FACULTY OF SOCIAL SCIENCES
Department of Economics
University of Copenhagen

Master Thesis

Christian Langholz Carstensen

An agent-based model of the housing market

Steps toward a computational tool for policy analysis

Supervisor: Peter Stephensen

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Summary

Intro

This thesis develops a beta version of a new agent-based model of the Danish housing market, which is well suited to throw light on a range of aspects concerning business cycles and macro-prudential regulation. In particular, the model is employed to analyze the effects of income shocks, mortgage rate shocks and regulation on debt-to-income limits.

The thesis first presents an introduction to the subject of agent-based modeling. Agent-based models are computer simulations of markets or entire economies, in which the agents are equipped with a set of rules to govern their behavior. An agent-based model therefore constitutes a virtual market, which is potentially a 1:1 representation of a real market. By this property, agent-based models distinguish themselves from traditional economic models, which only holds a single or a few representative agents (if not a continuum of identical ones). As shown in the review, agent-based modeling have shown promising progress in several fields of economics, but it seems particularly relevant for analysis of systemic risk and market transitions, because the dynamics are not conformed to an a priori calculated equilibrium. The model developed in this thesis has been programmed from scratch in the object oriented language C#.

The model

The model comprises around 5,000 heterogeneous agents at each point in time, who are equipped with enough autonomous behavior to keep the housing market evolving endogenously throughout a simulation run. The agents include one bank and the rest is households, who decide to move and thereby trade on the housing market depending on age specific probabilities. Every time a household moves, it chooses between moving to a rental or a freehold. The latter is of heterogeneous quality while rentals only come in one kind. Whenever households move, they have the option of taking a mortgage loan with fixed installments. In each period after the purchase, borrowers pay back on their loans, which increases their home equity. The income process of households is stochastic with a fixed growth term, which means that they have a probability of being unable to service their mortgage debt. If households go into delinquency, they are forced to sell off their freehold while the bank looses money. Moreover, when putting a freehold up for sale, households add a minor premium to the estimated market price. If the freehold is not readily sold, they start downgrading the price. This behavior together with the stochastic income creates continuing fluctuations in the market price. Moreover, the age

dependent probability of moving, income process and freehold quality is calibrated to a Danish context so the behavior of the model can be qualitatively compared with Danish data.

Results

The model turns out to be well functioning in several respects. It generates fluctuations in prices within a plausible range as well as selling times that correspond to the empirical. Housing prices also display auto correlation due to the backward looking pricing mechanism. The model generates plausible age distributions of residents in the two kinds of dwellings. Empirically, young households have a much greater propensity to live in rentals, which declines steadily with age. The same is found in the model. It is also found that low-quality freeholds are priced higher relative to their quality than freeholds of better quality because of a strong demand by young, credit constrained households. These patterns emerge endogenously from the dynamics of the model. The model is shown to be in equilibrium in a statistical sense, as the time series of prices are

The model is analyzed by delivering two different kinds of shocks; interest rate and income shocks. A negative and a positive shock is given in both cases, and it turns out that the effects are not symmetric. The negative interest rate shock does not move prices much whereas the positive yields a marked fall. The interest hike implies a strong fall in the estimated home equity of owners and has a stronger adverse effect on the selling price of low-quality freeholds compared to high quality freeholds. Similarly, the negative income has a pronounced effect on prices and selling time, but the same does not go for the positive. This may be ascribed to the fact that speculation is not implemented in the model. The negative income shock is also shown to have a severe effect on the bank's income stream, which is otherwise not suffering from losses.

Finally, a virtue of the model is its endogenous dynamic in prices. The role of macroprudential regulation on the mortgage market, which has gained much attention in the wake of the financial turmoil of the 00s, is to curb destabilization of housing prices and indebtedness of households. The model is therefore utilized to test if higher debt-to-income limits on mortgage loans result in greater fluctuation of housing prices. When considering the whole cycle of build-ups and busts, the model shows a small but significant tendency for higher volatility when the debt-to-income level is increased. A more pronounced result is obtained when considering only the propensity for collapses. In this case, a debt-to-income limit of 4.5 has a notable destabilizing effect compared to a limit of 3. Such a result is noteworthy for policymakers as the limit seemed to have moved exactly from 3 to 4.5 during the heydays of the 00s.

Preamble

I wish to thank my advisor Peter Stephensen for very enthusiastic and generous guidance during the process. Furthermore, I would like to thank Stephanie Kofoed Rebbe and Allan Carstensen for reading and commenting on the thesis.

All code material for the agent-based model and data analysis is available for download by following link: `http://1drv.ms/1Q7yPw3`

Contents

1 Introduction

The boom and bust in housing prices in Denmark during the 00s were dramatic. The boom was accompanied by a substantial leveraging of households, enabled by financial innovation and lucrative borrowing conditions. When the financial crisis hit the Danish economy in 2008, housing prices were already slowing down, but they plummeted when credit conditions tightened. The ensuing adjustment phase of the housing market was far from smooth. For example, the number of available houses nation wide rose from 15,000 in 2006 to 40,000 in 2008, which has been the level since. The average sales time for houses similarly increased threefold without falling subsequently. And from the second to the fourth quarter of 2008, the number of sold houses fell by 50%. These statistics cover strong regional differences, as the market in the capital area experienced the most severe heating up, but also a fast recovery, whereas the slump lingers on outside the urban regions.[1]

Such gyrations call for economic models that are apt for analysis of policies to promote stability on the housing market. These models would do well in capturing (i) the wide degree of heterogeneity in houses and households acting on the market (ii) frictions that make the adjustment to equilibrium time-consuming (iii) endogenous booms in prices and (iv) credit limitations. (i) is important because empirical analysis have shown that various subgroups have different ways of responding to key variables on the housing market[2] and the effects of policy on subgroups of the population may be interesting to know for the policy maker; (ii) matters because understanding the adjustment process better may prove to be valuable input to macroeconomic forecasts and general analysis; (iii) and (iv) are obvious as we wish to regulate with the latter to avoid the former.

This thesis develops a proposal for a model that embodies (i)-(iv). More specifically, the proposal is an agent-based model, which is loosely speaking a one-to-one simulation of a market, where many heterogeneous agents act autonomously. The heterogeneity defining households on the actual market is readily incorporated into the model because of the flexibility of agent-based modeling that is thoroughly discussed in the following section. Furthermore, agent-based models are well-suited to analyze the out-of-equilibrium behavior of a market, because there is no explicit reference to such an entity in the decision making of agents. The model therefore encompasses (ii) and (iii). Lastly, the model includes a bank that sets borrowing criteria on mortgage credit in accordance with those of Danish banks, whereby (iv) obtains.

[1] Statistics obtained from the Association of Danish Mortgage Banks, `www.realkreditraadet.dk/Statistikker/Boligmarkedsstatistik/Data.aspx`

[2] Cf. Mian and Sufi (2011) and Landvoigt et al. (2015)

The model is found to be working well in several respects; it generates plausible patterns for the time it takes to sell a house, for the endogenous changes in housing prices and the age distribution of people living in freeholds and rentals. When delivering shocks to the market, the model is able to provide non-standard statistics on the transition phase such as the losses of the banking sector and the age of first time buyers. As such, the thesis aims at making a contribution to the agent-based literature by producing a calibrated model of a housing market for policy analysis, which there only exists one of at the moment, cf. Geanakoplos et al. (2012). As the field of agent-based modeling is young in economics, different specifications of models on the same subject are still needed to make clear how varying designs affect dynamics.

Second, the model is applied to test how varying the regulation on debt-to-income limits affect the stability of the market. By Monte Carlo simulations, it is found that raising the allowed debt-to-income from 3 to 4.5 increases the tendency of sudden collapses on the market. Since rising debt-to-income limits was the norm during the 00s, and lately as well, this thesis makes a case for following such a tendency closely.

The thesis is organized as follows: in section 2, I give an introduction to the concept of agent-based modeling and examples of important work within the field. Then a review on similar work on agent-based housing markets is provided. Section 4 presents the thesis' model and section 5 analyzes its endogenous dynamics. Two kinds of exogenous shocks are investigated in section 6, which reflect economic expansions and contractions. Thereupon, we turn to the subject of macroprudential regulation in terms of borrowing conditions, which is first discussed in light of empirical research. Then, the debt-to-income limit is experimented with in the model, to find the effects of varying this policy instrument.

2 A review of agent-based modeling

2.1 Introduction to agent-based modeling

This section presents the philosophy behind agent-based models (ABM) and a host of their applications. Since the agent-based approach to economic models poses a departure from traditional economic models, I will go to some lengths explaining the architecture of interesting ABMs and their usefulness. An ABM is in essence a complex, dynamical system comprising autonomously interacting agents. This system may be shaped to represent a market, an entire economy, the social dynamics in group of people, the behavior of a herd of animals or anything else where separate entities interact. A typical analogy for an ABM is a computational laboratory experiment or a Petri dish, because what we study via ABMs are simulations of real world phenomena like markets in small scale. As in the real world, an ABM of a market consists of a large number of buyers and sellers who act autonomously, making decisions of buying and selling based on their information and behavioral traits. The same goes for an ABM, so if an ABM consists of 10,000 agents, there will be 10,000 actual separate entities making trades with one another, each one acting on its own. Hence, the entire aggregate behavior of the system is out of the hands of the modeler. This implies that equilibrium in an ABM is not imposed or assumed, but must arise by it self, if it is to prevail in the model. This is the crucial departure from most economic models, so in the remaining of this chapter I will explain how simulations by ABMs are carried out, what notions of equilibrium can be applied and what kind of contributions ABMs can deliver to the economics profession.

To get a sense of the meaning behind 'agent-based', it is helpful to focus on the agents as individuals equipped with a set of properties and algorithmic behavioral rules. The properties reflect the state they are in and the rules determine their actions. For example, an agent in an economic ABM may have income and savings as properties, while its search strategy on the market is a behavioral rule. Agents do not necessarily know the behavioral rules or properties pertaining to each other, meaning that they can potentially be private information. Depending on the setting, the behavioral rules will typically make the agents goal-seeking and adaptive to changes in the environment of the model. This is as such no different from classic economic models where behavior is formulated as a maximization problem given the state of the model. The difference lies in the formulation of behavior, which in ABMs can range from very simple rules of thumb to adaptive learning strategies, e.g. as in Kirman (2011). Rules of thumb are implemented in an if-then fashion, e.g. a firm might have specified '*if* a neighboring firm increases prices with success *then* increase own prices by the same amount'. They might also include optimizing behavior such as choosing optimally among a range of alternatives, if a certain event occurs, or simple probabilistic manoeuvres. Note that there is no need for agents to have homogeneous behavior in terms of model design. One may simply decide that a share of agents behave in one way, while another share behaves differently.

When a large number of autonomous and adapting agents interact, the result can be defined as a complex, adaptive system, Tesfatsion (2006). The meaning of 'complex' in this setting is that the system displays emergent properties as a result of the interaction of its entities. Emergent properties are properties that arise endogenously without the designer of the model explicitly writing them into the model. In a model of a market, efficiency or Pareto optimality are such properties. Other examples are autocorrelation of prices, bubbles, crashes and coalitions in networks. Naturally, these phenomena will tend to feed back on the decisions made by individual agents, as they are adaptive to their environment. If a certain phenomenon such as a bubble is to be explained by an ABM, then the model must be dynamically complete in all relevant aspects. Dynamic completeness implies that the modeler only specifies the initial state of the model and then lets it run solely on a basis of the actions undertaken by the agents of the model, Tesfatsion (2006). Specifying the initial state of the model typically involves assigning distributions of endowments, productivity etc. to the agents such that all their properties have values which render them ready to take action. Requiring dynamic completeness does not amount to excluding exogenous shocks to the model in the course of a simulation run. If the shocks are plausibly seen as exogenous in the economic environment that the ABM mirrors, they should be exogenously occurring the simulation as well. One example is the interest rate in a small open economy with a currency peg, which could well have a time path exogenous to the state of the economy in a corresponding ABM.

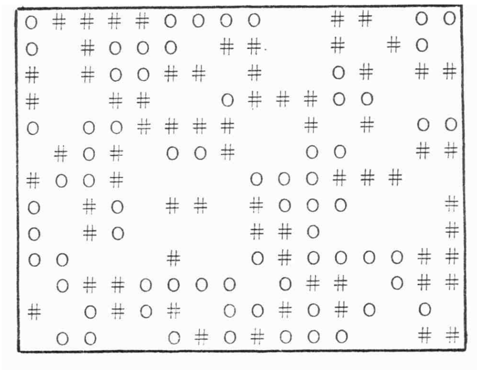
The methodology of agent-based modeling have been applied to a number of research areas which are quite different in nature. A good share of the literature is dedicated to testing economic theory in which the element of complexity and heterogeneity has been ignored or downplayed. Macroeconomic models often rely on the assumption of a representative agent or a continuum of identical agents, which, at least in the minds of many researchers within ABM, is not trivial. The fact that economic agents have to make their decisions based on information that is costly or time consuming to acquire, or even inaccessible, makes room for coordination failures in the market that representative agent models cannot easily encompass, cf. Delli Gatti et al. (2011) or Assenza et al. (2015). A related field in the ABM literature is that of generating new theoretical results, which analytical models have a hard time producing. ABMs have the flexibility that the behavioral rules of the agents can feature arbitrary bounds on their rationality or amount of information and they thereby lend themselves to testing the implications of bounded rationality in an economic system. More light will be shed on these aspects in the following exposition of notable ABMs, beginning with one the discipline's founding fathers.

2.2 A model of urban segregation

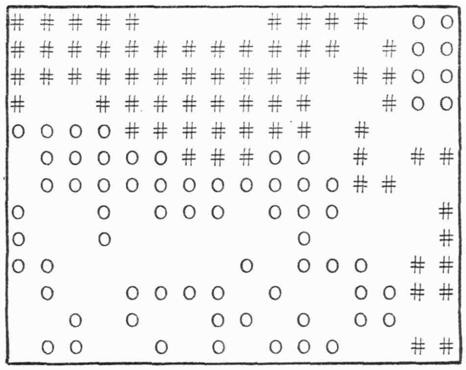
In the seminal work of Schelling (1971), he inquires into the dynamics of segregation between different kinds of individuals. He observes that racial separation into all white or all black neighborhoods in American metropolitan areas is a prevalent phenomenon, even when regarding the average economic difference between the two groups. He further contends that this might not be a result of a strong distaste for mixing with other ethnicities; the pattern of segregation can arise if either group has merely a distaste of being outnumbered in its own neighborhood. In fact, they might have a preference for some mixing, yet that might not be enough to secure non-segregation. To test this hypothesis, he constructs one of the first ABMs applied to social science in which two kinds of agents move around in a number of iterations until an equilibrium is reached. The result is that widespread segregation is indeed a possible equilibrium even under low mixing aversion.

In order to capture the spatial dimension of a city, Schelling employs a grid (or a 'checker board') where each cell of the grid can be either vacant or occupied by an agent. There are two types of agents, one represented by a circle and the other by a hash tag. Initially they are in equal number, but he also tests the model with uneven distributions of the two. All the agents are equipped with one very simple behavioral rule. If the surrounding neighborhood of the cell they occupy is inhabited by more agents of the other kind than their own, they move to the closest cell in any direction where that is not the case. Disregarding the cells along the edges of the grid, each cell has 8 other cells bordering. These 8 cells make up the neighborhood of a respective cell, so an agent wants to move if 5 of its neighboring cells are occupied by the other agent type (if no cells are vacant). At the beginning of the simulation, all agents are spread out at random over the grid. About 20% of the cells are left vacant so that it is possible to move around. Each round of the simulation begins by registering all agents wanting to move. They are then allowed to do so one by one in random order, which concludes the round. When the first round finishes, some of the agents that were previously satisfied with their neighborhood are now outnumbered. These agents are then registered in the second period, moves and so forth. The process continues until no agent is dissatisfied with its neighborhood, which establishes the equilibrium.

The distribution of agents on the grid at the beginning and at the end of the simulation is depicted in figure 1. There is a surprisingly high degree of clustering among the two types of agents. Initially, only 10% of the agents did not have neighbors of a different type while that



(a) Initial distribution



(b) Equilibrium distribution

Figure 1: Landscapes of segregation. Figures are copied from Schelling (1971)

number increases to 90% in the equilibrium, making for a very divided society. The element of randomness is of course important when considering these statistics. Every time a moving agent has more than one satisfying cell to choose from, it picks at random. A whole range of possible equilibrium distributions thus pertain to a single initial state, and their degree of segregation can vary. To demonstrate, Schelling runs the simulation again and finds this time that 40% of the agents live isolated from the other type. These days, a researcher would have performed a Monte Carlo study of the model, running it e.g. 1,000 times with different initial distribution to calculate average statistics. Such a sensitivity test presumably posed a computational burden at the time of Schelling's original research, but the stability of segregation as an outcome has been well established since, cf. Shin and Sayama (2014). It is further demonstrated in the paper, that segregation will still appear when one of the groups is more averse to mixing than the other. In that case, the more averse group end up more isolated from the other type because they cluster in areas without vacant cells. To put it differently, they isolate themselves in ghettos.

To recap on the terminology of the introduction, the agents of Schelling (1971) are adaptive and goal seeking because they constantly investigate their environment in order to obtain a certain state of affairs. They do not learn however, as their own or others' experiences with moving cannot affect their subsequent behavior. The simulation is dynamically complete because all dynamics play out solely on the basis of the agents' autonomous behavior, without any the control of the modeller. Hence, segregation can be said to be an emergent property of the model and inasmuch the model offers an explanation for it. That is, the model explains segregation as an outcome of only quite mildly segregationist preferences. These results, I believe, demonstrate some of the attractiveness of ABMs. The model is quite intuitive, flexible and delivers a result which could not easily have been derived analytically (although recent advances are coming close, see Shin and Sayama (2014)).

2.3 An ABM with learning

Where learning was altogether disregarded in Schelling's model, an implementation of such a behavioral pattern is exactly what drives the dynamics of model by Kirman and Vriend (2001). They build an ABM representing a specific market for fish to explain how the observed price dispersion and loyal relationships between buyers and sellers can emerge. Their model is interesting because the emerging behavior of agents in the course of a simulation resembles the observed behavior of the market participants, which by standard economic reasoning seem puzzling. Learning is however not an easy phenomenon to model, so the behavior of agents no longer holds the simplicity and elegance of Schelling (1971) .

The market they model is the fish market of Marseille, for which there is quite detailed data. This market is populated by fixed number of local sellers and buyers which trade repeatedly every day. The sellers of fish communicate their current prices in a take-it-or-leave-it fashion to interested buyers without posting them publicly. Therefore, sellers can, and do, discriminate prices between buyers, who are not aware of other traders' price offers. Sellers are even observed to charge different prices during a day to the same buyer. The supplied fish is categorized by a common quality measure, and since all retailers are professional, repeat participants, informational asymmetry concerning quality does not seem to be a likely distortion. Given this market structure, it is at first glance surprising to find that the majority of buyers are loyal to a single vendor meanwhile price dispersion is sustained.

The exact details of behavior in ABMs are often too many to outline in a literature review. This ABM is no different, so the following exposition cuts some corner, yet keeps the aspects of decisions and learning relatively detailed. The level of detail should expose the flexibility of agent based modeling to the reader. The model consists of 10 sellers and 100 buyers. In each day of the simulation, the market has two sessions - the morning and afternoon. Buyers first try to purchase their unit of fish in the morning, but some will not be served or reject the offered price and thus try again in the afternoon. The buyers then resell their purchased quantities outside the market at a common price p^b (in restaurants etc.). Likewise, sellers obtain their supply at a common price p^s. In each session, buyers gather around sellers who then determine the order in which they will be served. As will be clear, learning comes into play when sellers decide who to serve next, when reservation prices are determined and when buyers choose sellers to contact.

In every session, buyers and sellers each have a set of decisions to make. Buyers have to choose which seller to contact (they only contact one) and what prices to accept. Sellers have to choose which price to charge and how to order the buyers at their stand. Each of these decisions are formulated as a simple rule of action for a well specified range of possible scenarios. Whenever a choice is to be made, the agent will have competing rules of action in mind. In order to pick a specific one, he remembers the past performance of all rules, given by the value s_j for rule j, and makes a random choice, where the probability of each rule depends on s_j. The measure of performance varies between decisions, but one of the applied measures is simply the revenue after reselling the fish.

To be specific, consider buyers at the start of a session. They have the 10 competing rules in mind: 'contact seller 1', 'contact seller 2', ..., 'contact seller 10'. Every time a specific rule is followed, the buyer updates its corresponding performance measure in the subsequent period by the function

$$s_{j,t} = (1-c)s_{j,t-1} + c\pi_{t-1}, \quad c \in (0,1), \quad \pi_t = \max\{p^b - p_t, 0\} \tag{1}$$

To settle on a specific rule (which seller to buy from), buyers run through all rules and assign the value $b_{j,t} = s_{j,t} + \varepsilon_{j,t}$, $\varepsilon_{j,t} \sim N(0,\sigma)$ to each, letting the rule with the highest number win. This is the basic learning process of all the decisions made by agents. It ensures that agents will take actions that have proved to be profitable, though not in a deterministic fashion. In that way, they are still prone to test previously inferior rules and adapt if they now prove to be profitable.

Buyers also have to decide whether to reject or accept every possible price offer; if they reject an offer, they will walk away from the session empty handed. Prices only come as integers in the set {0,1,2,...,20}. For each of the possible prices, a buyer has one rule that accepts and one that rejects, because both actions are possible given an offer. This time $\pi_t = p^b - p_t$, if a buyer accepts a price in the morning. If he rejects it, the payoff value depends on the revenue from the afternoon session, such that the zero payoff in the morning session is not artificially downgrading s_j.

When sellers consider how much fish to supply, they choose between amounts of {0,1,...,30}, yielding 31 rules. They decide on a rule like buyers and whenever a specific rule j is followed, the seller updates s_j with π equal to total revenue. Next, sellers have to serve the buyers gathering at their stand in a certain order. They do so by keeping track of the loyalty displayed by buyer j with the index $L_{j,t} \in [0,1]$. This index is a weighted average of how many days in the past buyer j has visited the seller, where 1 represents a buyer who comes to the seller's stand every day. After calculating $L_{j,t}$ for all buyers at his stand, the seller makes a random choice of whom to serve next. He does so by assigning the value $(1+L_j)^w$, $w \in \{-25,-20,...,20\}$ to each customer, which is then proportional to the probability of getting served next. The choice set of w represents the rules that the seller needs to choose between when organizing the queue of buyers. If he sets $w < 0$, he puts loyal customers at the back of line, possibly trying to attract newcomers. He is indifferent if $b = 0$ and if $b > 0$, he rewards loyalty. Sellers determine the value of w the same way as in (1), though this time with π equal to the ratio of actual revenue to highest possible revenue.

Lastly, sellers must choose what price to charge a buyer. In this decision, a seller considers both the buyer's loyalty, how many buyers are left and the remaining amount fish in stock. He takes the ratio of remaining buyers and remaining fish and groups into three categories, 'high', 'medium' and 'low'. The same three categories are applied to the loyalty index. For every of the 9 combinations between loyalty and the ratio of remains, the seller has to choose between 21 rules of pricing - 'set $p = 0$', 'set $p = 1$' etc. This choice is performed as before by calculating b_j for each. In that way, customers at the same stand might experience different price offers. When a day concludes, sellers update the value of s_j attached to the rules they employed, using the share of times without rejection by buyers as the payoff value.

The model outline above features the distinct characteristic of agent-based modeling that there is no reference to aggregate measures in the behavior of agents. I.e., there is no single market clearing price and the total supply is not directly a function of the total demand, as is standard in economic models. Agents act decentralized and on the basis of their past experience. Whatever market outcome emerges in the model is thus a product of these completely decentralized actions. To that end, it is quite interesting that stable relationships arise between sellers and buyers, even as the model allows the opposite to happen. Loyalty is then truly an emergent property of the model because that concept is not even a separate feature in the mind of buyers, while sellers have the option of ignoring or discouraging it. The development of loyalty among buyers is depicted in figure 2, where we note that the constant level of loyalty is established around

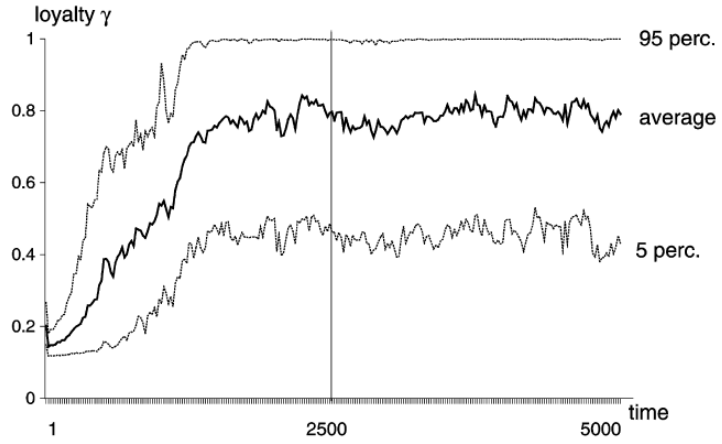


Figure 2: Evolution of loyalty among buyers in morning sessions. Figure taken from Kirman and Vriend (2001)

one third into the simulation. The reason for loyalty has of course a clear equilibrium-like explanation; both buyers and sellers are better off. As the authors demonstrate, average payoffs are higher for both buyers and sellers when buyers are loyal because fewer trades fail. Trades fail when buyers cannot accept the prices charged, but this misalignment is mitigated when buyers repeatedly visit the same seller because they both learn to coordinate on prices.

2.4 Macroeconomic ABMs

The objective of creating a full fledged ABM of an economy with a reasonable number of firms, credit institutions and households in which all behave autonomously and in a theoretically reasonable way is clearly challenging. As there is no need to impose general equilibrium and rational expectations in an ABM, this approach has been suggested as well suited to analyze the dynamics of economic crises. In that spirit, two major research programmes have been funded in the wake of the recent financial crisis to develop macroeconomic ABMs oriented towards policy analysis. The first was EURACE in 2007, which resulted in the EuraceUnibi model, and the second was CRISIS in 2011 which has yielded a number of recent publications on agent-based modeling and other topics of complex systems.[3]. One of the interesting features of the EuraceUnibi model is that it includes two separate regions (i.e. countries) engaged in trade between which workers can migrate. Thereby it lends itself to testing economic policy such as the cohesion policy of the EU, which is supposed to foster economic convergence between the

[3]For example: Aymanns and Farmer (2015), Klimek et al. (2015), Assenza et al. (2015)

member states. This is undertaken in the paper by Dawid et al. (2014), where they set up the model with one region lagging behind in productivity of workers and capital. Then policies of raising human skills and subsidizing investment in new vintages of capital in the lagging region are implemented. The authors find that the former policy only contributes positively to convergence if the labor force is not too mobile across regions. Otherwise, labor force is attracted to the leading region, which triggers wage inflation in the lagging and hence a loss in competitiveness. The policy of subsidizing investment is however effective as long as firms invest in capital close to the technological frontier.

One of the notable macroeconomic ABMs is the Mark I model, which is the foundation of much of the agent-based research in the CRISIS project. The Mark I model as presented in Delli Gatti et al. (2011) has mainly been developed by Delli Gatti a number of collaborators throughout the last half of the 00s. Since it features more or less all standard entities of an economy, it is in a sense an all purpose model for macroeconomic analysis. The model particularly features a market for credit with a number of active credit institutions, so investigating the role of the financial sector in macro dynamics is a primary appliance. It is indeed envisioned as an alternative to the DSGE-models of monetary and fiscal policy, but with a higher degree of richness of market interaction and no imposition of equilibrium and rational expectations. As stated by the authors of Delli Gatti et al. (2011), the ambition of the model is that its markets must be entirely self-organizing by the local interactions of agents and any equilibrium state must hence develop endogenously. The model holds three types of agents (workers, firms and banks) and three markets (consumption goods, labor and credit). The markets are organized such that the demanders draw a random sample of suppliers and contact them in descending order according to their posted price.[4] E.g., firms lacking retained profits to pay their wage bill will contact the cheapest bank they 'know' first and apply for credit. Banks do not have endless credit, so they reject the most financially unsound firms, who then contact the next on their list. Firms unable to pay their wage bills, or have negative net worth when the goods market closes, go bankrupt. Simulations of the model show that the aggregate statistics such as unemployment, inflation productivity and wage follow normal empirical patterns. The model further produces such regularities as the Phillips- Okun and Beveridge curve, and the distribution of firms is right skewed. Simulations further reveal endogenous recessions of the economy, which are triggered if a handful of large firms go bankrupt. When a firm goes bankrupt, it delivers a local demand shock to the economy as the employees of the firm now rely on subsidies, demanding less goods from other firms. If the firm was borrowing, it further hurts the equity of its bank which now

[4]The labor market is of course flipped, such that workers first contact the firm they know with the highest wage offer.

lends out less and thus decreases the survival probability of other liquidity constrained firms. In that way, a firm may drag down other firms and banks with it in the fall, especially if the economy is in a financially fragile state, escalating the process into a recession. One might then say that the business cycle of the Mark I model depends more on the systemic risk of the economy in comparison with classical DSGE-models, where a business cycle is initiated by a shock to productivity, interest rates or mark-ups, cf. Christiano et al. (2010). The Mark I has been further improved by Assenza et al. (2015), who add capital production to the model and let the bank optimize between investing in a safe assets or lending to firms. The inclusion of capital and the pertaining moderation of firms' decisions lead to an autocorrelation of simulated GDP, which is in the range of empirical results. Furthermore, the magnitudes of autocorrelation and volatility of GDP, consumption, and investments are in the correct empirical order. This model represents one of the most compelling macroeconomic ABMs for policy making to date, but as the authors note, the practice of econometric estimation of ABMs is in its infancy, so the model is still not completely operative for that purpose.

2.5 Other applications of ABMs

The agent-based approach in economics has further been employed in such areas as systemic risk and institutional design of markets, forecasting and econometrics. An example of an ABM as a forecasting tool is given in Geanakoplos et al. (2012), who note that an ABM developed on Wall Street performed very well at predicting foreclosures in the U.S. housing market in the 90s. Another example is Young et al. (2014), where an ABM convincingly forecast the spot price of electricity in the New Zealand Electricity market. In the field of econometrics, ABMs can serve as data generating mechanisms for aggregate times series on which the validity of representative agent models can be tested, see Chen and Yeh (2002) and Chen et al. (2012).

Its applicability on analyses of systemic risk and institutional design is obvious given the easiness of modeling herd behavior, externalities and feedback mechanisms. A few examples of research in this area is given by (i) Klimek et al. (2015); (ii) Tesfatsion (2010) and (iii) Bookstaber et al. (2014). In (i) the Mark I model is devised to encompass three different schemes of dealing with failing banks - either a bail-out where taxpayers fund the saving of the bank, a bail-in where the burden is carried by the remaining banks or lastly letting the bank fail with the remaining banks to take over its loans. The impact of one bank's failure on other banks turns out to depend on the state of the economy, because banks' credit supplies are burdened by firm bankruptcies in down turns. The authors find that the bail-in scheme is preferred in downturns while letting the banks fail is better in stable periods. The ABM in (ii) is used to test the implications of changing the pricing mechanism in the U.S. whole sale electricity market. The new pricing mechanism is supposed to mitigate congestion of the transmission grid by letting the price of providing power at a particular point on the grid represent the least possible cost to the system as whole. Thereby, the spatial dimension of the model is crucial, as congestion at one point of the grid can affect the costs inflicted upon the neighboring points. (iii) is a model of a financial market developed for the Office of Financial Research at the U.S. Treasury to assess how shocks in the financial sector propagate through the network of different actors such as hedge funds, dealers and liquidity providers. The model is reminiscent of network models used to study the systemic risk of the financial market, but its contribution is to let the elements of the network have the ability to act on the flows between actors and the associated risk so as to change them. Thereby, the model is able to integrate the behavior of market participants in a trading network and asses the contagion of the market if one or more participants experience a shock.

2.6 Object oriented programming and agent-based modeling

As the exposition of ABMs in the previous section should convey, an ABM is merely a set of rules that govern agents behavior together with their states. The way to 'solve' model is to write a number of algorithms in a programming language that represent each of these rules and then execute the program. The behavior of agents in a given ABM might have a number of different algorithms which are all valid alternatives, so it worthwhile to underline that an ABM as piece of economic theory is its set of rules and not the computer script that carries the simulation. This is not to say that the programming of an ABM is insignificant to the usefulness of the model. Implementing the wrong algorithms can slow down the simulation enough to render Monte Carlo analysis impossible or limit the number of agents to an uninteresting amount. Several platforms with well suited algorithms for many-agent systems have therefore been developed to implement ABMs efficiently and more pedagogically than by writing the source code from scratch. These platforms do tend to constrain the user's possibilities, as he must employ the predefined templates of their libraries. Further, the platforms are primarily designed for ABMs of engineering and biological systems, so most of the work in economics is carried out without the help of platforms. In theory, however, an ABM is invariant to the programming language in which it is formulated, yet it is no coincidence that the vast majority of the platforms and ABMs are written in languages belonging to the object oriented paradigm.[5] The principles behind object-oriented languages make these specially well suited for agent-based models, because they allow a quite intuitive mapping between the theoretical ABM and its implementation in code.

[5]See `https://en.wikipedia.org/wiki/Comparison_of_agent-based_modeling_software`

These principles are in a sense part of the methodology of agent-based modeling, as they help shape the thinking of the modeler. The ABM I present in the subsequent sections is developed in `C#`, a very widely used object oriented language, and therefore, the basics of object-oriented programming (OOP) is described below.

The philosophy behind OOP is that the underlying structure of software should mirror the structure of the real-world. One may regard the real-world as entirely made up of objects of many different types and each of these objects can be a composition of other types of objects. Consider as an example the King's Garden; it is an object of type 'garden'. It is also a composition of many different types of objects such as 'grass', 'flower', 'tree', 'hedge' etc. These object types may come in different sub-types, e.g. the trees are of sub-types 'oak' and 'chestnut', which again are compositions of objects such as 'leafs' and 'trunk' and so on. It is a philosophical question whether this ontology of the world is truly meaningful, but it is definitely a schema which lends itself to the logical structure and consistency on which computer programming must rely. By that token, everything the user actively interacts with in an application is an object of a type; this may be players of a game, menus of a text editor, the mouse cursor or buttons on a web page. Of course, the source code of an application will employ many more objects under the hood than the user has interaction with, such as databases, connections to other software and objects created to support the visible objects.

All objects have the same general architecture in that they consist of a set of methods which make up their functionality, and a set of properties which determine the state they are in. An agent of an ABM might e.g. have methods of buying, selling, searching and posting prices together with the properties of income, wealth, stock of supplies and so on. Every object is a instantiation of its class, which is the abstract definition of the functions and properties belonging to this kind of object. This means that a specific agent can be an instance of the class 'household', in which the code describes what actions the household can undertake and what states it possibly obtains. The ABM may have several households interacting, in which case they are all instances of the same class and have the same functionality. Yet their states, meaning the values of their properties, need not be the same as these are functions of the events experienced by the individual households. What it specifically means for an object to be an instance of a class, is that every time the code which makes up the class is executed, a new address in the memory of the computer is created together with a pointer to it. The address contains all the information of the object and the pointer is what other objects has access to when they interact with that object. In that sense, objects are said to be reference types, as one only works with the reference to an object. The beauty of this is that the reference can be invoked any number of times and reused in the code and in different contexts.

To operate with objects is as such not unique for OOP, but what distinguishes OOP as a design philosophy of software is the four main concepts applied to the classes of objects:

- Inheritance
- Abstraction
- Encapsulation
- Polymorphism

Inheritance means that classes have a hierarchical structure since one class may inherit the methods and properties of another class in addition to those specified in itself. This allows the programmer to avoid replication of code when two or more almost similar classes need to be defined, as she can write just the one and let the others inherit all the functionality they need. In the ABM example, buyers may have much in common with sellers, so it is only necessary to code that functionality in the seller class and have buyers inherit from sellers. Too much repetition of code quickly turns a program chaotic and difficult to change, so programmers duly respect the Don't-Repeat-Yourself principle. In connection with inheritance is the principle of *abstraction*. Abstraction means that the design of a program should focus on what makes each object special in order to reduce complexity in the code. To this end, abstract classes that only work as super classes for other classes are used to implement all the behavior that is merely back ground for their functionality. For example, the programmer may write one general class for market participants and let a class for buyers and sellers respectively inherit from it, such that the code in each class only exposes what makes these types distinct.

Encapsulation is the principle of hiding unnecessary or private information of an object. Details about the implementation of its behavior should not be exposed when interacting with the object; other objects should only know what it does rather than how it works. If a buyer agent, e.g., makes a purchase from a seller agent, its methods used to organize supply stock and balance accounts should not be revealed to the buyer, as this is private information to the seller. The buyer should only care about submitting a bit to the seller, which is to say that its class should only contain functions for that part of the transaction. The seller should take full responsibility of evaluating the bit and delivering the product, so its class needs only code for that part. Some of the properties of an object may as well be private information, willingness to pay for example, and in that sense, the objects preserve a level of integrity. In a market setting, agents might thereby be designed to withhold information from one another as they do in real life. As a more general programming philosophy, the desire for objects' autonomy stems from need to make code easily applicable. It makes it much easier for programmers to collaborate if it suffices to know only what objects do when applying them to the task they are designed for. *Polymorphism* is a little too technical to go in depths with here, but in short, it means that the design should seamlessly allow objects of different types fulfill the same task, depending on the setting, even though they may execute it in different ways. These four principles constitute the cornerstones of any object oriented software design, yet the exhaustive list of design patterns is manifold longer. It is not necessary to follow this philosophy to build an ABM, but as should be clear, it is an obvious choice.

3 Agent-based models of the housing market

The literature on agent-based modeling for housing markets is unfortunately scarce at this moment. A thorough literature search returned three attempts to develop an economic ABM with focus on the dynamics of the housing market. Nevertheless, the most significant of them, Geanakoplos et al. (2012)[6], is probably also one of the most ambitious ABMs to date with regards to behavioral detail and empirical relevance. It features no less than 9 authors and implements a range of interesting micro-data to model the housing market of Washington D.C. Metropolitan Area. The stated aspiration of the authors is to make the simulation a 1:1 mapping of the actual housing market. That is, they want to create a model that fully encompasses the empirical heterogeneity in households' expenditure on housing, borrowing conditions and demographic variables. The second model explained in this section is by Erlingsson et al. (2014)[7], who develop a macroeconomic ABM including a housing market which is susceptible to endogenous crashes due to over-borrowing by households. Lastly, there is the paper by Ge (2013) whose ABM of the housing market centers on the behavior of speculators and the leniency of the mortgage system.[8] The following exposition of these three papers will serve as motivation for some of the choices made in the construction of this thesis' ABM.

3.1 Investment in housing and consumption

The demand for housing is modeled differently in all three papers. The general modeling philosophy in Geanakoplos et al. (2012) is to fit decisions as closely to data as possible, such that (conditional) distributions of variables in the model resemble the empirical. For example, the authors use a panel of household expenditures for the years 1997-2009 to estimate the desired

[6] The details behind the model is explained in the companion paper Axtell et al. (2014)
[7] The model is also employed in Erlingsson et al. (2012)
[8] This is a working paper but the model is further employed in Ge (2014)

expenditure on housing given their income and a range of macro variables. This is motivated by the general rule of thumb, that households spend one third of their income on housing, just fitted to the actual empirical distribution. More precisely, desired expenditure for a household is given by

$$P^* = \varepsilon \times \frac{38.8 \times \text{income}^{0.56}}{\tau + c + \text{LTV} \times r - 0.16 \times \text{lag(HPA)}}, \tag{2}$$

where τ is housing taxes and fees, c is maintenance cost rate, LTV is average loan-to-value of newly issued loans for the past three months, r is the average 30-year mortgage rate and lag(HPA) is last year's appreciation in the Case-Shiller index. To fit the share of heterogeneity in housing expenditure not captured by income, the additional term $\varepsilon \sim \mathcal{N}(-0.13, 0.46^2)$ is added. Consumption of other goods and services is also determined by an empirical relation between income and consumption, though more simple than that of housing. Lastly, savings are determined residually when housing and non-housing expenditures are subtracted income. As such, household are not engaged in an explicit optimization procedure when choosing their amount of housing services and savings.

In the model by Erlingsson et al. (2014), households similarly do not choose their housing investments by a standard optimization problem. Instead, a random sample of households are selected in each round to buy a unit of housing on the market if its affordable. Houses in the model come as discrete, homogeneous units which can be owned in any number. These units are meant to represent rooms, so all households initially own five units constituting a house of average size. Thereby, investment in housing services is still 'lumpy', because arbitrarily small amounts of housing services cannot be traded, yet the units are still more fine grained than in the other models in this review, as well as in the real world. When selected to trade on the housing market, households browse through all posted housing units for sale on the market and picks the cheapest unit they can find. In their decision of how much to spend on housing and consumption, households are governed by a precautionary savings motive whereby they seek to hold constant their amount liquid, non-housing, savings relative to income. This ensures that the households have a constant buffer against negative income shocks.

A more classical approach is followed by Ge (2013), where households' preferred balance between investment in housing and other consumption is decided by optimizing a utility function. As with the other models, households are exogenously and randomly forced to buy a house with a fixed probability. In contrast to the other models, households are not saving up, but instead purchase the best possible dwelling out of their annual income.

3.2 Credit extension

The mortgage system is quite elaborately modeled by Axtell et al. (2014), where both fixed installment and adjustable rate loans are extended to home buyers. When applying for a loan to purchase a home, households state to the bank their desired expenditure on housing P^* and their desired LTV. The desired LTV is determined by a random draw from the empirical distribution of LTV conditional on expenditure. This distribution develops quite significantly throughout the time span of the simulation, as it is much more right skewed in 2007 where lending conditions were much looser compared to that of year 2000. After stating preferences to the bank, the bank and the potential home buyer engage in a negotiation about the LTV it will accept given the interest rate and the type of loan. The bank itself is restricted in its portfolio composition of loans by an empirical level of risk and will therefore offer a loan with associated LTV to match that level of risk. The interest rate offered is randomly drawn from the distribution of offered interest rates conditional on the loan type. This division of households' desired loan amount and the bank's objective to manage risk allows the authors to experiment with credit regulation, for example by forcing the bank to pursue a less risky credit extension than the empirical. They find that freezing the leverage of extended loans to the 2001-level, the housing bubble of the 00s is largely ameliorated.

Another interesting feature of this model is that households will convert their loans to a lower interest rate if it falls sufficiently to compensate for a conversion charge. They will do so with a probability that depends on the gap between the going interest rate and the one on their loans. When converting the loan, households may choose to cash out their saved up equity in the property by increasing the leverage of the loan back to the initial level of the mortgage contract. If a borrower experiences a fall in income which renders it unable to service its debt, it first tries to convert the loan to a lower interest rate. If that does not get the borrower above water, it may choose to perform a strategic default whereby the bank claims their property in compensation. Alternatively, the bank will foreclose the mortgage contract, claim the property and sell it off on the market. Either scenario happens with probabilities according to the respective empirical distributions.

Credit giving is less complex in Erlingsson et al. (2014), where loans only come as adjustable rate mortgages. The economy holds multiple, competing banks in the financial sector which conform to the Basel II regulation (the equity base must constitute 8.5% of total lending). Households are restricted in their borrowing by the requirement that debt services may not exceed a varying fraction of 0.2 to 0.4 of their income. If they receive a negative income shock that increases the burden of servicing debt to an excessive fraction of their income (60%), they put their units of housing on the market at a price below the going rate to quickly bring down debt. If a shock is so severe that debt services amount to 70% of a borrower's income, the bank is forced to write off the debt and take a loss. Since banks have to cover at least 8.5% of lending with their equity, the write-off reduces the bank's lending opportunity which potentially leads to declining loans to new potential borrowers. Each of the bank sets their interest rate as a markup on the discount rate decided by the monetary authority, who follows a classical Taylor rule of inflation and unemployment.

In Ge (2013) credit extension is governed by the forces of a perfect credit market on which a risk free asset must yield the same interest as mortgage contracts, corrected for the endogenous probability of default due to a no arbitrage condition. Hence, the bank estimates in each period the probability of default on mortgage contracts and sets its interest rate so as to be indifferent between investing in a safe asset or risky mortgage positions. One of the key mechanisms in the model is therefore, that rising property prices imply lower interest rates, because the value of collateral increases, which then which feeds back on housing demand. The collateral rate of newly extended mortgage contracts is determined by the required down payment, which is exogenous to the model. When experimenting with this parameter, it turns out that a down payment requirement of 0.1 of the sales price allows for repeated endogenous crashes in housing prices, while a rate of 0.2 ensures stable conditions on the market.

3.3 Income process

The income process of households is likely to be an important feature of a model of the housing market because houses become savings objects when borrowers pay back installments on their debt. Therefore, one would expect that housing investment and expectation of future income are interrelated. At least, the choice between renting and owning one's home is likely to be influenced by expectations of future income. For example, in the questionnaire by Andersen (2011) 60% of renters between 30 and 44 years report that the main reason for not owning a home is that they expect higher future income. Nevertheless, the three models under consideration do not model expectations of income. In Erlingsson et al. (2014), the income process is quite non-linear as it depends on the earnings of firms and employment status. The performance of firms depends on the demand for consumption goods which again is partly determined by the endogenous movements on the housing market. The housing market affects demand since there is a wealth effect on household consumption from changing housing prices. Hence, households cannot perfectly foresee their future income and when credit is extended to them, the only factor that decides their borrowing opportunities is their current ability to service debt. Similarly in

Ge (2013), where the income process is exogenous to the model and in each period drawn from a non-changing distribution. Expectations of future income might have proved interesting in Axtell et al. (2014) since household have a stochastic income process which could be forecasted. It is modeled as in Carroll et al. (1992), where income grows at a fixed rate but is exposed to inno vations that are transitory as well as innovations with lasting effects. The size of innovations are estimated on micro data for Washington D.C. This is also the same approach as will be pursued in the model of this paper.

3.4 Market protocol

On a housing market, one basic assumption on buyers that should seem reasonable is that they have bounded knowledge of the market. Searching the market is time consuming and costly, so they will settle with less than full information of potential trades when deciding on a purchase. It is not obvious how restrictive the bounds are, however, as crude information is readily available in newspapers and real estate agents provide counseling to buyers, which relieves the search burden. The compromise is that households are given close to full information in most of the housing market ABMs reviewed. The housing market of Ge (2013) is divided into distinct regions in each of which an auction is held every period. To find a suited house for purchase, buyers search through only the regions where last period's clearing price is within their budget restriction. They submit an offer for a house in their preferred region g given by

$$bid_{i,t}^{g} = (1 - \eta + \delta \times \mathrm{TOM}_{i,t}) \times P_{t-1}^{g} \tag{3}$$

where η is an initial attempt to undercut the market, $\mathrm{TOM}_{i,t}$ is the time the buyer has spent searching the market at time t so δ expresses the buyers' impatience as their willingness to pay increases with waiting time. P_{t-1}^{g} is the market clearing price in region g in the previous period, which is posted in a public protocol visible to all agents. Sellers submit asking prices symmetrically to (3), only with the signs on η and δ reversed. Based on the bids and offers, the 'real estate agent' then settles the market price at which all houses are traded in region g. Buyers and sellers who did not complete a trade in that round wait until next round to give it a new attempt. As such, this model relies on a local auction-structure in the design of its market.

The model by Erlingsson et al. (2014) holds a more simple market protocol, where households acting as buyers are queued in random order to be offered the cheapest unit of housing on the market. The average price of housing units in the previous period is public knowledge and sellers try to earn on the sale in so far they are not forced on the market due to foreclosure. Sellers therefore put a markup on their posted price, given by a factor uniformly distributed in the interval $[1, 1+\sigma]$. Conversely, if the seller is in foreclosure, it will post a price with a factor in the interval of $[1-\sigma, 1]$ times last period's average price.

The market in Geanakoplos et al. (2012) has a more classical flavor as there are no search frictions. Buyers are ranked according to their willingness to pay, P^*, whereupon the buyer with highest willingness is permitted its preferred dwelling on the market first. When sellers put their dwelling on the market, they attach an asking price by the metric

$$\exp\left(0.22 + 0.99 \times \log(\bar{p}^q) + 0.22 \times \log(s) - 0.01 \times \log(\overline{\text{tom}}) + \varepsilon\right), \tag{4}$$

where $\bar{p}^q$ is the average first asking price of the 8 dwellings with closest quality within the past six months. $\overline{\text{tom}}$ is the average time it takes to sell a dwelling, s is the average selling price relative to first asking price and ε is white noise. For each period that passes without getting their property sold, sellers choose between reducing their asking price or removing their dwelling from the market. The size of a reduction is stochastic and follows the empirical distribution of reductions, conditional on the time on market and prior reductions. The households who were looking for a freehold on the market, but were unsuccessful in getting one, either because their willingness to pay was below anything offered or because the bank denied them a loan, move to a rental. They reside in this rental for a random number of months, though not longer than a year, before they enter the market as potential buyers again. The 'rental market' of the model is relatively simple; rentals are either apartments, which all have the same quality, or houses rented out by speculators. Rents are the same for both kinds of rentals and the level is calibrated to make the number of available freeholds in the model match the empirical. In this way, the inclusion of rentals serves to catch those households who did not get a freehold in the market clearing process, and it restrains the development in housing prices by giving households an option outside of buying their home.

3.5 Dynamics of bubbles and bursts

The turmoil on housing markets during the 00s, that shook many of the western economies, renewed the economics profession's interest in the dynamics of housing booms. In that spirit, the three papers considered in this section all contain destabilizing forces of speculation or excessive leveraging. As for example in Geanakoplos et al. (2012), households are able to speculate in the housing market by acquiring additional freeholds and rent these out. The choice of investing in rentals is only based on the current month's yield on the investment and not the future income stream. Investors, which comprise about 9% of households, may take out new mortgages to finance their investments, whereby the ability of investors to leverage has implications for the attractiveness of investing. Specifically, less down payment requirements will tend to increase the yield on investments. High leverage limits for non-investing households should also matter greatly for the development in prices, as low down payment requirements will cause buyers to bid up prices. In line with the theoretical work of Geanakoplos on the effects of leverage standards in asset markets, cf. Fostel and Geanakoplos (2014), the primary objective of the model to test whether the overall increase in leverage among households during the 00s can account for the boom in housing prices. This test is implemented by changing the behavior of the bank such that it freezes the loan-to-value limit at the 1997 level throughout the simulation. It turns out that the price boom is ameliorated to a high degree when the bank does not decrease lending standards. For comparison, the same exercise is performed with the interest rate as it has been vividly debated whether the low interest rate during the 00s was to blame for the housing boom, cf. Glaeser et al. (2010). The model suggests that the falling mortgage rate did have some effect in pushing prices upwards, yet it was not nearly as powerful as that of leverage conditions. Furthermore, when both mortgage rate and leverage limits are frozen at their 1997 levels, the housing boom is almost erased from the simulation. This result is reminiscent of Heebøll and Larsen (2014), who investigates the effect of borrowing conditions on the Danish housing market during the 00s via an econometric model. He finds that the falling mortgage rate only accounted for a smaller part of the price boom, while the introduction of interest-only mortgages was responsible for the lion's share.

Speculation is in Ge (2013) undertaken by households who are randomly selected to care only about the expected return from reselling a house rather than its quality. The expected return is calculated via the change in market price in the property's region and the interest rate. As the regions experience different movements in prices, due to their differing quality, investors will search through all of them to locate the highest expected gain. Furthermore, investors have the option of investing in a safe asset which they will favor if its return is higher than the expected return on housing. As a consequence, there will be periods in which increases in housing prices are not sufficient to attract investors to the market. At some point, enough investors are attracted to get the prices accelerating until they reach an unstable, highly leveraged level. The situation becomes unstable because the bank adjusts the mortgage rate to keep the expected return on lending equal to that of the safe asset. That is, every time a household defaults on its loan, the bank increases the mortgage rate a bit. At the same time, home owners will strategically default if the size of their outstanding debt is greater than the value of their house together with the cost of defaulting.[9] When households and investors are highly leveraged, a minor increase in their borrowing costs or fall in housing prices can therefore trigger a collective selling out where

[9]Strategic defaults are legal in several American states, but generally not permitted in European countries.

foreclosures and increases in interest rates amplify each other. The model has an interesting implication explained in Ge (2014). The number of investors on the housing market does not affect price volatility following an income shock; instead, it increases the divergence between regions' recovery speed. Investors are more likely to invest in high quality regions, which causes them to bounce back faster after a crisis.

There is no explicit speculation in Erlingsson et al. (2014), but housing prices suffers from a persisting tendency to collapse if households are allowed to leverage too much. The probability of fire sales increases the more households are leveraged as income shocks can put them under water. Income shocks happen when a firm goes bankrupt and the workers cannot find new jobs, leaving them to rely on subsidies. If subsidies are not enough to pay mortgage costs, they need to quickly sell off their properties and therefore set asking prices below the going rate. This behavior causes the market price to drop. The drop in housing prices induces a negative wealth effect on all households, causing them to downscale consumption. Lower consumption leads to lower revenue for some consumption good producers, who now have to lay off workers; these workers experience a negative income shock and so forth. Through this mechanism, the model captures an interesting dynamic interplay between income and wealth of households that is subject to a herd-like externality and borrowing criteria. Nevertheless, it has to be said that the assumption of market cutting asking prices and a significant wealth effect are debatable. For example, Carroll et al. (2011) investigate the effect on aggregate consumption from changes in aggregate housing wealth in the U.S. since 1960 (which is the source of the model's wealth effect). They find that there is an immediate propensity to consume 2% of a wealth shock and a long run propensity of 9%. The estimate does not discriminate between positive and negative wealth shocks, which one might imagine be different in a way crucial to the model. Furthermore, as Carroll et al. (2011) also mentions, micro panel studies are not conclusive on the existence of housing wealth effects. For instance, Browning et al. (2013) use micro data of Danish households in the period of 1987-1996 and find no overall tendency to change consumption following housing wealth shocks except for young households.

4 Description of the housing market ABM

In the following section I explain the architecture of the ABM developed for this thesis. Some of its features are inspired by the models in the literature review, Geanakoplos et al. (2012) in particular, although it is not a direct extension of any of them. The model is calibrated to Danish data, and is as such minded to resemble the Danish housing market. Some important features of the market have not been incorporated yet, i.e. speculators and construction of new housing, so the current model is meant as a proposal for a more comprehensive ABM of either a specific city or the entire Danish market. This extended model could be applied for projections as well as detailed policy analysis. It should, however, be elaborate enough in its current state to make qualitative predictions of the short run dynamics pertaining to shocks and the effects of credit regulations. To give a sense of its current complexity in terms of code devoted to its implementation, note that it takes up about 2500 lines in C#.

4.1 Life cycle of households

The only actors trading on the model's housing market are households. They are accordingly equipped with enough behavior to keep the market evolving endogenously throughout the simulation. This behavior is specified in a number of methods prescribing when to move, how to search the market for suitable dwellings and how to engage other households in trade. Households are heterogeneous with respect to age, income, moving probabilities and their information on market developments. A household is instantiated at the age 20 and dies at the age of 100 the latest. Each year in the simulation counts 10 rounds ("months"), so every household participates at most 800 rounds in a simulation run. This represents the time span that people would usually be active on the housing market in the sense that they consider buying or selling real estate with some probability. Households possessing freeholds will always sell their property before leaving the simulation such that their saved up equity can be cashed out, and their freeholds passed on by trade. Hence, they do not leave the simulation until the house is actually sold, for which there is no time restriction.[10] Old households living in rentals just leave the simulation instantly when their time is up. A fixed number of households are instantiated every round such that the population is roughly constant, although it will tend to increase temporarily if the market freezes and old households cannot hand off their freeholds.

While the maximum age of households is 100 years before they leave the simulation due to death,

[10] In other words, if households are selected to pass away, putting their freehold up for sale is the last action they undertake. As soon as the freehold is sold, they are removed and they do nothing but wait in the mean time.

households die at varying ages. The hazard rate of dying is inspired by Gompertz law, which states that the death rate has both a constant and a age-dependent component, and the latter increases exponentially with age. Thereby, there is a constant doubling time for the chance of dying for households. The hazard function employed in the model is given by:

$$\mathsf{P}^{\mathsf{death}} = 0.0005 + 10^{[-4.2+0.038\times age]}, \tag{5}$$

where age is measured in years. The scaling parameter -4.2 is the average between that of men and women, as estimated on the Danish population by Hansen et al. (2006), p. 7. This hazard function implies that the probability of dying doubles every 8 years at ages above 60. Although every round of the model resembles a month in the real world, there is only one draw a year testing whether a household dies.[11]

Furthermore, there is a chance that households will sell their freeholds to move into a rented home for elderly (nursing homes etc.). The chance of doing so follows a rough approximation derived from aggregate statistics on people living in rentals dedicated to elder care. More precisely, it is specified as the increase in the share of people living in retirement homes from one age interval to the next.[12] The calculated probabilities are shown in table 1. These probabilities

Age	$\mathsf{P}^{\mathsf{nursing\ home}}$
67-74	0.10%
75-79	0.63%
80-84	1.14%
85-89	2.10%
90+	5.99%

Table 1: Annual probability of moving to a retirement home.

together imply that 21% of the population at age 89 will be living in retirement homes, which also goes for the Danish population.[13] If a household is already living in a rental when it is selected to move to a retirement home, it simply leaves the simulation as in the case of dying. In sum, a household has once a year a probability of being removed from the simulation, and this probability is given by the union of $\mathsf{P}^{\mathsf{death}}$ and $\mathsf{P}^{\mathsf{nursing\ home}}$.

[11]The annual draw is chosen because the Gompertz-specification cannot be parameterized to a monthly chance of retiring without putting too much of the probability mass in the last few years of the life span.

[12]The data is obtained from Statistics Denmark, table RESI01.

[13]The probabilities probably undershoots a bit, since the calculation rests on the assumption that people in retirement homes have the same probability of dying as people in private homes.

4.2 Income

The income of households is a stochastic process which includes a deterministic growing component capturing the returns to experience on the labor market. The specification is inspired by that of Carroll et al. (1992), where the stochastic income process is the sum of a fixed growth term, transitory and permanent innovations. The latter are permanent in the sense that the process is a random walk. In the present specification, innovations to the permanent part are also a random walk, but there is no drift as that comes in deterministic increases. Income is received every month, but only updated once a year. That is, households receive the same amount for 10 months in a row and then experience innovations to the stochastic components on the 11th. This model was chosen as there were no known estimates[14] on the variance of monthly income applying to the Carroll specification. When instantiated into the simulation, the permanent part is normally distributed as $I_i^P \sim \mathcal{N}(20\,000, 1\,000)$. After that, it changes once every 10 periods according to:

$$I_{i,t}^P = \begin{cases} 700 + I_{i,t-1}^P & \text{if } age_{i,t} \in [20, 45] \\ I_{i,t-1}^P & \text{if } age_{i,t} \in (45,\ 65) \text{ or } age_{i,t} > 65 \\ 0.8 \times I_{i,t-1}^P & \text{if } age_{i,t} = 65, \end{cases} \tag{6}$$

It says that a household expects to earn an extra 700 of the fiat currency every year from it is 20 years of age to 45. There are no deterministic changes in permanent income between age 45 and 65. The permanent income drops by 20% at that age, after which it is constant. This pattern is meant to resemble the process of labor income first increasing with experience until a satiation point. Income drops when the household retires from the labor market, as it relies on pension savings henceforth. The permanent innovations are a log-normal random walk:[15]

$$\tilde{\varepsilon}_{i,t}^P = \varepsilon_{i,t}^P + \sum_{l \in T_{i,t}} \varepsilon_{i,l}^P, \quad \varepsilon_{i,t}^P \sim \mathcal{N}(0,\ 0.113) \tag{7}$$

[14]The literature search performed during the writing of this section returned only annual estimates.

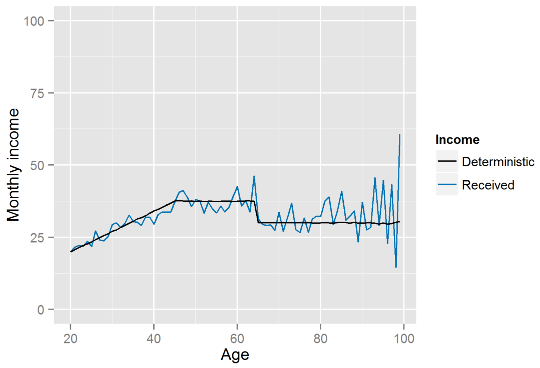
[15]The estimates are taken from Carroll et al. (1992), p. 71.

where $T_{i,t}$ is the set of update periods of the household so far, $T_{i,t} = \{t-10, t-20\ldots\}$. The actual income of the household, $I_{i,t}$, is then:

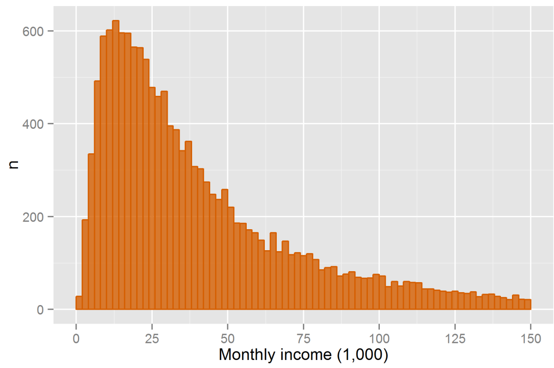
$$I_{i,t} = \begin{cases} \exp(\tilde{\varepsilon}^P_{i,t} + \varepsilon^{tr}_{i,t}) \times I^p_{i,t} & \text{if } age_{i,t} \leq 65 \\ I_{i,t-1} & \text{else} \end{cases} \tag{8}$$

$$\varepsilon^{tr}_{i,t} \sim \mathcal{N}(0,\ 0.155) \tag{9}$$

The income distribution and the average income by age is depicted below. As is evident, income is log-normally distributed due to the log-normal innovations.



(a) Median income by age



(b) Income distribution

Figure 3: Monthly income of households.

4.3 Preferences of households

The households of the model only regard two entities when making decisions throughout the simulation, housing services and non-housing goods. The market for non-housing goods is not modeled explicitly; it is just assumed that households can obtain any bundle of non-housing consumption goods at prices that are not affected by developments on the housing market. The preferences of households over housing services and and non-housing consumption are given by the log-transformed Cobb-Douglas utility function:

$$U = (1-\alpha)\log(C_{i,t}) + \alpha\log(Q_{i,t}), \tag{10}$$

where $C_{i,t}$ is consumption in current period and $Q_{i,t}$ is the quality of the dwelling possessed by household i in period t[16]. If maximizing utility with these preferences subject to monthly disposable income, households will spend a share α on housing services and the rest on other consumption. This is approximately what happens when they act on the housing market and α is therefore set to match housing consumption's empirical share of total consumption. In Dam et al. (2011) p. 45, it is documented that this share has been trending upwards in the past 50 years, and is currently around 20%. Accordingly, the choice parameter α is set to 0.2. Note that households do not have explicit preferences for one type of dwelling or the other; everything they care about is assumed to be captured in the quality index. Some models of the housing market, e.g. Iacoviello and Pavan (2013), do assume that households derive extra utility from owning their residence. This can be justified by surveys such as Andersen (2011), which shows that attaching value to having free disposal over one's residence is a frequent motive for buying. This consideration is circumvented in the current model by assuming that the rentals have lower quality than any freehold, so every household with enough income to pay for it prefers a freehold.

4.4 Moving

When instantiated into the simulation as young, all households start off living in a rental. Young households do not save up for their first purchase; instead they buy a freehold when the bank estimates that their income is high enough to carry the borrowing costs. Actual young households do normally have to save up some money before buying due to expenses of lawyers, registration fees and liquidity requirements by their mortgage provider. These costs are not included in the model, and the need to build up liquidity for the first purchase is therefore also abstracted from. Freehold owners automatically build up home equity by paying installments on their mortgage, which constitute their savings. Because savings are build up in this indirect manner, households only need to think about their preferences when they actively search for a new dwelling on the housing market. In all other periods, they simply just pay their mortgage costs and consume the rest.

All households will want to trade their dwelling, in so far they own it, and move to a new one with some probability in each period. This probability has an age specific component and a market dependent component. The age specific is very high for the young households

[16] The quality of freeholds is calibrated to make its distribution proportional to the income distribution. The factor between the two is chosen such that if the price of a freehold is equal to its quality, then there will be a household whose income is $\frac{1}{\alpha}$ times the mortgage expense of that freehold. In other words, there will be a household for whom the mortgage cost is the optimal budget share on housing. This creates the distribution of quality in figure 17, which is in fact empirically plausible, cf. Dam et al. (2011), p. 26

and declines relatively quickly from the mid-twenties to the mid-thirties. The falling chance of moving reflects that households are testing various houses and locations before settling on a preferred one. Moreover, real households experience a change of needs when starting a family and careers, which causes further relocating up till the mid-thirties. The probabilities used in the model are based on statistics from Hansen et al. (2013), that include singles as well as families.

	Probability of moving	
Age	Annual	Period
20-24	35.0%	4.216%
25-29	20.0%	2.207%
30-34	12.5%	1.326%
35-39	9.9%	1.037%
40-44	7.0%	0.723%
45-49	6.0%	0.617%
50-54	5.0%	0.512%
55-69	4.5%	0.459%
70-84	2.5%	0.003%
85+	1.6%	0.002%

Table 2: Age dependent probability of moving from dwelling.

These probabilities implicate that households on average move 5-6 times in their life-cycle due to age-specific motivation. Whether the probable event of moving happens for a household is practically resolved by drawing a uniformly distributed number between 0 and 1 in each period, and then command the household to move if the number is below the age specific probability. In case the household owns a freehold, it must first sell this before buying a new one, while renters can go to the market instantly. A freehold owner selected to move therefore submits its property to the market protocol and awaits being contacted by interested buyers. It may take several rounds before the owner succeeds in selling its property, depending on the market conditions. In the meantime, the household is not conducting further actions on the market. When selling its freehold, the household receives liquid equity

$$E^l_{i,t} = P_{i,t} - L_{i,t}, \tag{11}$$

where $P_{i,t}$ is the selling price of the freehold, and $L_{i,t}$ is the outstanding principal on the household's mortgage at the time of selling. In the first coming period after selling its property, the household searches the market for a new dwelling. Before doing so, it requests the bank to be informed about its budget limit for freehold purchase, which is determined by income history and $E^l_{i,t}$ - see below in section 4.5. In order to search the market for potential dwellings to buy, the household thereupon draws a random sample of n^{search} freeholds for sale and discards those whose offered prices are above the budget limit. The set of remaining freeholds is denoted Ψ. To evaluate the utility associated with each dwelling in the remaining sample, the household must translate the asking prices to monthly user costs in order to infer what monthly consumption will follow from each alternative. For each freehold k in the sample, the translation between user cost and asking price for household i is

$$uc^k_{i,t} = \varsigma(r_t) \times \left(P_{k,t} - E^l_{i,t}\right), \tag{12}$$

where $\varsigma(r_t)$ is the annuity on a mortgage defined in equation (22) below. That is, the household uses its total cashed out home equity from the previous sale to buy a new freehold. Households are assumed to be aware that their income is subject to random innovations, so from a precautionary motive, they do not want to leverage up to the point where they are unable to meet their debt commitments because of a modest negative shock. Hence, households need to form an expectation of the their underlying income process, and from that calculate how much consumption they will plausibly enjoy in the near future at a given level of housing costs. To be cautious, households make an expectation by exponential smoothing between past income and past expectation of income

$$I^e_{i,t} = \lambda I_{i,t-1} + (1-\lambda) I^e_{i,t-1}, \tag{13}$$

where $I^e_{i,t}$ is the expected income for the current period. Temporary shocks will by this expectation not have full impact on the perceived income. If the expression is substituted back as far as possible, which is the J years since household i was instantiated, the expectation of underlying income can be written as

$$I^e_{i,t} \simeq \sum_{j=1}^{J} \lambda(1-\lambda)^{j-1} I_{t-j\times 10} + (1-\lambda)^J \tilde{I}^e_{t-J\times 10} \tag{14}$$

Recall that income only changes once a year (10 periods), which is why the index is multiplied by 10. As a replacement for the expected income in the first period, the actual income received is simply used, $\tilde{I}^e_{t-J\times 10} = I_{t-J\times 10}$. The expected income is in other words a weighted average of previous income realizations, with the weights going exponentially towards zero for more distant lags.

With this expectation of income in mind, the household goes to market and chooses the dwelling with user cost-quality pair, (uc^*, Q^*), which yields the highest expected utility

$$U(I^e_{i,t} - uc^*_t,\ Q^*) \geq U(I^e_{i,t} - uc^k_t,\ Q^k), \quad \forall k \in \Psi \tag{15}$$

Having chosen its preferred freehold in the market sample, the household contacts the owner of the dwelling and submits a bid corresponding to the offered price.[17] The owner accepts the bid and transfers the freehold to the buyer, who is now registered as the owner. The previous owner is subsequently without a dwelling, so in it searches the market in its coming round. In so far the household had chosen a rental instead of any of the freeholds on the market, it would simply have moved to one such without further ado.

A household might also move because current market conditions or income are lucrative. In each period, there is a constant probability for owners, P^{O}, that they will investigate the market to check whether a better alternative than their current may plausibly be acquired. In the benchmark of the model, $\mathsf{P}^{\mathsf{O}} = 1\%$, such that there is an annual chance of investigating the market of 9.5%. They perform the investigation by first forming an estimate of their freehold's market value, and from that calculate the expected equity they cash out from selling. As when actually moving, they acquire their budget limit based on the expected equity and past income from the bank. Subsequently, they draw a random sample of freeholds on the market of size n^{search}, and keep only the affordable subset Φ. On this set, the number preferable alternatives is counted

$$A = \sum_{k \in \Phi} I\left(U(I^e_{i,t} - uc^k,\ Q^k) > U(I^e_{i,t} - uc_{i,t},\ Q_{i,t})\right), \tag{16}$$

where $I()$ is the indicator function. If the number of better alternatives is great enough, i.e., larger than a fixed parameter A^*, the households conclude that market conditions are well enough to move. Since owners have to actually hand off their freehold before buying a new one, it is important that they are relatively confident before moving voluntarily. Contrary to the owners, renters do not have age specific moving probabilities. They do however check the market for a better alternative more often as their monthly probability of doing so is $\mathsf{P}^{\mathsf{R}} = 0.1$, which amounts to an annual probability of investigating the market of 65%.

As the housing market is modeled with posted prices, owners need to form an asking price when deciding to put their freehold up for sale. In order to form an asking price that aligns with the going market price, they need to estimate a relationship between quality and price in

[17] This procedure could relatively easily be extended to include an actual bargaining.

recent sales. The algorithm employed to set the asking price is meant to resemble the real world scenario, where a real estate agent is aware of recent sales in the neighborhood of the seller. By his knowledge of the market, the real estate agent is capable of estimating the current worth of specific features such as size and number of rooms in the area of the seller. The resulting estimate of the dwelling estimate is then passed on to the seller, who posts it as the asking price. In the model, this job is delegated to the households themselves for simplicity. Their method to form an asking price is to draw a random sample of n^{ask} trades within the past 5 periods. To localize the freeholds resembling their own, they keep only the n^{avg} with closest quality. On this reduced sample, a seller finds the average price-quality ratio and sets a preliminary price as

$$\tilde{P}_{i,t} = Q_{i,t} \times \frac{1}{n^{avg}} \sum_{j=1}^{n^{avg}} \frac{P^j}{Q^j}. \tag{17}$$

If a seller happens to own a freehold of higher quality than any other in the sample, then $\tilde{P}_i$ is set by using only the ratio between the highest quality dwelling and the seller's freehold quality times the sales price. The opposite goes if the freehold is of lower quality than any in the sample. Furthermore, sellers are assumed to realize that they will probably not sell their property immediately due to search frictions. Therefore, they put a minor markup μ^u on the estimated price as a compensation for the expected waiting time.[18] Hence, the asking price attached to a seller's freehold when it is first put on the market is

$$P_{i,t} = (1 + \mu^u) \times \tilde{P}_{i,t} \tag{18}$$

If it takes sellers more than three months to sell their freeholds, they start downgrading their asking prices by a constant markdown in each month that passes to attract buyers. The asking prices accordingly evolves as

$$P_{i,t} = (1 - \mu^d) \times P_{i,t-1} \tag{19}$$

In the real world, sellers probably do not downgrade their asking prices in this continuous way, as repeated downgradings signal that there is something wrong with the property, or that they are in a poor situation to negotiate. In the absence of a more elaborate negotiation process, the specification above serves to create a link between excess supply, prolonged selling times and falling prices. The fact that such a link does exist can be motivated by figure 18a and 18b of the appendix. The decentralized buying and selling behavior of households constitutes so to say

[18] Another motive is that households are averse to missing out on capital gains, and therefore put a markup on the asking price. In that way, they are not cheated by the market if the estimate of sales price is too low.

the tâtonnement process of the market, in which sellers constantly test how much to charge for their offered commodities, while buyers act on a take it or leave basis. This means that prices are subject to a non-trivial, out-of-equilibrium correction phase when shocks hit the market. Such a phase is not present in traditional Walrasian economies, where prices are settled before supply or demand decisions are made.[19]

4.5 The bank and borrowing procedures

The model contains a single bank, which acts as if under perfect competition in the sense that it offers an exogenously given interest rate together with exogenous borrowing criteria. As part of the mortgage extension, the bank manages the loans of households by letting them create an account in connection with taking loans.[20] The loans are modeled as objects with properties revealing their remaining principal, interest rate, the ID of the borrower and due payments. These loan objects are managed by the bank object in that households request a bill from the bank in each period, which then does the job of calculating the due payments. When receiving payments, the bank reduces principals on the loans of paying households until the loan is fully paid. After that, the bank deletes the loan object and stops sending bills to the previous borrower. If a borrower cannot service its debt, it is forced to put its dwelling up for sale. When sold, the revenue is used to redeem as much of the outstanding debt as possible. If the revenue cannot cover the principal on the loan due to falling market prices, the bank incurs a loss. In each period, the bank calculates its earnings as:

$$\tilde{\pi}_t = \sum_{j \in B_t^p} r_j L_{j,t} - \sum_{k \in B_t^d} \left(L_{k,t} - p'_{k,t} \right) \tag{20}$$

where B_t^p is the set of paying borrowers at time t, while B_t^d is the set of defaulting. The first term represents the total interest on remaining principal for household j payed at time t. The interest rate on mortgages may differ among households, as it depends on the time of their obtainment (no subscript t). The second term in (20) represents the losses of the bank. $p'_{k,t}$ is the revenue from forced house sale by the delinquent household k, which is payed to redeem as

[19]For a comparison between the Walrasian market dynamics and that of an ABM, see Gintis (2007).

[20]Formally, the loans are stored in a dictionary, which is a data structure particularly well suited for situations with many repeated look-ups by an ID-key. In this case, loans are indexed by the households' ID number, which the bank employs to look up the corresponding loan object every time the household interacts with the bank. One might also have stored the loans in a list, a linked list or as properties in each individual household. All these options would, however, have rendered the actions undertaken by the bank slower. The bank needs to look up, add and delete loans many times in each iteration of the simulation and these operations are all executed in minimal time by a dictionary when its maximum size does not change. The same does not hold for the other options.

much of the remaining principal $L_{k,t}$ as possible. The loss incurred by a credit institution from a defaulting borrower is not exactly $L_{k,t} - p'_{k,t}$ in the real world, as the remaining debt is carried further by the household. However, the institution has to write off a considerable amount of the debt in its balance sheets and spend man hours on the foreclosure process, so the losses to the bank are by no means insignificant in case of a default.

4.6 Credit negotiation

When evaluating how much credit to allow a household, real world banks take in consideration a number of metrics regarding the financial soundness of the household. The number of recent overdrafts, income stability, liquid savings and insurance policies are among such metrics.[21] To capture these metrics in a simple rule for the purpose of the model, the bank has following algorithm to establish the maximal amount of money, it is willing to extend:

1. Find the minimum monthly income, I^{min}, in the past 4 years.
2. Calculate expenses on servicing debt, the household can afford given I^{min}, without its consumption going implausibly low.
3. Calculate the loan, $L^*(uc)$, which pertains to the maximal expense.
4. Calculate the maximum loan amount given the debt-to-income constraint, $L^*(I)$.
5. Offer the minimum of 3. and 4. to the household.

Since the household has saved up equity E, it will only take up a mortgage covering the remaining desired expenditure for a dwelling. The costs of borrowing, which a household must be able to accommodate without resorting to implausibly low consumption levels, are the period-wise payments on a 30 year fixed installment loan with fixed interest rates.[22]. The monthly costs of a fixed installment loan obtained at time τ are found by solving the differential equation

$$L_{i,\tau+t} = (1 + \tilde{r}_\tau)\, L_{i,\tau+t-1} - uc_{i,\tau+t}, \tag{21}$$

where $\tilde{r}_\tau = \frac{r_\tau}{10}$ is the interest rate fixed to the mortgage contract at the time of purchase divided by the number of periods pr. year and $uc_{i,t}$ are the constant monthly payments until the principal is repayed. Solving this equation such that $L_{i,t} = 0$ after 30 years (300 periods) to determine

[21] Cf. recommendation by the Danish FSA ('Finanstilsynet') in `https://www.finanstilsynet.dk/da/Indberetning/Vejledninger-og-informationer/Vejledninger/~/media/Indberetning/2012/System/AS_vejledning_20121217_2.ashx`.

[22] As in line with legal requirement, see paper from FSA above.

the size of $uc_{i,t}$ results in following payment schedule:

$$uc_{i,\tau+t} = \varsigma(r_\tau) \times L_{i,\tau}, \quad \varsigma(r_\tau) = \frac{\tilde{r}_\tau}{1 - \left(\frac{1}{1+\tilde{r}_\tau}\right)^{300}} \tag{22}$$

Taking the equity of the household into account when calculating the monthly expense on a loan, the bank derives following user cost of borrowing to purchase a freehold:

$$uc(P, E) = \varsigma(r) \times (P - E) \tag{23}$$

The bank holds an estimate of the minimal consumption, a household must enjoy to sustain itself, denoted C^{min}.[23] Combining the expenses of a loan with the minimal consumption, the requirement in 2. comes down to

$$I^{min} - uc(P, E) \geq C^{min} \tag{24}$$

To obtain the maximum price of a freehold, used in 3., the bank simply varies P in (24) from a low starting point until equality is reached with $P^* \mapsto L^*(uc)$.

Closely related to this constraint is debt-to-income, DTI, which is the ratio between total debt and annual income. The less readily a household can pay off its debt, the more risk is carried by the bank. Therefore, the bank further constrains how much debt a household can uptake given its income. Given the allowed DTI, ϑ^I, the bank calculates:

$$L^*(I) = \vartheta^I \tilde{I}^{min} \tag{25}$$

where $\tilde{I}^{min} = I^{min} \times 10$ is annual income. In the benchmark of the model, $\vartheta^I = 3$ but the effect of changing this constraint will be tested in the analysis of the model.

Combining the above constraints, the budget limit which the bank communicates back to the household applying for a loan is:

$$\min\{L^*(uc),\ L^*(I)\} \tag{26}$$

Due to the low income of young households, the income constraint is most likely to be binding in the early stage of life, while the DTI-constraint matters more for older households.

[23] The Danish FSA sets $C^{min} = 5,000$ for individuals and $C^{min} = 8,500$ for couples without children for medium to high quality loans.

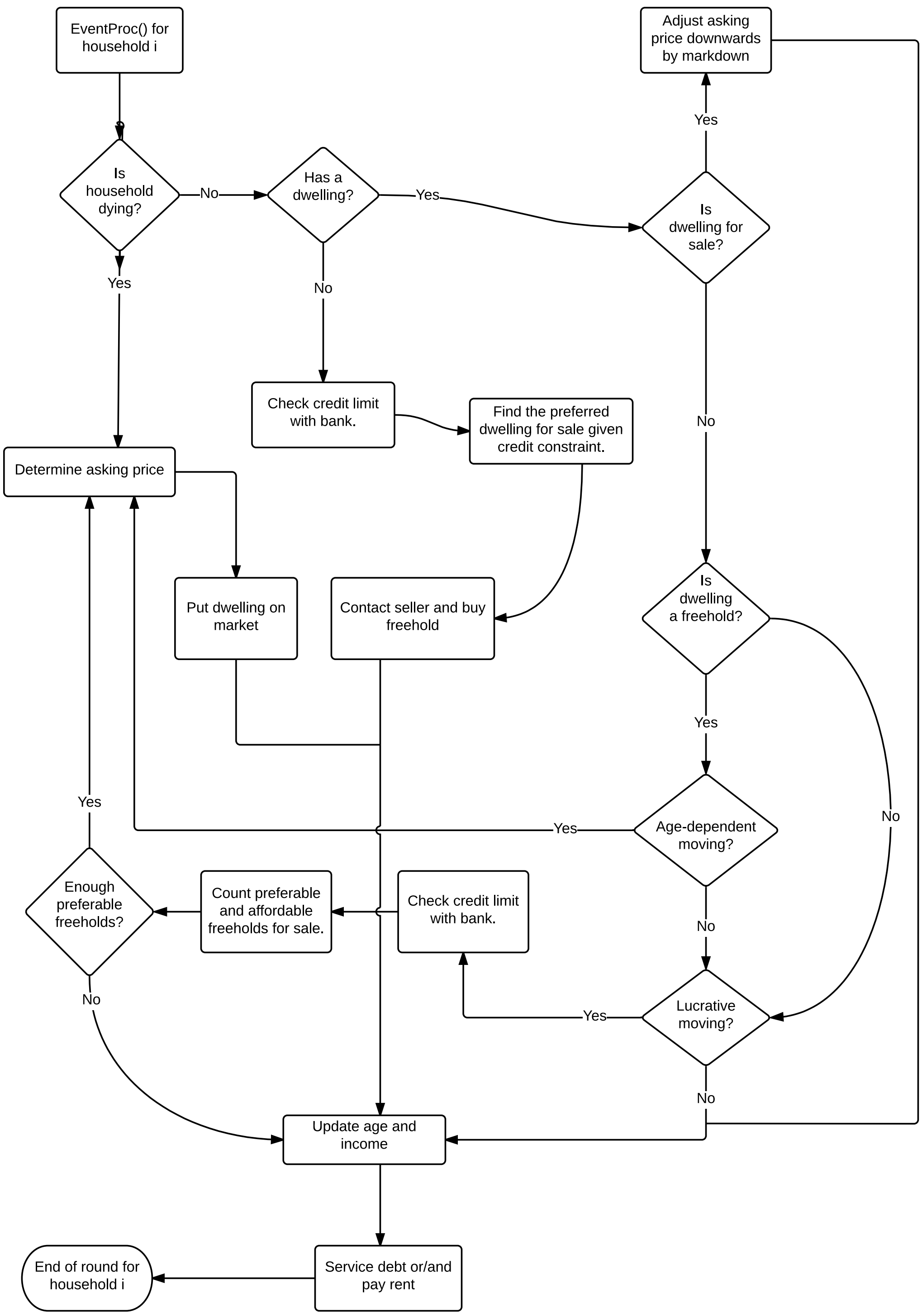
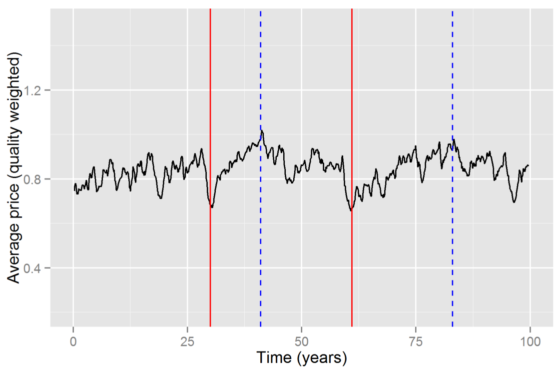


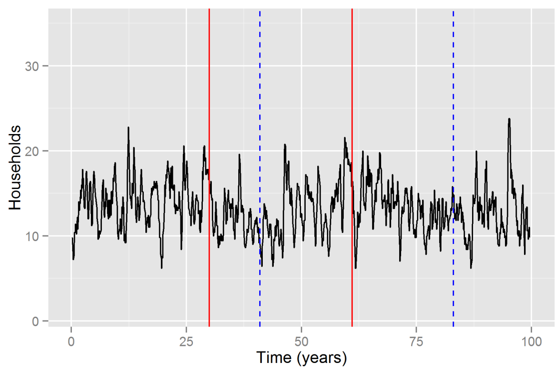
Figure 4: Flow chart representing one period in a household's life

5 Analysis of benchmark simulation

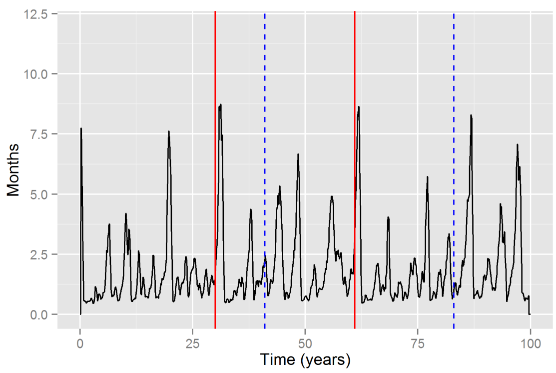
Having specified all the behavior and parameters of the model, the next step is to investigate whether the simulation of the housing market produces sensible outcomes. To that end, it really is possible to retrieve any imaginable data set pertaining to the decisions and behavior of agents in the model, as it is programmed from scratch. These are of course mostly interesting if we have some idea of what the empirical counterparts look like. In the following analysis of some key statistics from the model, it turns out that the model conforms quite well to several aspects of the Danish housing market.



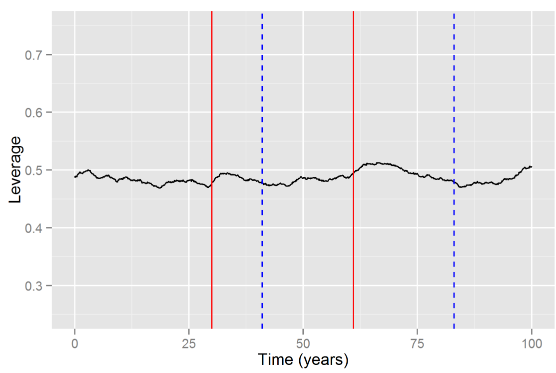
(a) Price index, quality weighted.



(b) Households selling due to lucrative prices.



(c) Average months on market before sale.



(d) Average leverage of homeowners.

Figure 5: Statistics of benchmark model. 5 months moving average except (d).

In figure 5 we see that the market is exposed to cycles of contractions and expansions. The price index is defined as the average of selling prices divided by quality, which is chosen to mimic standard indices of house prices, such as the one employed by Statistics Denmark and other statistics institutes called SPAR.[24] In SPAR, the index value is given by the sum of prices of dwellings traded in a given year divided by the sum of estimated prices of those same dwellings in a starting year. The expected prices of dwellings in the SPAR-index' base year is practically the same as the quality of dwellings in the current model, as households are instructed to trade at the quality of the dwellings for the first couple of rounds. Note that whether one takes the average of prices over quality or divides the sum of traded prices with the sum of quality, does not make a noticeable difference for the current context. We see that the price index keeps within a steady band around 0.8, but is repeatedly hit by sudden negative shocks followed by periods of steady growth. These contractions, indicated by red vertical lines, may be defined as market bursts. It is an interesting feature of the model that this behavior fits well with that of real housing markets, where upturns are longer and more persistent than downturns, see Bracke (2011).

The average time it takes to sell a freehold is related to the development in prices. Recall that sellers set prices by looking at other recently sold freeholds and downgrade if they cannot sell off their property. Hence, if demand drops they start downgrading their asking prices, which will negatively affect how newly posted freeholds are priced. When looking at figure 5c, it seems that busts are accompanied by prolonged waiting time before sellers get to trade their property. From an average of 2 months, the average time before a sell suddenly shoots up to 8.5 months. The reason why the incline shows up after the bust, on the right side of the red line, is that the statistic is collected from the period's traded properties. Meaning that it took the freeholds sold just after the bust around 8.5 months to be sold, and hence *they* were on the market without being sold during the downturn. When time on market is moderate, prices tend to be rising since sellers will put a markup on asking prices without needing to cut them later. The average time it takes to sell a freehold in the model is almost surprisingly well aligned with empirical data. For example, in the Copenhagen metropolitan area in second quarter of 2005, around the acceleration of the housing bubble, average time on market for a house was 76 days (2.5 months). When the bubble burst, average days on market rose to 189 (6.3 months) in 2008 Q2. As for the country average, days on market went from 105 in 2006 Q2 to 218 (7 months) in 2010 Q1.[25] In other words, the model generates plausible levels of time on market, although they are more volatile than empirical statistics.

In every period, a share of households are selected to check if the market offers better alternatives than their current dwelling. Before doing so, they update their expectation of the worth of their

[24]See `dst.dk/~/media/Kontorer/12-Priser-og-forbrug/Beregning-af-prisindeks-for-ejendomssalg.pdf`

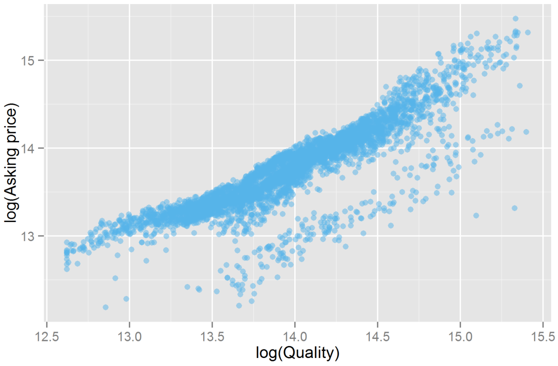
[25]See the Association of Danish Mortgage Banks' statistic BM030, `realkreditraadet.dk/Statistikker/Boligmarkedsstatistik/Data.aspx`

freehold, in so far they own one, as they will sell it first to use the proceeds when buying a new home. Therefore, falling prices will induce some households to put their property on the market if the value of their own homes have not fallen so much that the capital loss makes a better alternative unaffordable. An inflow of sellers to the market initially puts a further downward pressure on prices as the chance of selling decreases for each seller. However, straight after a seller concludes a trade, it enters the market as buyer, increasing the chance of a trade for sellers. Hence, bottlenecks with too many sellers on the market can arise, but they will be followed by an increase in demand such that 'excess supply' is brought down. Figure 5b shows the number of households that enter the market in each period because they decided to sell upon checking the market for better alternatives. It indicates that the fall in prices brings some sellers to the market, although the numbers are not in the extreme. In fact, market bursts still happen on the same scale with this behavior turned off. Figure 5b also shows that fewer households voluntarily enter the market because of lucrative prices when prices are peaking. It might be reasoned that rising prices imply expected capital gains to owners, giving households an extra incentive to invest when prices are booming; which is the opposite of what happens in the model. And certainly, it would be a natural extension of the model to include expectations of capital gains in the motives of households.

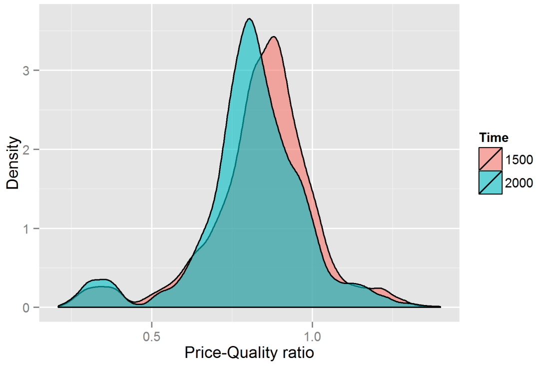
The leverage of home owners is defined as the sum of outstanding mortgage debt divided by the sum of freeholds' worth. Leverage in figure 5d is evolving around 48% during the simulation, which is close to average of actual leverage among Danish home owners in the period of 1990 to 2008, see Andersen et al. (2014). Leverage in the Danish housing market is, though, somewhat more volatile than in the model as it went from 43% in 2006 to 64% in 2013. The model weakly displays the reverse scenario of a typical housing booms. Normally, housing booms are associated with increased leverage, but the model shows downward trending leverage during the build-up of prices (that is, in the time span just before the intersection of the blue dashed lines). Leverage rises just after a collapse of prices because the collapse reduces equity cashed out from selling, whereby buyers with high incomes find it optimal to borrow more to afford their preferred freehold.

The price structure in the housing market can be assessed by the distribution of prices and the relation between prices and quality. Below in figure 6a is plotted the price-quality pair of each freehold in the simulation's period 2000. As is evident, prices increase approximately linearly with quality, but variance is somewhat alternating. Especially, prices are less spread out for low quality freeholds than for high quality, probably because the markup and markdown on prices are multiplicative factors. Therefore an expensive freehold will be both priced higher above the estimated market price in nominal terms (and subsequently cut more) than a cheaper freehold.

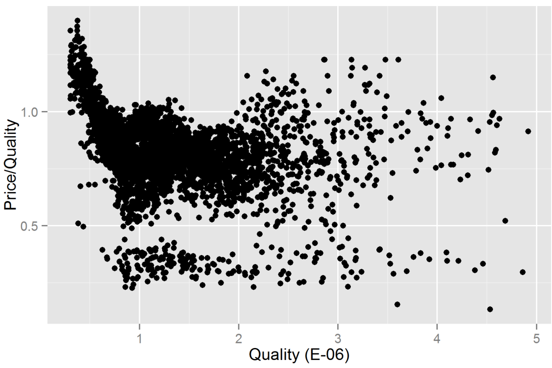
In figure 6b, the densities of prices divided by quality in period 1500 and 2000 are presented. They are approximately symmetrical around their modes, so there is no general strong tendency for prices to be excessively over- or undervalued. In figure 6c, however, we do see that particularly low quality freeholds are priced highly given their quality when compared to freeholds in the mid-range of quality. This should reflect that young households are all instantiated into a rental, which has the lowest quality of all dwellings, and then start looking for a freehold on a low budget. The constant inflow of young households living in rentals creates a stronger demand for the cheap low quality freeholds, which are thus allowed to be relatively highly priced. The



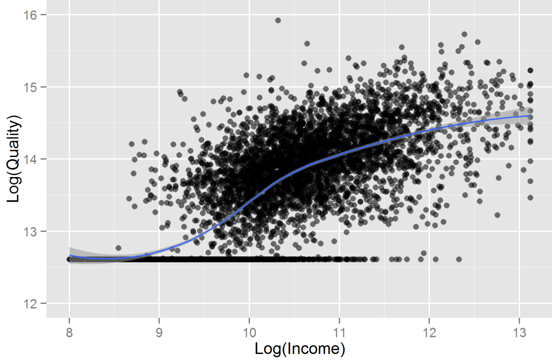
(a) Prices plotted against quality of sold freeholds.



(b) Distribution of price-quality ratio.



(c) Spread between price and quality given quality

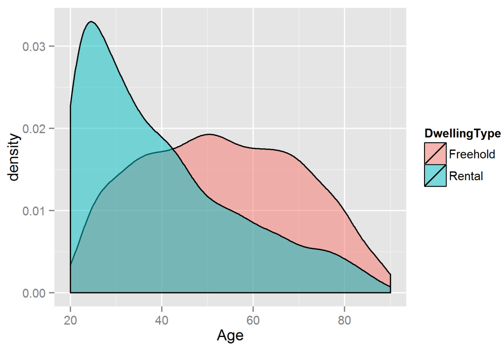


(d) Choice of dwelling quality depending on income

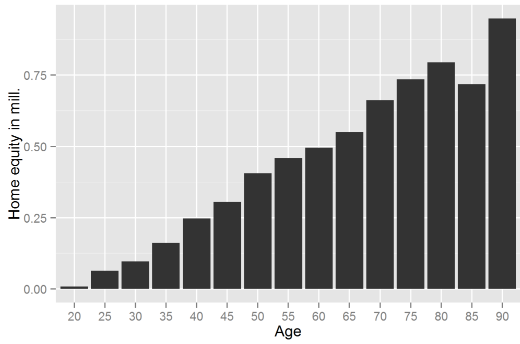
Figure 6: Price structure of benchmark model in the last period.

preferences of households are specified such that higher income should entail higher willingness to pay for housing services. Since higher quality of housing yields higher utility, we should expect freeholds of high quality to be allocated to households with high incomes and vice versa for low incomes. This relation is indeed what figure 6d captures, where higher income is associated with higher quality. It thus seems that the market of the model is well functioning.

A related relationship is that between age and type of dwelling. Given that (i) the income of households is increasing over their lifetime and (ii) rentals are less attractive but cheaper than freeholds, we should expect to see primarily young households residing in rentals. As their income increases, they should make a transition to become owners. This behavior is what figure 7a depicts. It shows the age distribution of households in rentals and freeholds respectively, and evidently, the probability mass for renters decline with age. Conversely, it increases sharply for home owners until the late 30s after which it is relatively constant. A quite similar pattern emerges among Danish owners and renters, which shows in figure 16 of the appendix. Also, what is not shown directly in the graph is that the average age of mortgage holders is 53 years, which is also the average age of mortgage holders in the Danish housing market for a range municipalities.[26] This should speak to the correctness of the model's specification of income process, since this mainly determines the age specific tendency of borrowing and buying.



(a) Density of the age given type of dwelling.



(b) Average home equity for each 5-year interval.

Figure 7: Age distributions and home equity at the last round of a simulation.

Figure 7b shows the average equity saved up in freeholds through paid installments. That is, it shows the latest update of expected market price minus principal on debt. The home equity[27] is increasing over the entire lifetime of households because they only get 30-year mortgages and have a fair chance of moving once after the age of 50. In the actual economy, debtors have the opportunity to refinance their mortgage and cash out some of the saved up equity, which means that actual home equity will not be growing as steadily as in the model.

Another statistic which needs to be regarded is the turnover rate. In the benchmark simulation,

[26]See Realkredit Danmark, `www.rd.dk/PDF/Om20os/Analyser/2014/realkreditlaan-og-alder.pdf` (Due to a compile error in the verbatim environment, a percent-sign is omitted between 'Om' and '20' in the http-address. Insert this to get the right address.

[27]In Danish 'Friværdi'.

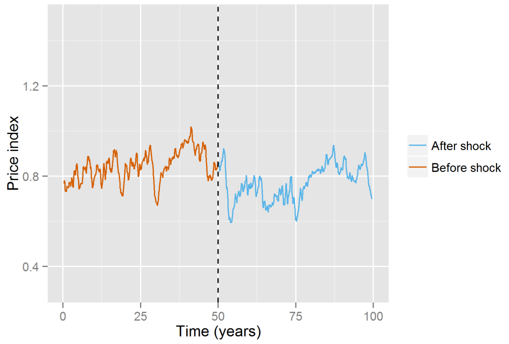
13% of the stock of freeholds are traded each year on average. That is probably somewhat above the actual market, as it is 5-6% for the US housing market, Duca et al. (2010). The explanation for the excessive trading is likely that the real housing market offers a more varied rental market, which absorbs more of the moving activity than that of the model.

6 Analysis of exogenous shocks

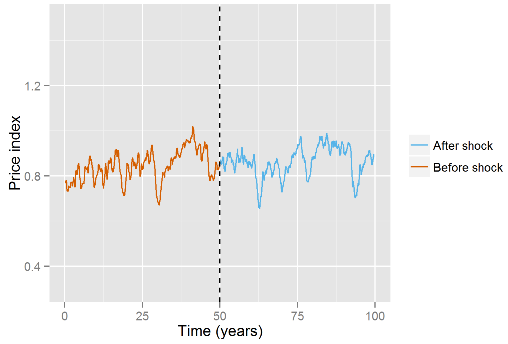
6.1 Mortgage rate

This part of the analysis investigates the impact on the housing market from changing the interest rate on loans taken after the shock. The interest rate figures in the cost of owning a freehold, which households calculates when considering whether to move and their desired quality in housing. We should therefore expect to see a fall in demand when it is increased and a rise when decreased. The benchmark interest rate is 3% and this is changed to 6% and 0.5% respectively. Given the behavior of households, there is no reason to believe that the effects of positive and negative shocks will be symmetric, as for example in the theoretical model by Glaeser et al. (2010). If demand is lacking and the time it takes to sell rises, sellers will keep downgrading their asking prices by 1% a month until buyers become interested. On the other hand, they put a 3% premium on the expected sales price when putting the freehold up for sale, which they reap if demand is high and sales time short. Upward adjustment in prices due to increased demand is therefore different from downward adjustment when demand drops, and the two could easily work a different paces.

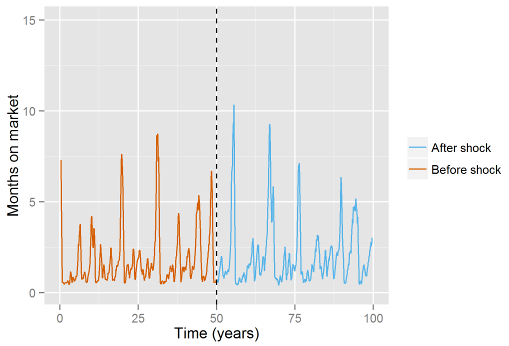
In figure 8a and 8b we find the effects of interest rate shocks on the price index of freeholds. The positive shock to 6% is followed by an immediate drop in prices of about 30%, which remain well below the pre-shock phase for the following 25 years. The effect is however not as significant when regarding the negative shock to the interest rate. Here, there is only a weak tendency of the price index to rise, and it suffers from collapses that bring the index down to the same level as before the shock. A comparison between the development in the model and real data really should not be pushed too far, but an econometric model for the Danish housing prices in Dam et al. (2011) shows that a 1 percentage point change in real interest rate is associated with a 15% change in real sales price in the absence of a change in supply. The impact on prices in the model from a positive 3 percentage points interest rate shock is by that estimate not extreme, but the effect of interest rate cut is very low.



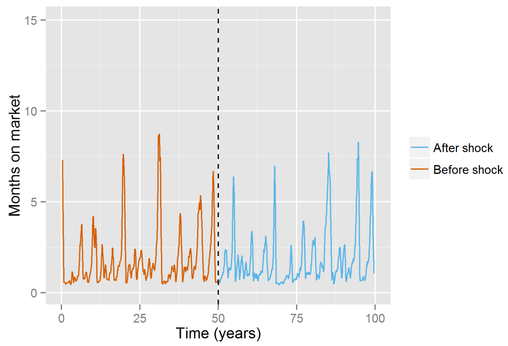
(a) Price index when interest rate is increased to 6%.



(b) Price index when interest is decreased to 0.5%.



(c) Time on market when interest is increased to 6%.

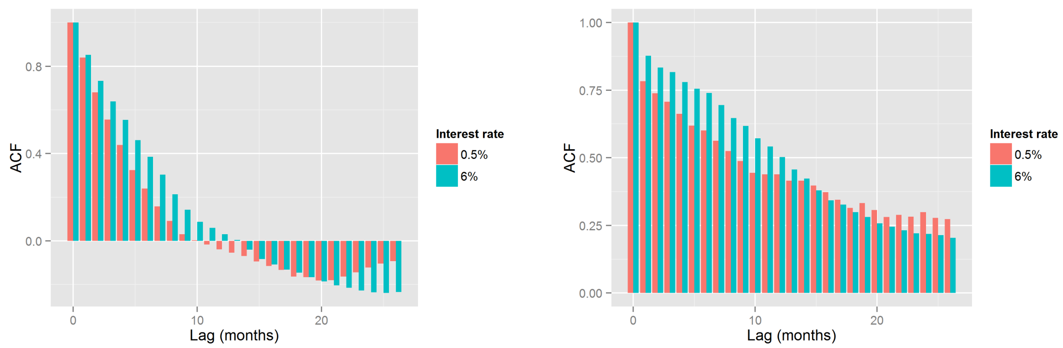


(d) Time on market when interest is decreased to 0.5%.

Figure 8: Reaction in prices and time on market when mortgage rate is changed in year 50.

The time it takes to sell a freehold is also affected by the changes in the interest rate, as shown in figure 8c and 8d. The positive shock results in a freeze that raises the average sales time of freeholds to 10 months, which is absent in when interest rate is lowered instead. It is of course this spike in sales time that causes the price fall described above. When cutting the interest rate, the average time on market does not seem change from the pre-shock phase, as the height and frequency of spikes in the process is roughly the same. What is perhaps less apparent from the figures is that the positive shock to interest rate creates higher auto correlation in time on market than the interest rate cut. This fact appears in figure 9a, which shows the ACFs of time on market for phase after the shocks. Clearly, the auto correlation in the 6% scenario is stronger at each lag. This is a noteworthy result because periods of sustained freeze overs in the housing market are problematic to home owners (of the actual economy) in several ways and should be avoided. For example, it inhibits their labor mobility, as they cannot move to take up another job while couples under divorce may be forced to live together. The price index followingly also shows some increased auto-correlation for the positive shock, although the relative magnitude is less.

Having seen that prices drop when the interest rate is raised, we might like to know if the drop impacts with equal force over the distribution of quality. That is, if freeholds of high and low quality are equally affected by the change in interest rate. Figure 10a features the scatter plot of price and quality for each freehold at the end of the simulation in the two shock scenarios. Furthermore, a spline is fitted to observations in each case. It turns out that the two splines coincide for high quality freeholds but separate in the low end, meaning that low quality freeholds are worse affected by an interest hike. This is likely explained by the fact that low-quality freeholds are typically bought by households without much saved up equity. These are more susceptible to lower demand when the borrowing cost rises, as they borrow relatively more than households with savings.

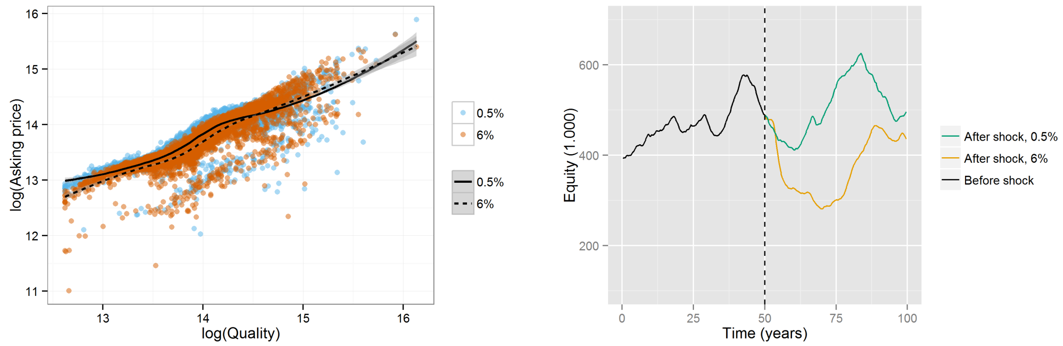


(a) ACF of time on market.

(b) ACF of prices.

Figure 9: Auto correlation functions of prices and time on market after shocking the interest rate.

A final aspect of the shocks to consider is the development in home equity. As the market price of freeholds shifts, so does the valuation of owners' properties and their next purchase on the market will therefore be affected by a 'wealth effect'. Households update the valuation of their freehold every time they are selected to check if the market offers preferable alternatives and this happens with a 10% chance every year. The valuations are therefore lagging a bit behind the development in prices - but that is also true of valuations in the real world. In figure 10b we observe home equity after each shock, which exhibits large differences. The positive shock to the interest rate creates a persistent slump in the valuation of freeholds in the subsequent periods, while the negative shock induces a steady boom. These reactions could have important implications in a more detailed model of financial stability on the housing market, where for example households are allowed to refinance their mortgages and extract the equity gain from increased market prices.



(a) Price versus quality at the end of simulation.

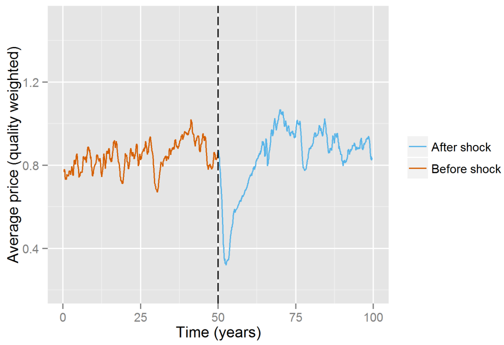
(b) Average equity of home owners.

Figure 10: Reaction in price structure and home equity when mortgage rate is changed in year 50.

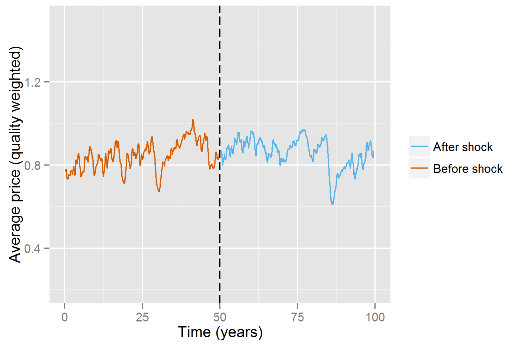
6.2 Income

We now turn to the attention towards the reaction of the model when the households are exposed to either an economic boom or a crisis. These are implemented as a year of either 50% higher or 50% lower income than in the previous year. For the same reasons as in the previous section, we should not expect symmetric effects from the income shocks. There is however an extra dimension to the reaction when shocking income rather than interest rate, because it affects both buyers looking for a freehold as well as owners. In case of a negative shock, buyers will naturally have a tighter budget restriction, but the some of the most leveraged owners may additionally not be able to meet their monthly debt payments to the bank. These owners are forced to sell off their property and cover as much of the remaining principal as possible from the sale. Until an owner 'in foreclosure' succeeds in selling, the bank is stuck with a non-performing loan whose missed out payments are accounted as losses. This inflow to the market of freeholds in foreclosure will furthermore amplify the fall in prices which is triggered by the lowered demand.

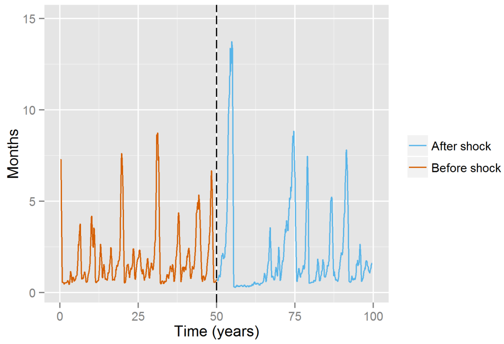
In figure 11a we observe the effect on prices from the income shock. They take a substantial hit, about a 50% reduction, which is followed by a 10 year recovery phase. The positive shock did, on the contrary, not affect prices particularly. That may be ascribed to the households precautionary way of assessing expected income, which is an exponential smoothing between previous expectations and previous income. Thus, receiving a large positive shock does not alter their inclination to act on the housing market as much as the negative shock. We might have seen a larger effect following the positive shock if households were allowed to own more than one dwelling in order to rent it out or resell for a profit. In that case, a moderate rise in prices might be amplified to a boom by wealthy households seeking profit. The fall in prices after the negative shock hits is of course fueled by a surge in time on market, which is depicted in 11c. Note that the surge is followed by a period of minimal time on market, causing prices to rise again, because the price level has shot below what the post-shock income level warrants.



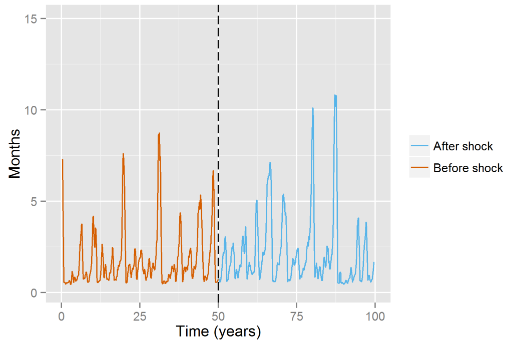
(a) Reaction to negative income shock in prices.



(b) Reaction to positive income shock in prices.



(c) Reaction to negative income shock in time on market.



(d) Reaction to positive income shock in time on market.

Figure 11: Prices and time on market when annual income is shocked by 50% for a single year.

As noted above, the negative income shock will force some households to default on their loans, as they are not able to pay their mortgage expenses together with a minimal amount of consumption. The losses endured by the bank when borrowers stop paying on their mortgage in connection with a negative income shock is shown in figure 12. Losses are in the absence of a shock very small, but they skyrocket when the crisis hits. Since the losses are measured as monthly missing payments, we need to sum all the monthly losses over the 6,3 years it takes before they are back to normal in order to estimate the total damage of the crisis to the bank.

These amount to 7.9 million of the model's currency. Since the freehold prices are roughly distributed as the actual Danish housing prices, we can use this estimate to assess the severity of the crisis for an actual bank. The bank of the model holds around 4,000 mortgage customers, which is a very small institution in a real world context. For such a small bank, 7.9 million DKK in missing income stream over six years would certainly be detrimental and likely put it out of business.

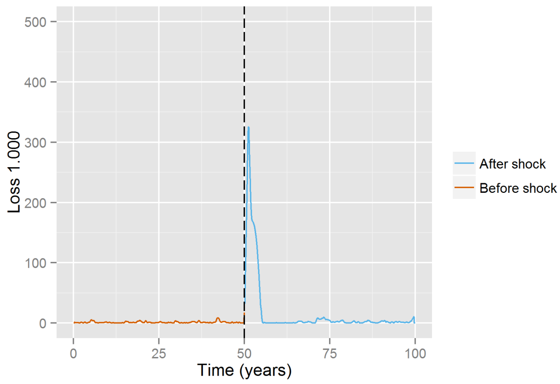
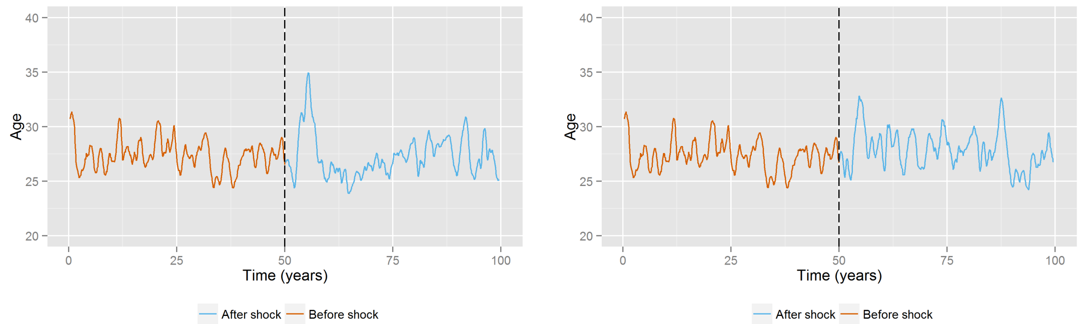


Figure 12: Reaction to negative income shock in losses endured by the bank.

From a welfare perspective, it may be interesting to investigate how the shocks affect young households' chance of getting a foothold on the market. Being cut off from the freehold market will, at least in a Danish context, be associated with a welfare loss for those who desire to live in a house rather than an apartment, as well as it limits the geographical scope of possible residences. The latter may be particularly problematic for families with children, who often prefer less urbanized areas, see Andersen (2011). Young households in the model, read 'families with children', are unfortunately also those whose options on the housing market are most adversely affected by a negative income shock. Before the negative shock hits, these households are already only just able to afford the freeholds of lowest quality, but this becomes even harder after the shock.[28] Older households with saved up home equity will of course loose more money than the young, because they loose both part of their equity and income. Yet due to their remaining equity, they still have the option of moving to a lower quality freehold than their current, in case they are forced to move. Young households are, on the contrary, without that opportunity, so they have to postpone the time of their first purchase. In figure 13a, we observe precisely that the average age of first time buyers rises to 35 in the period after the negative

[28]Especially, even if the prices of freeholds fall as much as income, it may still be impossible to purchase a freehold if monthly debt expenses of doing so are too high to also maintain a sufficient consumption of non-housing goods.

shock. Interestingly, the same thing happens on a smaller scale in the positive shock case. In figure 11b we saw that prices reacted with a period of stability and a level slightly above average. This may account for the seemingly increased age of first time buyers in the 20 years after the positive income shock, because they have to wait longer before their income has grown sufficiently to act on the market. In figure 21 of the appendix, it is evident that the number of households having to rent (because of insufficient income to buy a freehold) takes a long swing above average after the positive shock.[29] For comparison, when the negative shock hits there is a higher but shorter spike in the number of households renting. Note again, that the model at this stage is not well suited to account for long term effects of shocks, as the supply of dwellings is constant. The bottom line interpretation is still, that a negative income shock squeezes out young households from the market more severely and abruptly than a positive shock, but also for a shorter time.



(a) Negative income shock.

(b) Positive income shock.

Figure 13: Average age of first time buyers when shocking the income process.

[29]To be clear, when the number of renters rises, so does the total number of households in the simulation as there is fixed number of freeholds, which all must have an owner.

7 The statistical equilibrium of an ABM

The description of the model so far has been centered around a single representative simulation in which the central statistics seem to move around constant level. This could, in a preliminary sense, be lend the interpretation that the model is governed by equilibrating forces and as such represents a stable market. This is certainly what we want from an ABM, as the goal was to 'grow a virtual economy from the bottom-up'. Nevertheless, eyeballing a single simulation run is not sufficient evidence to conclude that the model in general attains an equilibrium in an economic sense. That follows from the fact that a simulation of an ABM inevitably relies on a degree of pseudo-randomness in the initialization and throughout the run. For example, dwellings need to be randomly assigned to household at the beginning of the simulation, and the order in which households are allowed to trade is randomized in every round. In order to implement randomness in the simulation, all stochastic variables depend on a predefined sequence of random numbers, which is defined by a seed and contained in the programming environment. Thereby, the exact same simulation can be run repeatedly if the seed remains unchanged, even if the agents are exposed to randomness. It is quite intuitive then, that the choice of seed for a simulation run should not have lasting effects for the model's statistics, if the analyst wishes to offer a structural interpretation of the setup. For example, the initial random distribution of dwellings among households could have an effect on which freeholds get traded in the first periods, which again could have persistent impact on the price level through out the simulation. But then the found price index would depend on the initial randomness and perhaps not primarily on the theoretically important aspects of the setup.

To be more exact about the equilibrium properties of an ABM, an agent-based equilibrium should be defined as statistical in the sense that the model's relevant statistics are stationary time series. Thereby, we know that the model is stable. Furthermore, the statistical equilibrium is unique if the relevant statistics are ergodic time series, because then the effect of initial conditions eventually vanishes. The ensuing method of testing whether the housing ABM obtains a unique statistical equilibrium follows the exposition of Grazzini (2012).

A stochastic process $\{X_t\}$ is said to be strictly stationary if the joint distribution of realizations does not depend on the specific interval in time. That is, let $F_X(x_{t_1}, \ldots, x_{t_n})$ be the cumulative distribution function of joint distributions of realizations up till time n. Then strict stationarity follows if $F_X(x_{t_1}, \ldots, x_{t_n}) = F_X\left(x_{t_{1+\tau}}, \ldots, x_{t_{n+\tau}}\right)$ for all n and τ. This means that all moments of the process remain constant. A stochastic process can be said to be stationary up to an order k if only its k first moments are independent of time. Most importantly, second order stationarity (or 'weak stationarity') implies that the mean and autocovariance are independent of time. That is to say, $\mathrm{E}[X_t] = \mu$ and $\mathrm{Cov(X_t, X_{t-\tau})}$ depends only on τ and not on t. Probably, the most well known test for stationarity in the economist's toolbox is the Dickey-Fuller test. It tests whether a unit root is present in a time series (non-stationarity) for which the data generating process is autoregressive and the error terms are i.i.d. The main issue with this test, and extensions of it, is the assumption of a parameterized data generating process. The complexity of an ABM means that its data output cannot necessarily be described by a parameterized function. Therefore, the test employed to assess stationarity needs to be non-parametric and as such, without assumptions on the data generating process.

One non-parametric test that lends itself to testing both stationarity and ergodicity of time series generated by an ABM is the Runs-test, or Wald-Wolfowitz after its authors. This test can be used to assess whether observations can be fitted to a a given function. The intuition behind the test is that if the time series can be described by the function, then the observations should be distributed randomly over and below the function. Therefore, the null-hypothesis is exactly that observations are randomly distributed over and below the given function. Observations above the function are assigned a 1 and observations below assigned 0. Define now a run as a sequence of identical symbols, i.e., when a subset of observations are consecutively either above or below the function. For example, the sequence $\{1, 0, 1, 1, 1, 0, 0\}$ has 4 runs $\{1\}, \{0\}, \{1, 1, 1\}, \{0, 0\}$, meaning that the given function was crossed 3 times by the observed series. Let n denote the number of observations above the given function and m the number beneath. The expected number of runs, R, is given by

$$\mathrm{E}[R] = \frac{2nm}{n+m} + 1 \tag{27}$$

and the variance is

$$\mathrm{Var}[R] = \frac{2nm(nm-n-m)}{(n+m)^2(n+m-1)}. \tag{28}$$

For a large enough sample, the distribution of R can be shown to be approximately normal. Therefore,

$$Z = \frac{R - \mathrm{E}[R]}{\sqrt{\mathrm{Var}[R]}} \tag{29}$$

is standard normal, $Z \sim \mathcal{N}(0, 1)$, and can thus be used to test the hypothesis that the observations fluctuate randomly around the given function.

Observe now, that to check the order k stationarity of a time series, one needs to check the constancy of its n first moments. If the time series is split into a number of windows W, of say 100 consecutive observations, stationarity would imply that the moments of each window are well represented by the overall moments of the entire series. Define therefore the uncentered moment of order k of window i as

$$\mu_{k,i} = \frac{1}{T} \sum_{t=T(i-1)}^{T} (x_t)^k, \quad i = 1, 2, \ldots, W \tag{30}$$

where T is the time span of the windows. In order to apply the Runs-test for stationarity, one needs simply to find the runs of the series $\{\mu_{k,i}\} = \{\mu_{k,1}, \mu_{k,2} \ldots, \mu_{k,W}\}$ when the given function is the corresponding moment k of the entire series. Given the chosen confidence level, the computed Z-statistic for $\{\mu_{k,i}\}$ determines if $k-$moment of the time series is independent of time. So to conclude, if all k moments are independent of time by this test, the time series is stationary of order k.[30]

As Grazzini (2012) shows, when choosing the length of windows, it is important that there are adequately many such that the distribution of runs is close to normal. On the other hand, the moments calculated in each window should also be accurate, requiring them to be of substantial length. Therefore, he shows that with a window length $T = 1000$ and a number of windows $W = 100$, the test successfully detects stationarity even in a time series close to non-stationarity. If one was investigating traditional time series of economics, a requirement of 100,000 observations for a test would normally render it completely useless. However, for the case of an agent-based simulation of a reasonable run-time, we can generate all the observations needed.

Next step is to test the time series for ergodicity. Loosely speaking, a time series is ergodic if the effect of an observation eventually dies out such that two distant observations are independently distributed. Formally, a stationary process $\{x_t\}$ is also ergodic if for two bounded functions $f : \mathbb{R}^k \to \mathbb{R}$ and $g : \mathbb{R}^l \to \mathbb{R}$

$$\lim_{n\to\infty} |\mathrm{E}[f(x_t, \ldots, x_{t+k})g(x_{t+n}, \ldots, x_{t+n+l})]| \tag{31}$$

$$= |\mathrm{E}[f(x_t, \ldots, x_{t+k})]| \times |\mathrm{E}[g(x_{t+n}, \ldots, x_{t+n+l})]| \tag{32}$$

To see the difference between stationarity and ergodicity, consider the following experiment. The process $\{y_t\}$ is generated by letting y_1 be drawn from a random distribution. All subsequent observations for $t > 1$ are then set equal to y_1. In that case, the joint distribution of any two subperiods of the series will be equal, but the effect of the first observation is everlasting. Hence,

[30] I programmed the implementation of the Wald-Wolfowitz tests in R, which is available via the link in the preamble.

it is stationary, but not ergodic. The importance of ergodicity is therefore that we can be sure that a single time series produced by an ABM converges to the statistical equilibrium no matter the seed used for randomization and initialization. Thereby, the moments of a *single* time series will represent the true moments of the data generating process in the model, Grazzini (2012).

The test for ergodicity is a slight modification of the test from before. The original Wald-Wolfowitz test is designed to test whether two samples come from the same distribution. Intuitively, with a little adaption, we can apply it to test whether a range of time series produced by different seeds all come from the same distribution - meaning that they are all converging to the same statistical equilibrium. The basic test is to consider two stochastic processes,$\{x_t\}$ and $\{y_t\}$, and suppose that they are drawn from the continuous distributions $f(x)$ and $g(x)$ respectively. Then we form the set Q as the union of the two processes, and sort it ascending order. In order to create runs for the test, the set V is created, where $v_i = 1$ if $q_i \in \{y_t\}$ and $v_i = 0$ if $q_i \in \{x_t\}$. A run in V is defined as sequence of only 1s or 0s like previously. The number of runs R is computed together with E[R] and Var[R], such that the hypothesis $f(x) = g(x)$ can be tested with the Z-statistic. The test is only left-tailed, because any lasting difference between the two samples will tend to lower R compared to E[R]. To implement the test for ergodicity of, say, the first moment, one long series is generated from the model. As before, it is split into W windows, each of length T, and the first moment is calculated for each window. Thereby the first sample. The second sample is generated by running the simulation W times with different seeds to generate a set of time series each of length T. Then the relevant moment is calculated for each of the W time series, which then constitutes the sample for comparison. The union of these two samples thus constitutes the set Q mentioned above, and the test for ergodicity can be executed as described.

To sum up on the discussion of ergodicity and stationarity, the need for these properties comes from issue that characteristic outcomes of an ABM are normally not possible to derive analytically. Followingly, we need a statistical notion of equilibrium which is not confounded by arbitrary initial conditions. Stationarity and ergodicity ensure that the moments of a single time series represent the statistical equilibrium of the model regardless of the initial conditions. In so far features of the statistical equilibrium has to be compared with real data, ergodicity and stationarity are thus crucial if the comparison is to be meaningful. Further, stationarity and ergodicity have implications for sensitivity analysis of parameter values. If these properties obtain, then variations over a parameter value can be implemented with a fixed seed in the sensitivity analysis. The test for stationarity of the price index in the benchmark specification of the model yields a P-value of 0.31273, whereby the null hypothesis of stationarity is easily accepted. Slightly disconcerting is it that the test for ergodicity gives a borderline case. The

P-value here is of size 0.04444, meaning that the initial state of the model seemingly has some effect on the level of prices throughout the model. This has however been improved from even lower P-values in earlier versions of the model by initializing it into a state closer to its long run behavior. In the current state of the model, it is furthermore movements in prices which are of primary interest rather than their level.

8 Macroprudential regulation

8.1 Empirical perspectives on macroprudential regulation

This section is devoted to the effects of regulation on borrowing and credit extension to the housing sector. The ABM developed in this thesis is well suited to test the effects on stability of the housing market when experimenting with macroprudential regulation, which is undertaken at the end of the section. In order to motivate what magnitude of effects we should expect from the experiments, I first present the most recent findings in the empirical literature on the subject.

Macroprudential regulation is in short all the non-interest rate instruments used to control developments in asset markets and the financial sector. Among these instruments are caps on borrowing, lending, exposure limits and taxes levied either on the supply- or demand side of the market. Academic interest in such policies have not bloomed until lately in the wake of the recent period of financial crisis and real estate booms, so literature on the subject is relatively sparse but growing. A handful of recent papers analyzes the empirical effects of macroprudential instruments (MPI) in panels of countries, providing some evidence that MPIs can curb destabilizing tendencies. Particularly, they suggest that easy credit conditions do affect credit growth and to some extent housing prices, which makes a case for regulation in that direction.

To be more specific about the implementation of MPIs on the supply side of the credit market, the most frequent of them are reserve requirements, liquidity requirements, risk weighting of loans and exposure limits. Exposure limits may be targeted directly on lending to the housing sector in order to embank credit expansions in this regard. An example of this is to restrict a bank's housing loans to a certain maximum fraction of the bank's equity. Policies affecting the demand side of the credit market are most commonly the LTV (loan-to-value) and DTI (or debt-service-to-income, DSTI, the cost of borrowing against income). Lastly, taxes that affect the cost of borrowing, buying and owning real estate also factor in as MPIs, as they work to modify the user cost of having a house.

One notable paper on the empirical effects of MPIs is Kuttner and Shim (2013), who create a data set consisting of 60 developed or emerging economies for the last 30 years. On this data, they investigate the connection between usage of MPIs and the development in housing prices and growth in housing credit respectively. The paper employs three different statistical approaches, (i) a dynamic fixed effects model; (ii) a mean group regression and (iii) an event study. All three types of models are estimated for both credit growth and housing prices separately, with either one or more MPIs included among the regressors in (i) and (ii). The collected effect of the models is that credit growth is better controlled by MPIs than housing prices, which generally are not much affected. For example, when including LTV and DSTI separately (i) on credit growth, they are both significant with DSTI having the larger effect of lowering credit growth by 6.8% when increased by one unit. When DSTI and LTV are included in the same regression, only DSTI stays significant and LTV goes to zero. The authors suggest that this may be caused by the two instruments getting implemented simultaneously when needed. The authors also distinguish between loosening and tightening of the MPIs in (i) on credit growth, which yields that DSTI-limits only have significant effect in case of tightening. When model (i) is implemented on housing prices, however, none of the MPIs have significant effects on a 5% level. The result of model (ii) is not promising either, as none of the MPIs have detectable effects here on either credit growth or housing prices. There a minor impact of DSTI contractions on credit growth in the event study (iii), where also tax increases seem to matter for housing prices.

Another related survey is that of Cerutti et al. (2015) which applies much the same approach as above in investigating the effects MPIs. They create one index based on borrower-targeted policies (DTI and LTV) and another based on financial institutions (reserve requirements and exposure variables). These are included in a fixed effects regression model of housing prices or credit growth on lagged GDP and interest rates. Their data consists of 119 countries in the period 2000-2013, including developing, emerging and advanced economies. As noted by the authors, the impact of macroprudential regulation may vary with the openness of countries and their state of development. Borrowers in more open economies may find it easier to circumvent the regulation by obtaining loans across borders or by non-financial institutions. The same argument applies to the explanation why advanced economies may see less effect of MPIs. The financial sectors in these economies are better developed and possibly offer better chances of avoiding the limits of MPIs. Consequently, regressions are also performed on subsamples based openness and level of income.

Part of the picture from Kuttner and Shim (2013) emerges in Cerutti et al. (2015). Putting both MPI-indices together, the effect of MPIs on credit growth gets significant and sizable for all subsamples. The effect is more than three times larger in emerging markets and developing economies than in advanced, where a one standard deviation change in the MPI-index reduces credit growth by 2.2 percentage points. The relative difference between open and closed economies is also notable, with almost doubling of the coefficient to MPI. When the MPI-indices are split into financial institution- and borrower-targeted respectively, the effect vanishes altogether for advanced economies. Closed economies and emerging markets still see strong effects from both kinds of MPIs. When regressing on the single MPIs instead of indices, DTI is more effective than LTV, and the effect is much stronger for emerging markets than advanced economies. Turning to the effect of MPIs on housing prices, the authors find little evidence that macroprudential policies have been able to dampen growth in housing prices. Coefficients to both kinds of MPIs are insignificant and close to zero in both emerging and advanced economies. The paper thus affirms the previous finding, that credit but not housing prices are effectively regulated by MPIs.

A different approach is taken by Claessens et al. (2013), who create a large panel of individual banks from a range of both advanced economies and emerging markets to investigate the effect of MPIs on the banks' growth in lending and leverage. Their analysis shows especially that both LTV and DTI-caps are effective in limiting the vulnerability of banks, as they reduce both growth in total assets, leverage and non-core to core liabilities. Overall, the DTI has a greater dampening effect, where an introduction of this policy implies a 2 percentage point lower growth in assets compared to 0.49 when LTV limits are implemented. As Cerutti et al. (2015) also notes, it is likely that the effects of MPIs may be larger in boom periods than in busts. This notion is affirmed when the analysis is differentiated by expansionary and contractionary phases of the credit cycle. Many of the coefficients then turn from significant in the boom phases to insignificant in the busts.

The paper by Funke and Paetz (2012), which studies the effectiveness of active LTV-regulation in Hong Kong, stands out by being more theory driven than the other papers. The authors construct a DSGE-model representing the economy of Hong-Kong, where LTV-limits have been employed actively to control housing prices since the 1990s. The model is based on Iacoviello (2005), who first introduced collateral constraints in a DSGE-model that featured a housing market. The dynamics of the model hinges on the inclusion of two kinds of households, savers and borrowers, where borrowers discount future consumption heavier. Borrowing is constrained by an LTV limit through the intertemporal condition, that current debt cannot exceed a given fraction of the expected worth of the borrower's collateral in the coming period. The novelty of the model is that the central bank tightens the LTV limit endogenously when quarterly growth housing prices exceeds a threshold of 4%. This reflects the fact that the monetary authority historically has not adjusted it incrementally or linearly, but abruptly and only when the market is evidently heating up. The impulse-response functions of the model are obtained by delivering a persistent shock to the preferences of households, such that they demand more housing than in steady-state. The impulse-responses show that lowering the LTV-limit even by 15 percentage points from 70% does not strongly dampen the inflation in housing prices. The other variables of the economy are however kept in less deviation from steady-state after the shock when this policy is conducted. Taken together, the paper does recommend an active, non-linear rule on LTV-limits although the effects are on a moderate scale.

Rounding off this review, there are a few more papers worth mentioning. Glaeser et al. (2010) ask the question whether too easy credit conditions, i.e. absence of due macroprudential regulation, can explain the boom in housing prices in the U.S. in 1996-2010. When taking their model to data, they do not find that the upward trend in approval rates or downward trend in collateral requirements for mortgages are large enough to account for the large swing in housing prices. So even if MPIs might have softened the boom, the paper suggests that there were more forces at play than lenient credit conditions. This paper, as well all previous papers of the current section, only analyze the effects of macroprudential policies in the aggregate or on a representative household. Two papers suggest however that MPIs may impact households differently depending on their income and propensity to consume. In Landvoigt et al. (2015), the price dynamics of the housing market of San Diego is investigated during the 00s boom by an assignment model. The authors are therefore able to investigate how credit conditions affected households at different points on the distributions of age and income. When fixing the interest rate and LTV to the year 2000-level, the rate of housing value to income drops sharply for young households, but not for the elder. In other words, young household are those most exposed to macroprudential regulation. The paper by Mian and Sufi (2011) is an empirical investigation of the tendency to extract home equity from increasing housing prices among U.S. households. One of their findings is that especially households with low credit scores and high credit card utilization tend to borrow against increases in their property value. The extracted equity was primarily used for consumption or improvements in the existing property, but not for new real estate. The same households were also subject to a large relative increase in default rates and drop in auto loans after the housing market collapsed. In that sense, even if macroprudential policies targeted at borrowers are not fully able to curtail the development in housing prices, they may still be valuable instruments to protect households with high consumption propensity against over-borrowing.

8.2 Experiments with DTI-limits in the ABM

The take away from the empirical literature was that MPIs seemingly have not had strong effects on housing prices in advanced economies. The empirical studies did have certain limitations of limited sample sizes, selection biases and measurement errors in the instruments. Furthermore, they are generally not able to discern the effects of varying the intensity of a certain regulatory instrument; for that we need a more structural approach. In the following analysis, the thesis' ABM is applied to test the effectiveness of the DTI instrument in limiting the endogenous fluctuations of the housing market. The model is in principle just as eligible for experiments with LTV limits, except that it would require a more elaborate saving behavior of young households than the current model features.

It is a common rule of thumb in the Danish banking sector that the debt-to-income on housing loans to households should be in the range of 3-3.5, but that it rose above 4 during the 00s, cf. Liliegreen (2014). It was particularly high income families who were allowed to borrow more relative to their income, as 25% of households in the top quartile of the income distribution had a debt to income ratio above 4, which only applied to 15% of the general population, cf. Andersen et al. (2012). As for legislation, there is no official DTI limit on mortgages in Denmark, but borrowers with a DTI above 3.5 are considered as low quality by the FSA in their calculation of banks' risk exposure (for which there is a strict limit). Since policies with the power to avert new turmoil in the housing sector are in high demand, it is interesting to test if changing the DTI criteria in the ABM will matter for its stability. Here, stability is defined as the absence of large swings or sudden collapses in the price index. Three scenarios are studied and compared, namely one with a DTI limit of 2, 3 and 4.5 respectively (and with all other parameters at their benchmark value). A limit of 2 is of course very low in a Danish context, but it is included to highlight if there are non-linear effects in the model from raising the limit at different points.

The analysis rests on a Monte Carlo exercise where the model is run 200 times in each scenario with different seeds. For each time the model is run, the cyclic behavior of the price index is calculated together with its tendency to display collapses. Following the method utilized in the empirical investigation of Bracke (2011), the cyclic behavior is characterized by (i) the duration between two peaks in the price index, which defines the length of a full cycle; (ii) the relative increase from a local minimum to the next local maximum. The latter is denoted the amplitude of a cycle. Thus, we need to identify all the local minima and maxima of the price index. With a slight modification of the algorithm in Bracke (2011),[31] turning points of the price index is identified by following:

[31] I programmed it in R, and it is available via the download link for the model in the preamble.

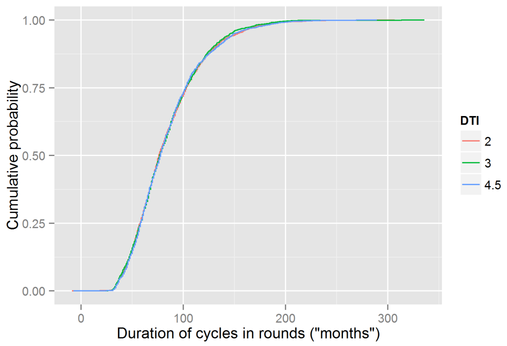
- When iterating through all data points, each data point y_t is compared to a window of data points on both sides of it. A window consists of 30 months. Thereby, a maximum y_t^+ fulfills that $(y_{t-30}, \ldots, y_{t-1}) < y_t^+ > (y_{t+1}, \ldots, y_{t+30})$. Similarly for a minimum, $(y_{t-30}, \ldots, y_{t-1}) > y_t^- < (y_{t+1}, \ldots, y_{t+30})$.
- A minimum must be followed by maximum and vice versa. If two turning points of the same kind follows each other, the last one is selected to get the full breadth of the cycle.

This algorithm is applied to each of the 200 price indices in the three scenarios after smoothing them with a 5-month moving average. Then the amplitude is calculated between every two following turning points together with the duration between every two consecutive maxima. This exercise yields a collection of cycle amplitudes and durations for each scenario. Next step is to compare these statistics, which I do by comparing their empirical distributions.[32] Figure 14a and 14c display the empirical cumulative probability functions for amplitude and duration.

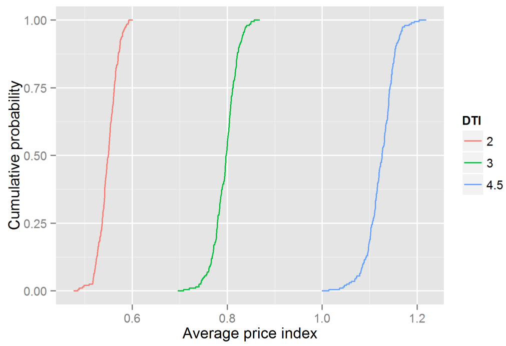
Obviously, the CDFs of cycle duration from each scenario are flat on top of each other. Hence, it does not seem that DTI limits in the chosen range have an effect on the duration of cycles. The case is, however, different for amplitudes. We see that the CDF of amplitudes in the 4.5 limit scenario is placed furthest to the right in the plot. This implies that each quantile of the distribution is associated with a higher amplitude than that of the two other distributions. Note also that the CDF of the scenario with DTI limit of 3 is at the right of the least lenient scenario, although closer. This suggests that effect of increasing the DTI limit is more disturbing when it happens at high levels. Figure 14b shows that price levels are quite apart in the three scenarios. When recalling the econometric findings of the previous section, that housing prices were not clearly affected by DTI limits, this may seem excessive. The statistical models were nevertheless focusing on the growth in housing prices and not levels, which makes a meaningful comparison difficult.

The statistical significance of the difference between CDFs can be gauged by the Kolmogorov-Smirnov test (KS test). It tests whether the unknown distribution of a sample is statistically the same as a particular distribution. In the present case, we test if the empirical distribution of amplitudes under one scenario is the same as that of another scenario. Table 3 contains three KS tests which confirm the claims above. They are one-sided, so the alternative hypothesis is that a given value of amplitude in distribution $\mathbb{A}_j$ is associated with a lower quantile than in distribution $\mathbb{A}_k$. Thereby, the CDF of $\mathbb{A}_j$ is to the right of the CDF of $\mathbb{A}_k$. The P-values strongly rejects the null hypothesis of equality in all three tests, so the distributions are indeed positioned in the same order as their associated DTI-limits.

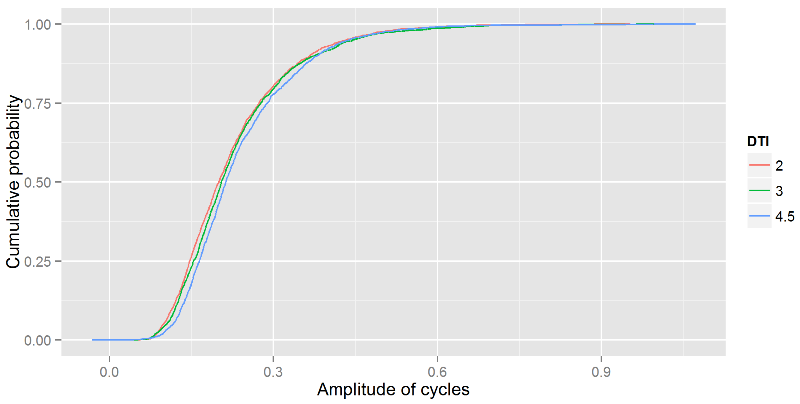
[32]Inspired by the Monte Carlo exercise of Gaffeo et al. (2014)



(a) CDFs of duration of cycles.



(b) CDFs of the average price index.



(c) CDFs of amplitude in freehold price cycles.

Figure 14: Cumulative distribution functions of behavior in the price index under different DTIs in Monte Carlo experiments

H_0	H_A	P-value
$\mathbb{A}_{4.5} = \mathbb{A}_3$	$\mathbb{A}_{4.5} < \mathbb{A}_3$	0.00114
$\mathbb{A}_{4.5} = \mathbb{A}_2$	$\mathbb{A}_{4.5} < \mathbb{A}_2$	0.00000
$\mathbb{A}_3 = \mathbb{A}_2$	$\mathbb{A}_3 < \mathbb{A}_2$	0.01519

Table 3: Kolmogorov-Smirnov tests for the equivalence of CDFs of amplitudes in price cycles under different DTI limits. $\mathbb{A}_j$ is the distribution of amplitudes when DTI limit is j.

In sum, the model suggests that there is a destabilizing effect of increasing the allowed DTI from a level of 3 to 4.5 - albeit not a very strong one (as in the empirical findings). Another important aspect of stability to shed light on is the model's tendency to generate sudden crashes which resemble crises on the housing market. The above analysis regarded both the tendency of build-ups and bursts in the market. Since bursts are more problematic for the economy than build-ups, we want to investigate whether the scenarios result in different propensities for sudden drops in prices. To do so, the same identification procedure as above is utilized to identify only minima. Then, for each minima, the relative decrease from the price index one year ahead of the crash is calculated. More precisely, the amplitude is given by

$$A_t = \left| \frac{y_t}{\bar{y}_{t-10}} - 1 \right|, \quad \bar{y}_{t-10} = \frac{1}{10} \sum_{i=1}^{10} y_{t-10-i} \tag{33}$$

Collecting the amplitudes of crashes for each scenario leads to figure **??**. Here we note that the intuitive order of the CDFs from before is preserved, but the 4.5 limit CDF is now even more separated from the others. This is an interesting result, as the model thus suggests that the market becomes more vulnerable to specifically collapses when high DTI limits are permitted. Again, the effect is not overly dramatic, but it does amount to adding 2-3 percentage points to the amplitude of collapses at each quantile above .25 in the lenient scenario. The effect is probably at the lower edge of what an extended model with speculators would have yielded. Speculators are characterized by being highly leveraged and aggressive bidders. Haughwout et al. (2011) show that the presence of speculators on the American market during the 00s amplified the bust exactly because of their leverage, which made them prone to go into delinquency and following foreclosure when prices first started falling. A sudden inflow of sellers in the thesis' model would similarly have an escalating effect on diving prices. Excessive supply of freeholds increases sales time which decreases asking prices. Newly posted freeholds are priced in accordance with those recently sold, prolonging the fall.

Whether an increased tendency for crashes should be traded off with easier borrowing conditions is nevertheless a matter the policy maker's preferences. Letting households borrow more against their income facilitates consumption smoothing, all else equal, which is beneficial especially for young buyers. Yet as Andersen et al. (2014) have shown, the most leveraged Danish households also experienced the largest drop in consumption in the backwash of the financial crisis; implying that their consumption was not well smoothed.

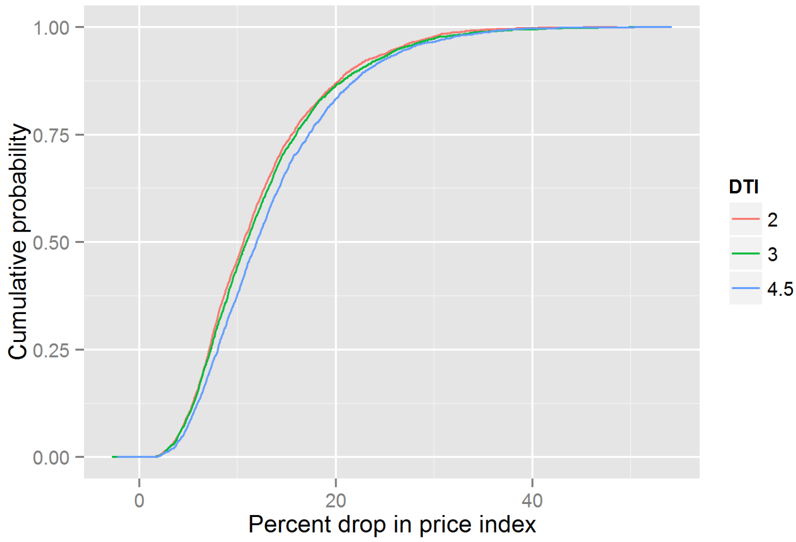


Figure 15: Empirical distributions of bursts in price index in Monte Carlo samples.

9 Conclusion

This thesis has presented a novel agent-based model of a housing market, which is tailored to a Danish context. As such, the model is a simulation of the Danish housing market which, with the sufficient computational power, could be scaled up to a 1:1 representation. The model includes a bank, dwellings of varying quality and a substantial number of households who are heterogeneous with respect to income, age, moving probabilities and home equity. Households are modeled such that their income and propensity to move, i.e. trade on the housing market, is age specific. Because of the life-cycle specification of households, the model is able to provide detailed analysis of the impact from market gyrations on subgroups of the population. The impact on the financial sector can also be evaluated due to the inclusion of a mortgage bank. The model is in its present state still meant as a step towards a more comprehensive version, which is capable of delivering forecasts as well as policy analysis of the housing market on a fine grained level. This model would need to be calibrated more closely to micro data than the current and feature some element of speculation.

The model was analyzed by delivering a positive and negative mortgage rate as well as income shock. In both cases, it turned out that the effects of positive and negative shocks were not symmetric. A doubling of the interest rate from 3% to 6% had a clear negative consequence for the market in terms of falling prices and prolonged selling time, whereas the cut to 0.5% did not alter the dynamics much. Increased interest rate also resulted in higher auto correlation in selling time and a non-linear affect on the price structure. The negative income shock produced a dramatic fall in prices whereas the positive only had a minor effect. The model has in its current state probably a more realistic behavior in downturns compared to upturns, as none of the agents are trying to turn a profit on increasing prices. The positive income shock therefore did not result in an inflow of investments to the housing market, that would drive up prices. The negative income shock was furthermore shown to be particularly adverse to young households, as it pushed the age of their first purchase by several years.

The thesis' model is characterized by a continuing dynamic of expansions and contractions on the housing market. It therefore lends itself to analyzing the possibly ameliorating effect of macroprudential regulation. To that end, the behavior of prices was investigated by a Monte Carlo study of varying the debt-to-income limit on mortgage contracts. The study showed that allowing the debt-to-income limit to reach 4.5, as it did during the 00s, is associated with moderately more severe collapses in prices compared to keeping it at 3. This result nods at the importance of macroprudential regulation.

10 Appendix

Table 4: Parameters of the model.

r	0.03
α	0.2
λ	0.6
P^{O}	0.01
P^{R}	0.1
n^{search}	100
n^{ask}	100
n^{avg}	6
ϑ^{I}	3
C^{min}	8000
μ^{u}	0.03
μ^{d}	0.01
$N_{t_0}^{households}$	5000
Inflow of households	13

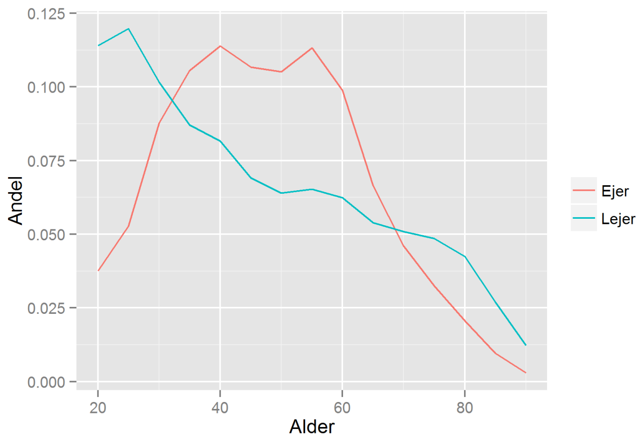


Figure 16: Distribution of age for owners and renters respectively in 2010. Table BOL66 of Statistics Denmark

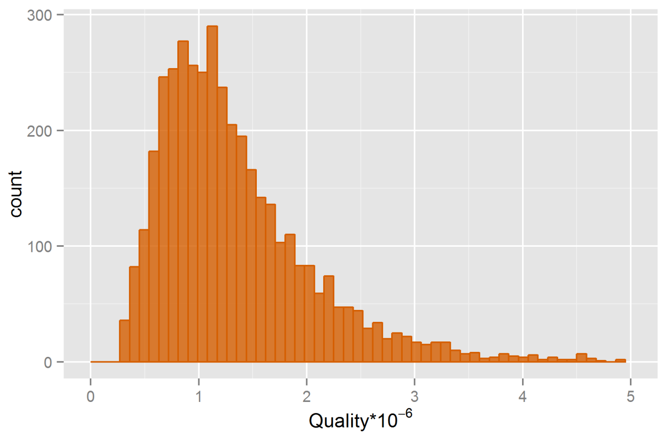
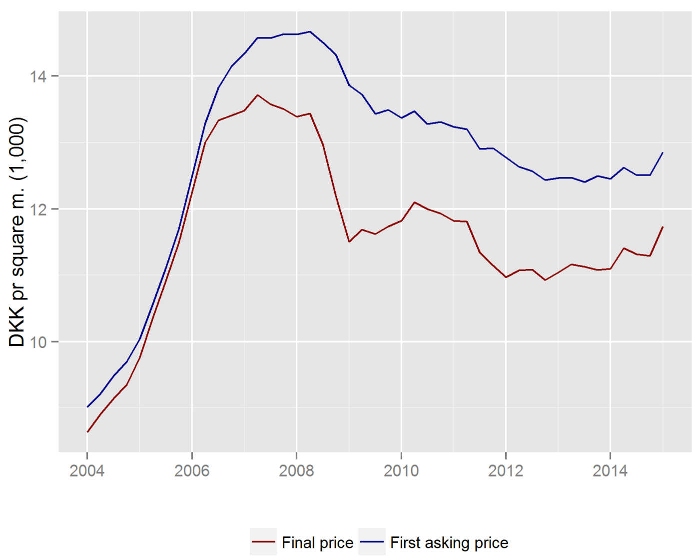
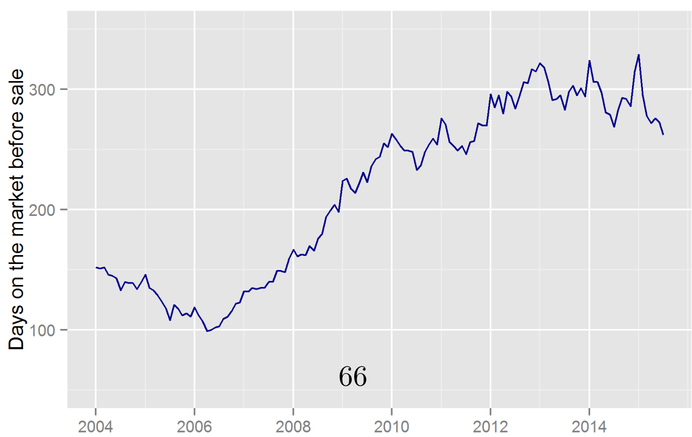


Figure 17: Distribution of freehold quality (fixed during simulation).



(a) Average first posted asking price versus final selling price, source: Association of Danish Mortgage Banks (BM010).



(b) Average days on market before selling, source: Association of Danish Mortgage Banks (UDB030).

Figure 18: Empirical developments on the market for houses in Denmark.

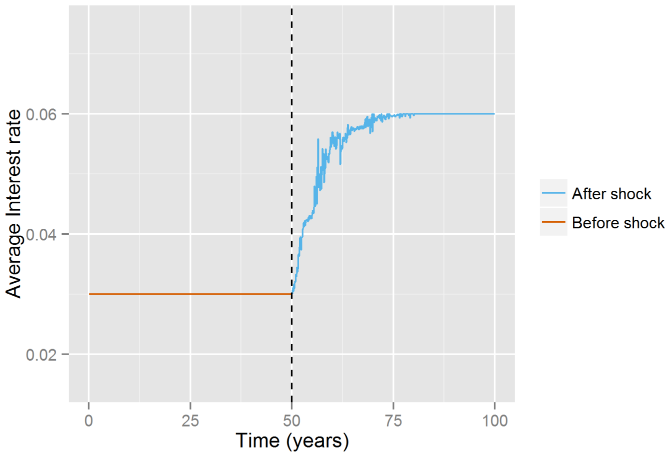
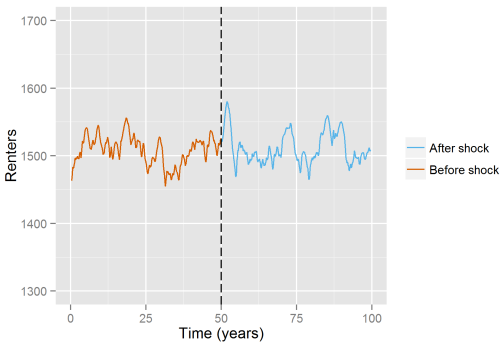
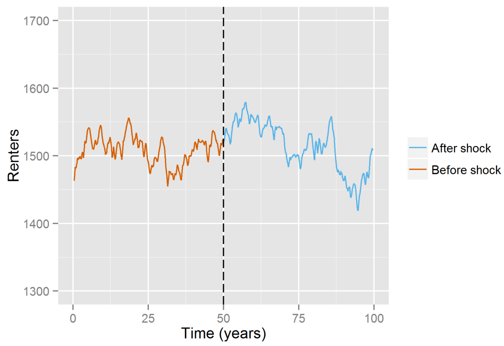


Figure 19: The average interest rate payed by mortgage holders when the interest rate is shocked to 6%.

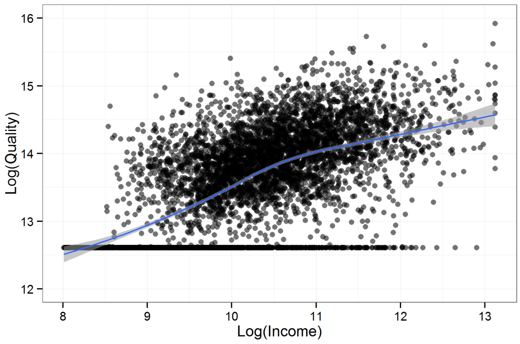


(a) Negative income shock.

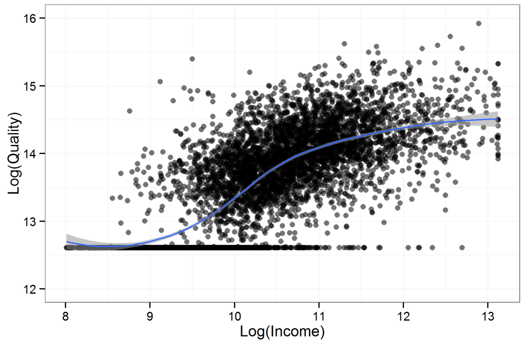


(b) Positive income shock.

Figure 20: Number of households living in rentals when shocking the income process.



(a) Negative income shock.



(b) Positive income shock

Figure 21: Income-quality relation 10 years after income shocks.

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