested in Kiuila-Rutherford does not smooth the curve very well. To see this, I have generated a step-curve with such a shape and fitted the curve using both methods. The result is seen below.

Figure 5.3

Smoothing of S-shaped MAC curve



The curve resulting from using the method suggested in Kiuila-Rutherford (2013) is simply not flexible enough to replicate the shape of the step-curve properly. Again, choosing a lower value of rho, would bring the Stephensen et al. curve even closer to the original MAC curve.

6. Discussion and Concluding Remarks

This section compares the two methods in more detail and concludes the memo

6.1 Better Fit

The main benefit of the Stephensen et al. (2020) approach is that it makes it possible to obtain a MAC-curve as close to the original step-curve as one desires. In some cases, the Kiuila-Rutherford method yields a MAC-curve that is far from the original step-curve. This difference in fit is very noticeable when the step-curve is "s-shaped".

6.2 Endogenous Technology Catalogues

An advantage of the method suggested in Stephensen et al. (2020) is that it is capable of handling technology catalogues with endogenous unit costs. Even though the unit costs might not be explicitly stated in the technology catalogue but rather determined inside the CGE-model, the Stephensen et al. (2020) approach will still be applicable. On the contrary, the approach in Kiuila-Rutherford (2013) does not render endogenous determination of unit costs possible. In case of a change in the unit costs of one of the technologies in the catalogue, the calibration process outside the CGE model must be carried through once more.

6.3 Transparency

Another advantage of the approach proposed in Stephensen et al. (2020) is that the technologies are implemented directly into the CGE-model. Doing this makes models with technological abatement opportunities more transparent for all involved parties. For instance, it is not possible to specify which exact technologies are installed by firms at a given level of taxation in *model 1*, cf. section 5. This means that one could argue that the approach suggested in Stephensen et al. bridges the gap between engineering and economics to a greater extent.

6.4 Heterogeneity

One interesting aspect of the method proposed in Stephensen et al. (2020) is that heterogeneity is introduced. In the world that we are modelling, firms are heterogeneous. One farm may be able to install a given technology at a cheaper cost than another. For instance, a large farm may be able to acquire sulphuric acid for acidification of manure at a cheaper rate than a smaller farm since the large farm would need a lot more of it. The introduction of heterogeneity thus offers the modeller room for aligning the model more with the real-world. This is an advantage of the method proposed by Stephensen et al. since it provides the modeller with more flexibility. Alternatively, the heterogeneity parameter can be fixed at an infinitely low value if it is convenient that all firms act the same way in the CGE-model. This would ensure that either all firms use a given technology or do not.

6.5 Hypothetical Baseline Abatement Level

Baseline abatement is required to have a non-zero level in order to calibrate the parameters of the abatement function using the approach proposed in Kiuila and Rutherford (2013). More precisely, one needs to assume a hypothetical level for abatement in baseline. Since there is typically no basis for setting such a level, an advantage of the Stephensen et al. approach is that no such assumptions are required to implement the information from the technology catalogue in the model.

6.6 Practical Matters

The explicit inclusion of abatement technologies in *model 2* is not an issue in terms of solution time. I have expanded *both models* with 100 fictive technologies to examine the impact on solution time. I find that *model 2* based on the Stephensen et al. (2020) approach takes slightly longer to solve, but the difference in solution time is negligible. However, the calibration process that takes place outside of the model in the method suggested by Kiuila and Rutherford (2013) is rather time consuming. The solver must be provided with plausible starting values in order to solve the non-linear problem and one might encounter a range of problems during the process. Using the approach suggested by Stephensen et al. (2020), the modeller can include more abatement technologies into the model simply by extending the existing model with abatement potentials and marginal costs for the new technologies. Similarly, updated data can easily be included in the model. Updating data or extending the model with more abatement technologies is slightly more difficult following the technique presented in Kiuila and Rutherford (2013). In this case, the modeller would have to calibrate the parameters of the abatement function by feeding the algorithm with reasonable starting values and solving the non-linear problem once again, cf. section 3.2. The considerations mentioned in this section are thus relevant when choosing by which procedure the abatement estimates should be integrated.

6.7 Limitations of the Approach in this Memo

In order to model the firms' choices of abatement capital in a general equilibrium framework and in order to compare the two selected approaches, an array of assumptions have been made. The approach in this paper has a few weaknesses that I elaborate on below. Firstly, the models in this paper are fairly simple. The modelling of the real world could be more profound. This would require more data and equations. For instance, the distribution parameters could be estimated empirically by utilizing data for inputs in Danish agriculture. Moreover, the composite farms could be split up into different types of farms. The integration of the method with the GreenREFORM model achieves exactly this.

6.8 Concluding Remarks

This memo has explored two different methods of incorporating abatement technologies into a computable general equilibrium model applying data from Denmark's Energy and Climate Outlook 2020, Dubgaard & Ståhl (2018) and Denmark's National Inventory Report (2020). The approach proposed in Stephensen et al. (2020) makes it possible to get infinitely close to the step-curve consisting of the marginal costs of the technologies by setting a low value of the smoothing parameter. Moreover, this method works better than the one suggested in Kiuila-Rutherford (2013) when the step-curve is "s-shaped". Additionally, the implementation strategy described in Stephensen et al. (2020) is more suitable for continuous updating of technological data and endogenous technology catalogues. Lastly, more of the information from the technology catalogue is retained when applying this method.

7. References

1. Stephensen, P., Beck, U.R. and Berg, R.K.: "End-of-pipe emissions: abatement technologies in a CGE-model". DREAM, Economic memo, 25 August 2020.

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A. Derivation of the Central Sorting Function

I assume that the costs for firm j from implementing technology t are log-normally distributed:

$$\ln\left(c_{j,t}\right) \sim N\left(\ln(c_t) - \frac{\rho^2}{2}, \rho^2\right) \tag{31}$$

where ln is the natural logarithm. In order for firm j to implement technology t, the costs must be lower than or equal to the CO2e tax, i.e.

$$c_{j,t} \leq \tau \tag{32}$$

Taking logs of (32) yields:

$$\ln c_{j,t} \leq \ln\tau \tag{33}$$

The share of firms who chooses to install technology t must be the probability that the condition given in (32) is satisfied. Since (32) and (33) are equivalent, the share of firms that chooses to use technology t is

$$Q_t = P\left(\ln c_{j,t} \leq \ln\tau\right) = F(\ln\tau) = \Phi\left(\frac{\ln(\tau) - \ln(c_t) + \frac{\rho^2}{2}}{\rho}\right) \tag{34}$$

where F is the cumulative distribution function for the normal distribution in (31). By subtracting the mean and dividing with the standard deviation of $\ln\left(c_{j,t}\right)$, I obtain:

$$Q_t = \Phi\left(\frac{\ln(\tau) - \ln(c_t) + \frac{\rho^2}{2}}{\rho}\right) \tag{35}$$

where Φ is the cumulative distribution function for the standard normal distribution.