Chapter 5

Households

5.1 Overview

This chapter describes the representative households in the DREAM model. We start in section 5.2 by explaining the construction of households as well as the general assumptions made in relation to households concerning timeline, composition, etc. In section 5.3 we present the specification of households' preferences concerning period utility (preferences over the consumption bundle and disutility from work) and the intertemporal aspects of consumption, savings and bequest decisions. As a prerequisite for deriving the optimal intertemporal choices, in section 5.4 the composition of household income in DREAM and the accumulation of household wealth is described in detail. Section 5.5 finds the solution to the consumption and saving problem of households, while section 5.6 deals with labour supply. Section 5.7 describes the behaviour of old (i.e. non-planning) households, and finally in section 5.8 we describe how households split the overall value of consumption in each period between the different components of the consumption bundle.

5.2 Construction and assumptions

In DREAM the population is organized in representative households in the sense that each household represents one specific generation, i.e. men and women who are born during the same year and their children under 17 years of age. These households are the decision-making units in relation to consumption, savings and investments in owner-occupied dwellings. The representative nature of households does not mean, however, that individuals of a household are assumed to be identical in every respect. Individuals of the same age are different in two respects: gender and (ethnic) origin. While the gender distinction is obvious, the origin distinction is a bit more involved.

The DREAM model uses DREAM's population projection, which disaggregates the Danish population into the following groups based on people's origins: Immigrants from so-called less developed countries, immigrants from more developed countries, descendants of immigrants from less developed countries, descendants of immigrants from more developed countries, and the rest of the population[1]. Within any of DREAM's representative households, the origin and gender distribution of individuals matches that of the corresponding "true" generation in DREAMs population projection. In this way each household encompasses 10 representative individuals: Males and females of the 5 origins. This formulation is made in order to enable DREAM to capture differences in observed economic behaviour among the 10 population groups mentioned above. For instance, DREAM uses RAS and Law Model data to model the different population groups' participation rates in various government transfer schemes. DREAM, with the aid of its population projection, is thus able to take into account the effect on government finances of a changing ethnic composition of the population[2]. Other examples of different economic behaviour across both gender and origin are the supply of labour, where differences in productivity and disutility of work results in differences in labour supply, and the take-up rates of individual government consumption.

The life of a household begins when children aged 16 turn 17 and leave the parent household to form a household of their own. A household exists until the end of the year in which its members are 101 years, at which point it ceases to exist. However, even though the household continues to exist, the inhabitants of the household pass away over time. Hence, the size of a representative household initially increases as parents have children and more people immigrate and then gradually falls as individuals pass away. It is assumed that surviving individuals retain undivided possession of the assets of the household until and including the age of 76 years.

The timeline of the representative households in DREAM is divided into two phases of life. From the household is formed and until the end of the period in which its members are 76 years old, the household supplies labour[3], enjoys consumption and makes savings decisions. During this phase of life, a household is said to be a planning household due to the forward-looking (life cycle) behaviour underlying its decisions. Although the household exists through the age of 101 years, it ceases to exist as a planning household at the end of the period when it is 76 years old, at which point it is dissolved as an economic entity and the remaining assets are given to the heirs as a bequest. From this point onwards, the household no longer makes intertemporal economic decisions. Rather, it simply consumes all income (pensions and government transfers) in each period. One can think of a person in a nursery home for elderly people.

[1] More developed countries are USA, Canada, Japan, Australia, New Zealand and all European countries excluding Turkey, Cyprus and some former Soviet republics, cf. Koch et al. (2004).

[2] For a detailed description of DREAMs population projection see Koch, Stephensen and Schou (2004).

[3] Though labour supply is very low from the age of 62 and onwards.

Many variables in the following will be expressed in units per adult in the household. Some variables are instead expressed per adult-*equivalent* due to the fact that we do not assign separate values to adults and children for the same variable, e.g. consumption. For instance, in the case of income mentioned above, this is given per adult and not per adult-equivalent since income is only received by adults. Consumption, however, is given per adult-equivalent since children consume as well as adults. The measure of adult-equivalents is obtained by summing over all individuals of the household, while assigning the weight $\frac{1}{2}$ to children, since it is assumed that children do not require the same amount of consumption as adults.

5.3 Preferences

5.3.1 Period utility

A household in DREAM obtains utility from consumption of goods and from leaving a bequest to its heirs, while it incurs disutility from time spent working. By $C^H_{a,t}$ we denote the overall *utility* in units per adult-equivalent obtained from consumption of goods by household a in period t. This overall utility is given by a nested CES subutility function defined over the different components of household consumption and presented in subsection 5.3.2 below. $Z_{a,t}$ denotes disutility from work of household a in period t. It is an average over disutilities of work of individual representative members of the household and, like $C^H_{a,t}$, it is given in units per adult-equivalent.

By combining utility from consumption and disutility of work, we obtain the period utility per adult-equivalent $u_{a,t}$ of household a in period t, which is defined as being additively separable in $C^H_{a,t}$ and $Z_{a,t}$:

$$u_{a,t} = C^H_{a,t} - Z_{a,t}. \tag{5.1}$$

Average disutility of work per adult-equivalent $Z_{a,t}$ is given by

$$Z_{a,t} = \sum_o \sum_s adj^{Hours}_t r^{LabFull}_{o,s,a,t} \frac{N^{Ind}_{o,s,a,t}}{N^{AdultEq}_{a,t}} \left(\zeta_{o,s,a,t}\right)^{-\frac{1}{\gamma}} \left(\frac{\gamma}{1+\gamma}\right) \left(1 + adj^{LS}_t\right) \left(L^S_{o,s,a,t}\right)^{\frac{1+\gamma}{\gamma}} \quad \text{for} \quad 17 \leq a < 7 \tag{5.2}$$

where $L^S_{o,s,a,t}$ is labour supply per adult-equivalent of origin o and sex s. In section 5.6 γ will be shown to be the elasticity of labour supply wrt. the after-tax real wage, while $\zeta_{o,s,a}$ is a level parameter in disutility of work. The reason that the above expression for disutility of work looks rather lengthy is simply due to the representative nature of households encompassing representative individuals of different genders and origins. The actual disutility of work per adult of origin o and sex s is represented by the term

$$adj^{Hours} r^{LabFull}_{o,s,a,t} \left(\zeta_{o,s,a}\right)^{-\frac{1}{\gamma}} \left(\frac{\gamma}{1+\gamma}\right) \left(1 + adj^{LS}_t\right) \left(L^S_{o,s,a,t}\right)^{\frac{1+\gamma}{\gamma}}, \tag{5.3}$$

which can be seen to be increasing and convex in labour supply. $r^{LabFull}_{o,s,a,t}$ enters in (5.3) because $L^S_{o,s,a,t}$ is labour supply conditional on the individual being in the labour force. $r^{LabFull}_{o,s,a,t}$ is the fraction of individuals of origin o and sex s who are in the labour force measured as full time employed people. As for adj^{Hours}, it is an exogenous parameter with the initial value of one that can be used to model an institutionally agreed decrease or increase in working hours.

Since individual disutility of work is disaggregated according to origin and sex, finding $Z_{a,t}$ is a matter of finding the average disutility of work across origin and gender. This explains why $\frac{N^{Ind}_{o,s,a,t}}{N^{AdultEq}_{a,t}}$ enters (5.2): Multiplying by the number of individuals of origin o and sex s $N^{Ind}_{o,s,a,t}$ gives total disutility of adult individuals of origin o and sex s, and dividing this by the total number of adult-equivalents $N^{AdultEq}_{a,t}$ in the household and summing over gender and origin consequently gives average disutility of work per adult-equivalent.

5.3.2 CES subutility

As mentioned, the utility value of consumption $C^H_{a,t}$ used in (5.1) comes from maximization of a CES utility function defined over the different components of household demand. While the actual derivation of demand for each component is deferred until section 5.8, the CES utility function is presented here for completeness. Figure 1 presents the nest tree of the CES function and will be explained after a few remarks on the notation.

In figure 1 we have suppressed the subscripts a and t and introduced 3 new subscripts indicating the type of consumption, producing sector and country of origin. This means that $C^H_{e,k,c,a,t}$ denotes consumption (expenditure) of type e, delivered from production sector k in country c, and consumed by household a in period t. Superscripts 1 and 2 are used to distinguish between intermediate goods and final goods, respectively. Consider first the set E indicating the type of consumption and defined as:

$$E = \{D, R, G, P, H, N\}\,. \tag{5.4}$$

Starting from the left, the elements in E represent the following types of consumption: consumption of dwellings (D), i.e. household demand for residential buildings and land, housing repair consumption (R), government sector goods consumption (G), private manufactoring goods consumption (P), housing consumption (H - aggregates over D and R as is evident from figure 1) and non-housing consumption (N - the aggregate of G and P).

Turning towards the delivering production sector we define the set K:

$$K = \{C, P, G, D\}\,,$$

where the elements denote private construction sector, private manufacturing sector, government sector and dwelling sector respectively.

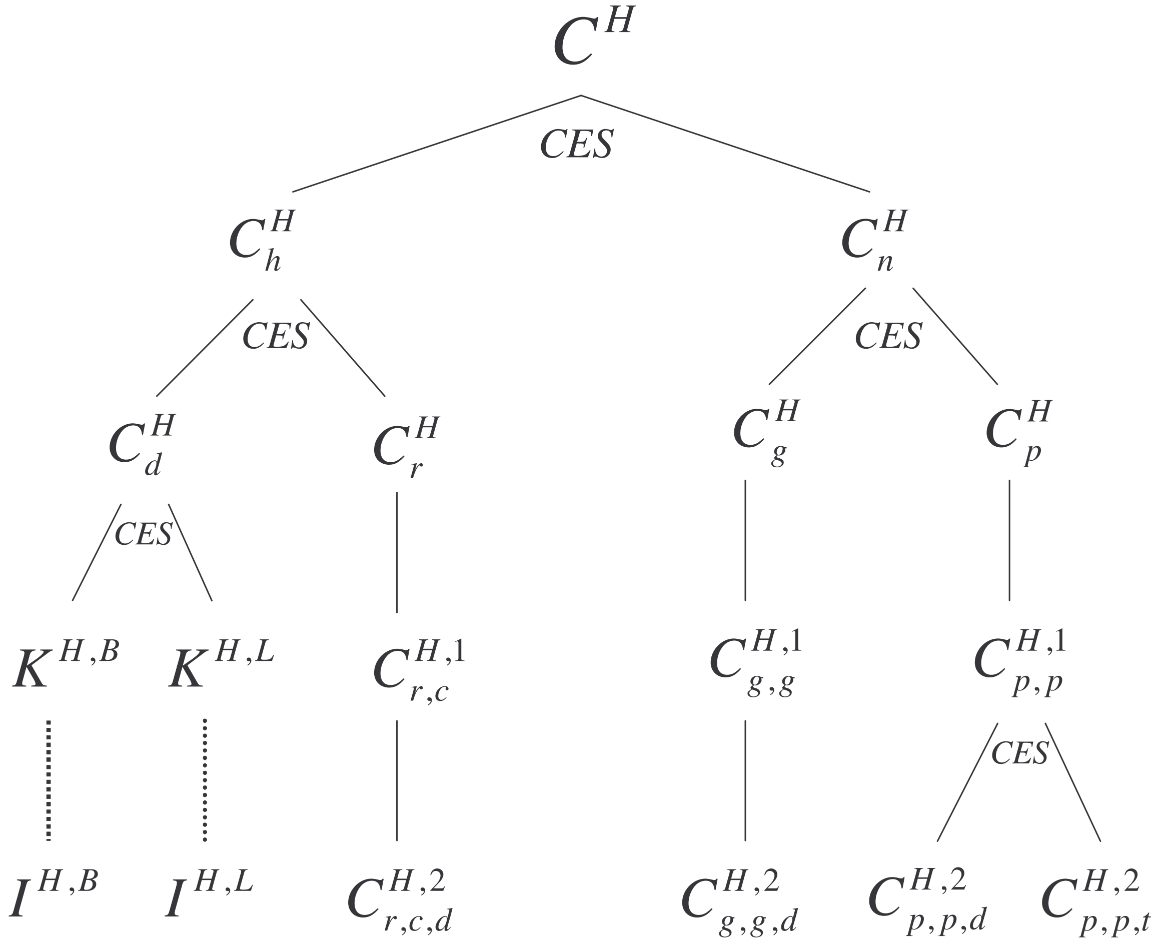

Figure 5.1: Consumption nest structure

Set C denotes country of origin and contains 2 elements

$$C = \{D, F\},$$

denoting domestic and foreign respectively.

Consider the nest structure in figure 1. Households substitute between consumption related to housing C_H^H and consumption related to all other goods, here labeled non-housing goods C_N^H. Non-housing consumption is an aggregate of consumption of privately produced goods and consumption of goods from the government producer, where private consumption consists of intermediate goods produced in the private manufactoring sector $C_{P,P}^{H,1}$ and government goods consumption consist of intermediate goods from the producer of government services $C_{G,G}^{H,1}$. While goods from the government producer are solely domestically produced $C_{G,G,D}^{H,2}$, goods produced in the manufacturing sector originate from the foreign sector $C_{P,P,F}^{H,2}$ as well as the domestic sector $C_{P,P,D}^{H,2}$.

Utility from housing consumption is an aggregate of utility from ownership of dwellings on the one hand (C_D^H) and utility from consumption related to housing repair (C_R^H) on the other. Housing repair consumption consists of goods produced in the private construction sector, which are supplied by domestic producers only ($C_{R,C,D}^{H,2}$). The remaining part of the diagram shows how utility from owning real estate is found by combining the building capital $K^{H,B}$ and land capital $K^{H,L,Ind}$ belonging to the household. Finally, household demand for dwelling capital gives rise to investments in buildings $I^{H,B}$ and land $I^{H,L}$.

5.3.3 Lifetime utility for planning households

After having defined the period utility function $u_{a,t}(C_{a,t}^H, Z_{a,t})$ and described the preferences underlying its two arguments, we will now characterize *lifetime* utility of households.

Consider a specific generation or representative household aged a at a given point in time t. This means that the household members are born in the year $t-a$. At the beginning of year t, where the household turns a years, the rest-of-life discounted utility for the representative household in question, $U_{a-1,t-1}$, is given by

$$U_{a-1,t-1} \equiv \left[\sum_{i=a}^{77} (u_{i,t-a+i})^{\frac{\nu-1}{\nu}} \upsilon_{a-1,i} N_{i,t-a+i}^{AdultEq} \right]^{\frac{\nu}{\nu-1}}, \tag{5.5}$$

where $u_{i,t-a+i}$ is period utility per adult-equivalent from (5.1) and $\upsilon_{a-1,i}$ is a discounting factor discounting from period i to period $a-1$ as defined in (5.6). (Remember that individuals are born at the beginning of a period whereas all other stock variables including utility are ultimo-dated. This is why we approximate life-time utility for generation a at time t with $U_{a-1,t-1}$.)

(5.5) demonstrates that we specify rest of life utility as a CES function over period utilities per adult-equivalent $u_{i,t-a+i}$ with intertemporal distribution parameters $\left[\upsilon_{a-1,i}N_{i,t-a+i}^{AdultEq}\right]$ and ν denoting the intertemporal elasticity of substitution. The multiplication by $N_{i,t-a+i}^{AdultEq}$ in the utility function means that the criterion applied in the objective (5.5) is the Benthamite one of maximizing total household utility also known as classical utilitarianism. An important implication of this criterion is that period utility per adult-equivalent in any period is given more intertemporal weight the higher the number of adult-equivalents in the household at the time. In this way, rather old households with few members do not weigh as heavily as younger households with more members.

5.3.4 Bequest motive

The modelling of the bequest decision may in general take two very different routes. First, bequests may be **altruistic**. This implies that the present household takes the utility of future generations directly into account by including the utility functions of these generations in the object function of the household's maximization problem. This leads to a model of "dynasties" where each dynasty maximizes utility over an infinite horizon. Assuming that no new dynasties enter the economy, this implies that the overlapping generation structure collapses and the model converts into a Ramsey type model. In this type of model the gradual dynamic transition of the economy, which is due to the entry of new households in the overlapping generations model, is replaced by intergenerational transfers within each dynasty.

Although intergenerational transfers do play a role in most families, it appears that such transfers are less perfect than is the case given by the altruistic assumption. Therefore we have in this model chosen to model a "**joy of giving**" motive for bequest. In this case the parent household obtains utility simply by leaving a gift to its heirs. The effect on the utility of the heirs does not directly enter into the bequest decision of the parent household. Therefore e.g. the number of heirs does not affect the bequest decision. This implies that the overlapping generation structure of the model is maintained although some intergenerational transfers take place.

Technically, the bequest motive is implemented by modelling the bequest as an extra fictitious period of consumption after the household has ceased to exist as a planning household at the age of 76. This can be seen explicitly from (5.5). Thus the assumption is that the parent household obtains utility from leaving the bequest as if it was an extra year's consumption. Since this fictitious period of consumption enters the life time utility function (5.5), the household has an incentive to save resources to finance the extra "consumption", and these savings will eventually amount to the bequest $A_{t-1}^{H,Beq}$ given to the heirs.

However, the household may attach a different intertemporal weight to the utility of the bequest $u_{77,t-a+77}$ compared to the consumption of previous years, which can be seen from the specification of the discounting factor $\upsilon_{a-1,i}$:

$$\begin{aligned} \upsilon_{a-1,i} &= \xi_i \left(\frac{1}{1+\theta}\right)^{i-a+1}, \\ \xi_i &= 1 \text{ for } i = a, ..., 76, \\ \xi_i &= \xi \text{ for } i = 77, \end{aligned} \tag{5.6}$$

where θ is the rate of time preference. The value of ξ determines the relative importance of utility from leaving a bequest compared to utility from previous years of consumption. A value of 1 means that the household considers utility from leaving an extra unit of bequest a perfect substitute for an extra unit of own consumption disregarding changes in household size and impatience. ξ is calibrated to match the observed profile of household wealth across households (generations) in the calibration year. Currently $\xi = 120, 6$.

5.4 Income and budget

The specification of disposable household income is very detailed in DREAM. Besides wage and capital income, the household receives bequests, several kinds of funded pension benefits and more than 20 different government transfers and pays several kinds of taxes. The following subsections describe these income components in detail; the subdivision into various groups is mainly motivated by the differentiated treatment in the Danish tax system of various kinds of income. Afterwards, the evolution over time of household wealth is explained.

5.4.1 Non-capital income

The Danish income tax system is a hybrid between a dual and a global income tax system (the difference being whether capital and non-capital (mainly labour) income are taxed in the same way or with different rates). For various technical and data reasons, DREAMs treatment of personal income follows a more consistently dual system, so that in DREAM taxation of capital income (in DREAM defined to include interest income, capital gains on shares and dividend income) is calculated separately and independently from non-capital income (labeled ”personal income” as in the official Danish tax rules) at flat proportional rates.

In this subsection we focus on the modelling of personal non-capital income. Hence, consider the following expression for personal non-capital income denoted $Y^{H,Pers}_{o,s,a,t}$:

$$\begin{aligned}
Y^{H,Pers}_{o,s,a,t} = & \left(1 - t^{Payroll}_t - q^{SP}_{a,t}\right) \\
& \times \left(\; adj^{Hours}_t r^{LabFull}_{o,s,a,t} \left(1 + adj^{LS}_t\right) L^S_{o,s,a,t} \left(\rho_{o,s,a,t} W_t \left(1 - q^{ZF}_{o,s,a,t} - q^{ZP}_{s,a,t}\right) - q^{ATP}_{a,t}\right) \; \right) \\
& + adj^{Hours}_t r^{LabFull}_{o,s,a,t} \left(o^{G,H,Unemp}_{o,s,a,t} - w^{q,ATP,Unemp}_t q^{ATP}_{a,t}\right) \left(L^{Max}_{o,s,a,t} - \left(1 + adj^{LS}_t\right) L^S_{o,s,a,t}\right) \\
& + \left(r^S_{o,s,a,t} + r^{LabS}_{o,s,a,t}\right) o^{G,H,S}_t \\
& + r^{LA}_{o,s,a,t} o^{G,H,LA}_t \\
& + r^{MB}_{o,s,a,t} o^{G,H,MB}_t \\
& + \left(r^{SB}_{o,s,a,t} + r^{Lab,SB}_{o,s,a,t}\right) o^{G,H,SB}_t \\
& + r^{BB}_{o,s,a,t} o^{G,H,BB}_t \\
& + r^{PEW}_{o,s,a,t} o^{G,H,PEW}_{o,t} \\
& + \left(r^{AP}_{o,s,a,t} + r^{Lab,AP}_{o,s,a,t}\right) o^{G,H,AP}_t \\
& + \left(r^{OAP}_{o,s,a,t} + r^{Lab,OAP}_{o,s,a,t}\right) o^{G,H,OAP}_{s,a,t} \\
& + r^{CA}_{o,s,a,t} o^{G,H,CA}_t \\
& + r^{AB}_{o,s,a,t} o^{G,H,AB}_t \\
& + r^{IB}_{o,s,a,t} o^{G,H,IB}_t \\
& + o^{G,H,AgeDepTax}_{s,a,t} \\
& + b^{ATP}_{s,a,t} + b^R_{o,s,a,t} + b^D_{o,s,a,t} + b^S_{o,s,a,t} \\
& + \frac{N^{Able}_{s,a,t}\left(b^{SP}_{s,a,t} + o^{G,H,CS}_{a,t}\right)}{\sum_o N^{Ind}_{o,s,a,t}}.
\end{aligned} \tag{5.7}$$

Broadly speaking, personal income can be divided into wage income, government transfers and funded pension receipts.

Wage income

The first two lines of the above equation represent wage income net of payroll taxes $t^{Payroll}_t$, contributions to SP $q^{SP}_{a,t}$, contributions to ATP $q^{ATP}_{a,t}$ and contributions to labour market pension schemes $q^{ZF}_{o,s,a,t}$ and private pension schemes $q^{ZP}_{s,a,t}$. $L^S_{o,s,a,t}$ is individual labour supply of a person of given origin, gender and age. The parameter $\rho_{o,s,a,t}$ captures time-dependent differences in productivity between workers of different origin, sex and age. W_t is an endogenous term representing changes in the general wage level as the result of changes in demand and supply on the labour market.

Government transfers

The third line above represents receipts of unemployment benefits by an individual. Note here that this builds on an assumption of underemployment where each individual shares origin-, gender- and age-specific unemployment equally and consequently is partly employed and partly unemployed. Any labour supply $L^S_{o,s,a,t}$ below the full employment level $L^{Max}_{o,s,a,t}$ (to be defined later) thus constitues a certain amount of unemployment $\left(L^{Max}_{o,s,a,t} - L^S_{o,s,a,t}\right)$. $o^{G,H,Unemp}_t$ is the level of unemployment benefits for a full-time unemployed person.

The remaining part of the above expression for personal income reflects pensions and government transfers. As can be seen, the expressions for transfers are for the most part made up of 2 components: The rate of the population group receiving the particular transfer, e.g. maternity benefits $r^{MB}_{s,a,t}$, times the amount to each receipient $o^{G,H,MB}_t$.

Transfers include financial aid to students (SU) $o^{G,H,S}_t$, leave allowance (orlovsydelse) $o^{G,H,LA}_t$, maternity leave (barselsorlov) $o^{G,H,MB}_t$, sickness benefits (sygedagpenge) $o^{G,H,SB}_t$, bridging benefit schemes (overgangsydelse) $o^{G,H,BB}_t$, post-employment wage benefits (efterløn) $o^{G,H,PEW}_t$, anticipatory pensions (førtidspension) $o^{G,H,AP}_t$, old-age pensions (folkepension)$o^{G,H,OAP}_{s,a,t}$, cash benefits (kontanthjælp) $o^{G,H,CA}_t$, activation benefits (aktiveringsydelse) $o^{G,H,AB}_t$, introduction benefits to immigrants (introduktionsydelse) $o^{G,H,IB}_t$, and finally non-income-compensating age-dependent government transfers $o^{G,H,AgeDep}_{s,a,t}$.

Pension receipts

The last two lines of (5.7) represent pension receipts: ATP pension receipts $b^{ATP}_{s,a,t}$, retirement pensions from the pension fund $b^R_{s,a,t}$, SP receipts $b^{SP}_{s,a,t}$, civil servants' pension (tjenestemandspension) $o^{CS}_{a,t}$, disablement pension (invalidepension) $b^D_{s,a,t}$ and spouse pension (ægtefællepension) $b^S_{s,a,t}$. For a detailed description of the modelling of pensions in DREAM please refer to chapter 6 of this documentation.

As can be seen the above expression is pre-tax personal income related to wage income, government transfers and pension payouts. In a moment we will convert this income into an after-tax disposable income, but before we do that it is essential to understand how taxation of personal income is modelled in DREAM.

Progressive taxes

While payment of bottom-bracket taxes and local-government (i.e., municipal, county and church) taxes are modelled in the standard way of multiplying personal income by a rate, constructing the payment of middle-bracket and top-bracket taxes for our representative agents is somewhat complicated. These taxes are progressive, and we are obviously unable to represent the heterogeneity in income of tax-payers for a given age and gender in the model itself. Instead, these taxes are represented in DREAM as a polynominal approximation of a tax function which is calculated in a pre-model using micro-data for a representative sample of Danish tax-payers.

The pre-model calculates a polynomial approximation (using a 6th-degree polynomium) of average tax payments as a function of average income. These polynomial approximations are used inside the main model to determine these tax payments as a function of average income, which is equal to the income of our representative individuals:

$$\begin{aligned} TR^{Mid,Ind}_{o,s,a,t} &= t^{Mid}_t \Big[\; k^{Mid6}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^6 + k^{Mid5}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^5 \\ &+ k^{Mid4}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^4 + k^{Mid3}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^3 \\ &+ k^{Mid2}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^2 + k^{Mid1}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^1 + k^{Mid0}_{a,t} \; \Big], \end{aligned} \tag{5.8}$$

$$\begin{aligned} TR^{Top,Ind}_{o,s,a,t} &= t^{Top}_t \Big[\; k^{Top6}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^6 + k^{Top5}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^5 \\ &+ k^{Top4}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^4 + k^{Top3}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^3 \\ &+ k^{Top2}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^2 + k^{Top1}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^1 + k^{Top0}_{a,t} \; \Big], \end{aligned} \tag{5.9}$$

where t^{Mid}_t and t^{Top}_t are the respective tax rates, and k-parameters like $k^{Mid6}_{a,t}$ and $k^{Top0}_{a,t}$ are determined in the micro-data pre-model. The variables k^{Mid}_t and k^{Top}_t are determined in the calibration process when the total middle-bracket and top-bracket tax revenues are equated to the actual revenue in the base-year. If the approximation were perfect and the micro-data and macro tax revenues were perfectly consistent, these variables would both be zero, but in practice they deviate somewhat from this. One reason is that the tax system does not take into account the middle-bracket and top-bracket tax payments from capital income, as capital income taxation in DREAM is treated completely separable from taxation of personal income (cf. above).

Returning to the derivation of household disposable income we will now use the tax variables derived above to subtract taxes from personal income derived in (5.7). This is explained in detail on the following pages, but the short version is that in (5.10) we subtract taxes and contributions to social benefit schemes and add transfers and pensions which are either not taxed as personal income or not taxed at all. Note also that (5.10) involves a summation over the origin of each individual so that $Y^{H,DispGender}_{s,a,t}$ is distributed over gender, age and time. In (5.13) we aggregate further to get disposable income distributed over age and time only, giving us disposable income per adult of household a. At this point we will also include a number of lumpsum transfers to and from the government and the foreign sector.

So consider the following gender-distributed expression for disposable income per adult:

$$
\begin{aligned}
Y^{H,DispGender}_{s,a,t} = & \frac{1}{N^{Adult}_{a,t}} \Bigg[\sum_{o \in oLab} N^{Ind}_{o,s,a,t} \Big[\; Y^{H,Pers}_{o,s,a,t} \\
& - r^{Unempq}_{s,a,t} k^{Unemp}_t w^{Unemp}_t Unemp^{Max}_t - r^{PEWq}_{s,a,t} k^{PEW}_t w^{PEW}_t Unemp^{Max}_t \\
& - \left(t^{Cou}_t + t^{Mun}_t + t^{Chu}_t \right) \Big(\; Y^{H,Pers}_{o,s,a,t} - k^{Source}_t - allow^{Pers}_t - allow^{Assess}_t \\
& \quad - r^{Unempq}_{s,a,t} k^{Unemp}_t w^{Unemp}_t Unemp^{Max}_t - r^{PEWq}_{s,a,t} k^{PEW}_t w^{PEW}_t Unemp^{Max}_t \; \Big) \\
& - t^{EITC,Eff}_t \cdot adj^{Hours}_t r^{LabFull}_{o,s,a,t} \left(1 + adj^{LS}_t\right) L^S_{o,s,a,t} \left(\rho_{o,s,a,t} W_t \left(1 - q^{ZF}_{o,s,a,t} - q^{ZP}_{s,a,t}\right) - q^{ATP}_{a,t} \right) \\
& - t^{Bot}_t \Big(\; Y^{H,Pers}_{o,s,a,t} - allow^{Pers}_t - k^{Source}_t \; \Big) \; \Big] \\
& - \sum_o N^{Ind}_{o,s,a,t} \left(TR^{Mid,Ind}_{o,s,a,t} + TR^{Top,Ind}_{o,s,a,t} \right) \\
& + \sum_o N^{Ind}_{o,s,a,t} \left(o^{G,H,AgeDepNoTax}_{s,a,t} + o^{G,H,NonAgeDep}_t \right) \\
& + N^{Able}_{a,t} \left(b^{ZPR}_{a,t} + b^{LD}_{a,t} \right) \left(1 - t^{CapPen}_t \right) \quad \Bigg] \\
& + \left(1 - t^{Beq}_t \right) \left(1 + i^H_t \right) A^{H,Beq}_{t-1} \left(\frac{N^{AdultEq}_{apu,t-1}}{N^{Adult}_{a,t}} \right) Dist^{Heirs}_{s,a,apu,t}.
\end{aligned}
\tag{5.10}
$$

First we subtract contributions to the post employment wage scheme:

$$
r^{PEWq}_{s,a,t} k^{PEW}_t w^{PEW}_t Unemp^{Max}_t, \tag{5.11}
$$

and the unemployment benefit scheme

$$
r^{Unempq}_{s,a,t} k^{Unemp}_t w^{Unemp}_t Unemp^{Max}_t, \tag{5.12}
$$

from personal income $Y^{H,Pers}_{o,s,a,t}$. Following the actual rules in place, annual post employment wage scheme contributions are modelled as 7 times maximum daily unemployment benefits $w^{PEW}_t Unemp^{Max}_t$. Since not all individuals contribute to the scheme we multiply by the gender- and age-specific rate of the population that participates $\left(r^{PEWq}_{s,a,t} \right)$. Finally, the multiplicative factor k^{PEW}_t is calibrated such that aggregate contributions in the model match the observed contributions in the calibration year. The treatment of unemployment benefits is similar.

Taxes are levied on the taxable part of personal income, i.e. $Y^{H,Pers}_{o,s,a,t}$ net of various tax allowances: the basic lump-sum personal tax allowance $allow^{Pers}_t$, the term $allow^{Assess}_t$ which represents those assessment-oriented allowances that are not explicitly modelled in DREAM, allowances for payments to unemployment benefit and post-employment wage arrangements, the effective earned income tax credit, and finally the correction term k^{Source}_t which is calibrated in order to reconcile the tax base of DREAM with actual source tax receipts in the calibration year. The expression starting in line 3 represents county, municipal and church taxes while the following term represents taxation of bottom-bracket income at rate t^{Bot}_t, the tax base of which is broader than the local-government taxes. The remaining part of the expression represents both taxes and transfers. First, we subtract taxes applicable to middle- and top-bracket incomes $t^{Mid}_{o,s,a,t}$ and $t^{Top}_{o,s,a,t}$. Then we add age-dependent and non-age-dependent non-taxable government transfers $o^{G,H,AgeDepNoTax}_{s,a,t}$ and $o^{G,H,NonAgeDep}_{t}$ as well as receipts from private pension schemes $b^{ZPR}_{a,t}$ and LD pensions $b^{LD}_{a,t}$ (lønmodtagernes dyrtidsfond) net of taxes (these pension benefits are treated as once-and-for-all payments and consequently do not enter (taxable) personal income, but are taxed with the specific tax rate for capital pensions).

The final line in (5.10) is the inheritance per adult-equivalent, which consists of two main components; $A^{H,Beq}_{t-1}$ is the bequest per adult-equivalent left ultimo period t-1 by the households that turn 77 in period t. Since this bequest is received by individuals as inheritance one period later in period t, it earns one period of interest. Furthermore, $A^{H,Beq}_{t-1}$ is the bequest left per adult-equivalent of the old generation, whereas we wish to operate with the inheritance per adult of the receiving generation. Hence, we correct for the difference between the number of adult-equivalents belonging to the old generation $N^{AdultEq}_{apu,t-1}$ and the number of adults $N^{Adult}_{a,t}$ belonging to the generation for which we are specifying disposable income. The parameter $Dist^{Heirs}_{s,a,apu,t}$ contains the distribution of heirs over all potentially receiving generations (those that are from 22 to 51 years old), ensuring that the correct share of the total bequest is inherited by the different generations. Finally, inheritance is taxed as represented by the factor $\left(1 - t^{Beq}_{t}\right)$.

What remains now is to include a number of lump-sum transfers to and from the households and to aggregate over gender, giving us household disposable income per adult, which will be the relevant measure of per period non-capital income in the asset accumulation equation. We get:

$$
\begin{aligned}
Y^{H,Disp}_{a,t} = & \sum_{s\in\{m,f\}} Y^{H,DispGender}_{s,a,t} \\
& + o^{F,H}_{t} + o^{G,H,LumpSustain}_{t} - o^{H,G,LumpRev}_{t} \\
& + o^{G,H,LumpExp}_{t} - \frac{t^{H,Weight}_{t} P^{C,H}_{C,t} N^{AdultEq}_{a,t} C^{H}_{C,t}}{N^{Adult}_{a,t}} \\
& - o^{H,G,Soc}_{t} - o^{H,G,SocOpt}_{t} - o^{H,G}_{t} - o^{H,G,Cap}_{t} \\
& + o^{G,H}_{t} + o^{G,H,Cap}_{t} \\
& - \frac{1}{N^{Adult}_{a,t}} \left(\sum_{o} \sum_{s\in\{m,f\}} o^{H,G,LumpPayroll}_{o,s,a,t} + o^{H,G,LumpDwe}_{a,t} + \sum_{s\in\{m,f\}} o^{H,SP,LumpSP}_{s,a,t} \right) \\
& + o^{G,H,LumpInt}_{t} + o^{H,G,LumpGrOS}_{t} + o^{G,H,LumpGI}_{t},
\end{aligned}
\tag{5.13}
$$

where $o^{F,H}_{a,t}$ are net transfers from foreigners and $\frac{t^{H,Weight}_{t} P^{C,H}_{C,t} N^{AdultEq}_{a,t} C^{H}_{C,t}}{N^{Adult}_{a,t}}$ is the vehicle excise duty rate (vægtafgift) modelled as a fraction of total household consumption expenditures (Note here the difference between adults and adult-equivalents: The fraction $\frac{N^{AdultEq}_{a,t}}{N^{Adult}_{a,t}}$ enters because consumption is given per adult-equivalent including children, while $Y^{H,Disp}_{a,t}$ is defined per adult, cf. the discussion in section 5.2). The remaining elements are lump-sum transfers to and from the government, many of which are included for calibration purposes to replicate features of the Danish economy, e.g. the revenue side of the government sector. These are described in more detail in chapter 7 on the government sector.

5.4.2 Capital income and asset accumulation

Having derived household disposable non-capital income per adult in each period, we will now specify how household wealth develops between periods. This involves deriving the savings identity or asset accumulation equation. Specifying the dynamics of household wealth will enable us to state the dynamic consolidated budget of the household and so obtain the relevant constraint for the intertemporal optimization problem of the household. To ease understanding, we first describe asset accumulation in a simple situation with only financial assets. Then we include trade in residential assets as well, and finally we introduce the treatment of unexpected capital gains in the event of surprise shocks.

Asset accumulation with financial assets (only)

Consider first the following asset accumulation equation in a situation where household wealth consists of financial assets only, i.e. holdings of bonds and shares:

$$A_{a,t}^{H,Fin,Ind} = \left(1 + i_t^H\right) A_{a-1,t-1}^{H,Fin,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + Y_{a,t}^{H,Disp} \frac{N_{a,t}^{Adult}}{N_{a,t}^{AdultEq}} - P_t^{C,H} C_{a,t}^{H}, \tag{5.14}$$

where $A_{a,t}^{H,Fin,Ind}$ represents household financial assets per adult-equivalent. (5.14) states that accumulation of financial assets is simply given as the previous period's financial assets plus an after-tax rate of return plus the current period's disposable income derived in (5.13) minus the period's consumption expenditure.

As can be seen from (5.14), this formulation makes no explicit distinction between holdings of bonds and shares, just as dividends do not explicitly enter (5.14). The different after-tax yields for these different sources of capital income are instead modelled in the variable i_t^H representing the effective rate of return. This is a weighted average of the different after-tax yields with the weights determined by the relative magnitude of bonds vs. shares in the composition of household financial wealth.

Household capital income arises from three sources: Capital gains on shares, dividend payouts from firms and finally interest income from holdings of bonds. They are represented by the following expressions:

$$\sum_{j\in\{C,P\}} \left(1 - t_t^{H,Gain}\right)(V_{j,t} - V_{j,t-1}) \tag{5.15}$$

represents after-tax capital gains, where $t_t^{H,Gain}$ is the rate applied to capital gains on shares and $V_{j,t} - V_{j,t-1}$ is the increase in the value of the representative firm in sector j. Note that these are *expected* capital gains, foreseen by all agents of the economy. Unanticipated capital gains occurring because of a surprise shock to the economy are treated later on.

$$\sum_{j\in\{C,P\}} \left(1 - t_t^{H,Div}\right) DIV_{j,t} \tag{5.16}$$

represents after-tax dividend income from holdings of shares, where $t_t^{H,Div}$ is the tax rate applied to dividend income, and $DIV_{j,t}$ is dividend payout from the representative firm in sector j.

$$\left(1 - t_t^{H,Int}\right) i_t \tag{5.17}$$

represents after-tax interest income from holdings of bonds, where $t_t^{H,Int}$ is the tax rate applied to interest income and i_t is the bond interest rate. The tax rates $t_t^{H,Gain}$, $t_t^{H,Div}$ and $t_t^{H,Int}$ are all calculated in various pre-models computing average capital tax rates from the rather complex Danish capital income tax system.

Since i_t^H is an after-tax *rate* of return, we convert the incomes from (5.15) and (5.16) into rates of return from holding shares by dividing both by total firm value $\sum_{i\in\{C,P\}} V_{i,t-1}$. Average after-tax returns on households' financial assets can then be expressed as in (5.18) where the weight w^{Assets} is applied to after-tax capital income from holding shares, while the weight $1 - w^{Assets}$ is applied to after-tax bond interest income:

$$\begin{aligned} i_t^H &= \frac{w^{Assets}}{\sum_{j\in\{C,P\}} V_{j,t-1}} \sum_{j\in\{C,P\}} \Big[\left(1 - t_t^{H,Gain}\right)(V_{j,t} - V_{j,t-1}) \\ &\quad + \left(1 - t_t^{H,Div}\right) DIV_{j,t} \Big] + \left(1 - w^{Assets}\right)\left(1 - t_t^{H,Int}\right) i_t. \end{aligned} \tag{5.18}$$

The value of w^{Assets} is exogenously fixed and presently set to 1/3. Hence, the households do not optimally choose the portfolio composition of their financial wealth. As explained in the preceding chapter, optimization by households as well as institutional investors would create corner solutions when the tax system discriminates between capital income for different types of investors.

An important implication of the tax structure in DREAM is the identical taxation of positive and negative capital income of household a in period t. $A_{a-1,t-1}^{H,Fin,Ind}$ are net financial assets and actually negative for a number of representative households in the model, partly because they smoothe consumption over the life-cycle, and partly because they also own real estate, which we shall introduce in a moment, but the same tax rate always applies.

Introduction of residential capital

We now introduce residential capital as well. In DREAM all dwellings are owner-occupied. Households in DREAM demand and own residential capital assets which are systematically divided into holdings of land and holdings of buildings (because of the different tax treatment and supply conditions of these two assets). In order to treat household income and expenditure streams consistently, DREAM computes also the imputed income and expenditure consisting of the returns to residential capital. That is, the household acts as its own landlord and pays an imputed rent beside the actual expenses in its capacity as a consumer and at the same time receives this payment in its capacity as the letter of a capital good.

To proceed we first subdivide total consumption expenditure into three sub-categories:

$$P_t^{C,H} C_{a,t}^H = P_{N,t}^{C,H} C_{N,a,t}^H + P_{R,t}^{C,H} C_{R,a,t}^H + P_{H,t}^{C,H} C_{H,a,t}^H, \tag{5.19}$$

The first term on the right-hand side is non-housing-related consumption expenditure. The second term is expenditure for housing repairs (a rather modest amount). The third term is the value of the actual consumption of land and residential buildings during one period. By definition it is equal to the user-cost times the stock of each of the two residential assets owned by the household in question:

$$P_t^{C,H} C_{H,a,t}^H = P_{j,t}^{K,H,B,User} K_{a-1,t-1}^{H,B} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + P_{j,t}^{K,H,L,User} K_{a-1,t-1}^{H,L,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}}. \tag{5.20}$$

The usercost expressions $P_{j,t}^{K,H,B,User}$ and $P_{j,t}^{K,H,L,User}$ are the sum of actual and imputed costs caused by the ownership of one of the respective asset units during one period. These costs are:

1) Taxes (the tax on owner-occupied dwellings levied on both land and buildings and the land tax levied on land only).

2) depreciation (in the case of residential buildings).

3) a foreseen capital gain (which enters the user-cost expression negatively).

4) an opportunity cost in the form of lost interest, which in theoretical economic models is usually set equal to the returns to financial assets.

In a market equilibrium with perfect competition a person renting a dwelling would pay a rent which is exactly equal to the sum of these 4 elements.

When calibrating DREAM, the above four elements can be calculated directly. At the same time, the figures for total consumption expenditures are taken independently from the National

Accounts so that (5.20) does not hold immediately. Because of this the correction factor k_t^{User} is introduced so that the user-cost expressions are as follows:

$$P_{j,t}^{K,H,B,User} = \left(t_t^{Dwe} + k_t^{User} + i_t^H\right) P_{j,t-1}^{K,H,B} + \delta_{j,t} P_{j,t}^{K,H,B} - \left(P_{j,t}^{K,H,B} - P_{j,t-1}^{K,H,B}\right), j = D, \quad (5.21)$$

$$P_{j,t}^{K,H,L,User} = \left(t_t^{Dwe} + t_t^{H,Land} + k_t^{User} + i_t^H\right) P_{j,t-1}^{K,H,L} - \left(P_{j,t}^{K,H,L} - P_{j,t-1}^{K,H,L}\right), j = D. \quad (5.22)$$

Hence, the variable k_t^{User} is a fictitious additional imputed dwelling income (which may be negative, depending on the actual calibration). It should be interpreted as a supplement to the financial rate of return i_t^H generated by residential assets in the same way as the risk premium creates a difference between the returns to bonds and shares. There are various reasons why dwelling-owners would require another rate of returns to residential than to financial assets for being indifferent between the two. These include another risk profile for residential assets, a liquidity premium or some additional non-pecuniary pleasures of owning one's own dwelling.

The imputed income of dwelling-owners is equal to the share of dwelling expenditures which is not used for actual tax payments:

$$Y_{a,t}^{H,Dwe} = P_t^{C,H} C_{H,a,t}^H - t_t^{Dwe} P_{j,t-1}^{K,H,B} K_{a-1,t-1}^{H,B} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} - \left(t_t^{Dwe} + t_t^{H,Land}\right) P_{j,t-1}^{K,H,L} K_{a-1,t-1}^{H,L,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}}. \quad (5.23)$$

This expression is hence also equal to the returns to the residential assets including the expected capital gain and a compensation for the depreciation of the building.

We now want to rewrite the financial accumulation equation (5.14), taking into account the fact that the household also is a dwelling-owner. This influences the equation in three ways:

1) Some of the expenditure is now imputed, though this does not formally affect the equation as dwelling consumption is still a subset of total consumption expenditure $P_t^{C,H} C_{a,t}^H$.

2) The dwelling-owner receives the imputed income $Y_{a,t}^{H,Dwe}$.

3) The financial wealth is now also affected by the acqusition or sale of real property. The value of the (possibly negative) gross investment in buildings and land, respectively, during the period must be added. Note that this last term is a matter solely of the portfolio distribution between real and financial assets and does not in itself affect total wealth of the household.

The result is

$$\begin{aligned} A_{a,t}^{H,Fin,Ind} &= \left(1 + i_t^H\right) A_{a-1,t-1}^{H,Fin,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + Y_{a,t}^{H,Disp} \frac{N_{a,t}^{Adult}}{N_{a,t}^{AdultEq}} - P_t^{C,H} C_{a,t}^H \\ &\quad + Y_{a,t}^{H,Dwe} - P_{j,t}^{I,P,B} I_{a,t}^{H,B} - P_{j,t}^{K,H,L} I_{a,t}^{H,L}, \end{aligned} \quad (5.24)$$

where $I_{a,t}^{H,B}$ represents the acquisition of new buildings by the household and $I_{a,t}^{H,L}$ correspondingly the net acquisition of land. Of course, either may be negative.

Parallelly, an equation for the accumulation of residential capital of the household can be stated as:

$$\begin{aligned} P_t^{K,H} K_{a,t}^H &= P_{j,t-1}^{K,H,L} K_{a-1,t-1}^{H,L,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + (1-\delta) P_{j,t-1}^{K,H,B} K_{a-1,t-1}^{H,B} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} \\ &+ \left(P_{j,t}^{K,H,L} - P_{j,t-1}^{K,H,L}\right) K_{a-1,t-1}^{H,L,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} \\ &+ \left(P_{j,t}^{K,H,B} - P_{j,t-1}^{K,H,B}\right) (1-\delta) K_{a-1,t-1}^{H,B} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} \\ &+ P_{j,t}^{I,P,B} I_{a,t}^{H,B} + P_{j,t}^{K,H,L} I_{a,t}^{H,L}, \end{aligned} \tag{5.25}$$

which states that the residential value of household a at the end of period t is equal to the residential value at the end of period t-1 net of depreciation (the first line) plus price rises during period t (lines 2 and 3) plus net investment (the last line). In (5.25) we use the fact that definitionally

$$P_t^{K,H} K_{a,t}^H = P_{j,t}^{K,H,L} K_{a,t}^{H,L,Ind} + P_{j,t}^{K,H,B} K_{a,t}^{H,B}. \tag{5.26}$$

Total household wealth $A_{a,t}^{H,Ind}$ is equal to the sum of financial and residential wealth:

$$A_{a,t}^{H,Ind} = A_{a,t}^{H,Fin,Ind} + P_t^{K,H} K_{a,t}^H = A_{a,t}^{H,Fin,Ind} + P_{j,t}^{K,H,L} K_{a,t}^{H,L,Ind} + P_{j,t}^{K,H,B} K_{a,t}^{H,B}. \tag{5.27}$$

Using this, we can arrive at a comprehensive accumulation equation by summing (5.24) and (5.25) on the left- and right-hand sides:

$$\begin{aligned} A_{a,t}^{H,Ind} &= A_{a-1,t-1}^{H,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + i_t^H A_{a-1,t-1}^{H,Fin,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + Y_{a,t}^{H,Disp} \frac{N_{a,t}^{Adult}}{N_{a,t}^{AdultEq}} - P_t^{C,H} C_{a,t}^H \\ &+ Y_{a,t}^{H,Dwe} - \delta P_{j,t-1}^{K,H,B} K_{a-1,t-1}^{H,B} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} \\ &+ \left(P_{j,t}^{K,H,L} - P_{j,t-1}^{K,H,L}\right) K_{a-1,t-1}^{H,L,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + \left(P_{j,t}^{K,H,B} - P_{j,t-1}^{K,H,B}\right) (1-\delta) K_{a-1,t-1}^{H,B} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}}. \end{aligned} \tag{5.28}$$

Finally, inserting the user-cost expressions (5.21) and (5.22) into (5.20), the result into (5.23) and the new result into (5.28) and then using again (5.27) and rearranging, we arrive at

$$\begin{aligned} A_{a,t}^{H,Ind} &= \left(1 + i_t^H\right) A_{a-1,t-1}^{H,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + Y_{a,t}^{H,Disp} \frac{N_{a,t}^{Adult}}{N_{a,t}^{AdultEq}} - P_t^{C,H} C_{a,t}^H \\ &+ k_t^{User} \left(P_{j,t-1}^{K,H,B} K_{a-1,t-1}^{H,B} + P_{j,t-1}^{K,H,L} K_{a-1,t-1}^{H,L,Ind}\right) \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}}. \end{aligned} \tag{5.29}$$

Compared to the structure of (5.14), the only non-standard term in (5.29) is the last one, which represents the supplementary rate of returns to dwelling-owners.

Extraordinary capital gains in the event of a surprise shock

As described in the introduction, a shock to the economy announced in period t+1 will result in an extra round of trade in assets and consequently generally affect asset prices already at the end of period t. All generations consequently experience a capital gain on their residential buildings and land, and all *planning* generations have a capital gain on their shares.

The capital gains on shares for planning generations is the following:

$$Gain_{a,t}^{ShareShock} = \sum_{j\in\{C,P\}} \left(V_{j,t} - V_j^{NoShock}\right) w^{Assets} N_{a,t}^{AdultEq} \frac{A_a^{H,FinIndNoshock}}{\sum_{j\in\{C,P\}} V_{j,t}^{NoShock}} \tag{5.30}$$
$$\times \frac{\left(\sum_{a\in A} N_{a,t}^{AdultEq} A_a^{H,FinIndNoshock} + \sum_{a\in AX0PU} N_{a,t}^{AdultEq} A^{H,BeqNoShock}\right)}{\sum_{a\in AX0PXU} N_{a,t}^{AdultEq} A_a^{H,FinIndNoshock} + \sum_{a\in AX0PU} \left(N_{a,t}^{AdultEq} A_a^{H,BeqNoShock}\right)}$$

for a shock announced in period t+1. All variables with the name "NoShock" have the value of the variable in question right before the shock is announced. The first factor of the expression $\left(\sum_{j\in\{C,P\}} \left(V_{j,t} - V_j^{NoShock}\right)\right)$ is the total change in the stock market value in the economy. The expression in the remaining first line $\left(w^{Assets} N_{a,t}^{AdultEq} \frac{A_a^{H,FinIndNoshock}}{\sum_{j\in\{C,P\}} V_{j,t}^{NoShock}}\right)$ is the share of the total stock market which the household owned before the shock. The fraction below is a scaling factor which ensures that the planning generations together are awarded exactly the capital gain corresponding to the value of shares of all households: The numerator is the total financial wealth of all generations, and the denominator is the financial wealth of the planning generations (including the bequest).

For nonplanning generations (including the premature generation) we have:

$$Gain_{a,t}^{ShareShock} = 0.$$

For residential assets it becomes necessary to introduce different prices of buildings and land before and after the shock. We have

$$P_{D,t}^{K,H,B} = \left(P_{D,t}^{K,H,B,Gain} + P_{D,t}^{I,P,B}\right), \tag{5.31}$$

where $P_{D,t}^{I,P,B}$ is the price index of building investment. $P_{D,t}^{K,H,B,Gain}$ is the difference between the price of the traded buildings before and after the shock and consequently equal to zero in all periods which are not associated with a surprise shock (arbitrage will normally ensure that the price on existing buildings must equal the price on new buildings, i.e. the investment price index).

Concerning land, there is naturally no production of "new" land, but we can correspondingly write

$$P_{D,t}^{K,H,L} = \left(P_{D,t}^{K,H,L,Gain} + P_{D,t}^{K,H,L,NoShock}\right), \tag{5.32}$$

where $P_{D,t}^{K,H,L,Gain}$ likewise only deviates from zero right in the period right before a shock to the economy.

The residential capital gain of each household is then equal to

$$\begin{aligned} &P_{j,t}^{K,H,L,Gain}\left(K_{a-1,t-1}^{H,L,Ind}\frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + I_{a,t}^{H,L,NoShock}\right) \\ &+P_{j,t}^{K,H,B,Gain}\left((1-\delta)K_{a-1,t-1}^{H,B}\frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + I_{a,t}^{H,B,NoShock}\right). \end{aligned} \tag{5.33}$$

Taxation of extraordinary capital gains on shares

Finally, also extraordinary capital gains on shares should be taxed (whereas extraordinary capital gains on residential assets are untaxed like ordinary residential capital gains), and this taxation must be included explicitly in the accumulation equation as it is not captured by the tax on ordinary capital gains on shares in the expression (5.15). The tax is paid in the period *after* the capital gain has materialized and looks as follows:

$$\begin{aligned} &J_{a,t}^{GainTax} = d^{TGainIni} \\ &\times\left[\frac{\left(\sum_{a\in AX0} N_{a-1,t-1}^{AdultEq}A_{a-1}^{H,FinIndNoshock} + \sum_{a\in APU} N_{a,t-1}^{AdultEq}A^{H,BeqNoShock}\right)}{N_{a,t}^{AdultEq}\sum_{a\in APX0}\left(N_{a-1,t-1}^{AdultEq}A_{a-1}^{H,FinIndNoShock}\right)}\right] \\ &\times\left[t_{t-1}^{H,Gain}\sum_{j\in\{C,P\}}\left(V_{j,t-1} - V_j^{NoShock}\right)w^{Assets}N_{a-1,t-1}^{AdultEq}\frac{A_{a-1}^{H,FinIndNoshock}}{\sum_{j\in\{C,P\}}V_{j,t-1}^{NoShock}}\right] \quad if\ 1st\ shock\ period. \end{aligned} \tag{5.34}$$

Starting with the bottom line, the part $t_{t-1}^{H,Gain}\sum_{j\in\{C,P\}}\left(V_{j,t-1} - V_j^{NoShock}\right)$ is the aggregate economy-wide taxation of the surprise jump in firm value, while the remaining expression is a matter of distributing this aggregate value between households. Hence, $w^{Assets}N_{a,t}^{AdultEq}\frac{A_{a-1}^{H,FinIndNoshock}}{\sum_{j\in\{C,P\}}V_{j,t-1}}$ is the share of aggregate firm value that household a owns, which is used to attribute the correct share of the aggregate taxation to household a. The top line in the expression finds the share of aggregate wealth that belongs to the planning generations and is used to scale the tax payment by each planning household.

Including the terms originating from surprise shocks (5.30), (5.33) and (5.34) in (5.29) we finally arrive at

$$\begin{aligned}
A_{a,t}^{H,Ind} &= \left(1+i_t^H\right) A_{a-1,t-1}^{H,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + Y_{a,t}^{H,Disp} \frac{N_{a,t}^{Adult}}{N_{a,t}^{AdultEq}} - P_t^{C,H} C_{a,t}^H \\
&+k_t^{User} \sum_{j\in\{D\}} \left(P_{j,t-1}^{K,H,B} K_{a-1,t-1}^{H,B} + P_{j,t-1}^{K,H,L} K_{a-1,t-1}^{H,L,Ind}\right) \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} \\
&+Gain_{a,t}^{ShareShock} - J_{a,t}^{GainTax} \\
&+P_{j,t}^{K,H,L,Gain} \left(K_{a-1,t-1}^{H,L,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + I_{a,t}^{H,L,NoShock}\right) \\
&+P_{j,t}^{K,H,B,Gain} \left((1-\delta) K_{a-1,t-1}^{H,B} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + I_{a,t}^{H,B,NoShock}\right),
\end{aligned} \tag{5.35}$$

which is the household asset accumulation equation actually used in DREAM. The whole expression consequently states the average household member's assets at the end of the period as last period's assets including interest plus non-capital income minus consumption expenditures during the period.

By now we have specified households' preferences, households' non-capital income, capital income and the dynamic accumulation of wealth. With that we are ready to start solving for the optimal behaviour of households.

We begin by noting that the entire problem can be decomposed into an optimal labour supply decision and an optimal consumption/saving decision. From (5.1) it can be seen that, if we ignore intertemporal effects in labour supply, then any choice of labour supply in any period must be such that the marginal unit of labour supply incurs as many units of disutility of work as it earns utility from consumption. If this were not the case, period utility $u_{a,t}$ could be increased by either increasing or lowering the supply of labour in the given period.

Furthermore we may define

$$Y_{a,t}^{H,Disp,Net} = Y_{a,t}^{H,Disp} \frac{N_{a,t}^{Adult}}{N_{a,t}^{AdultEq}} - P_t^{C,H} Z_{a,t}, \tag{5.36}$$

$$\left(1+i_t^{H,Net}\right) = \left(1+i_t^H\right) \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}}, \tag{5.37}$$

$$\begin{aligned}
Gain_{a,t}^{Shock} &= Gain_{a,t}^{ShareShock} - J_{a,t}^{GainTax} \\
&+P_{j,t}^{K,H,L,Gain} \left(K_{a-1,t-1}^{H,L,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + I_{a,t}^{H,L,NoShock}\right) \\
&+P_{j,t}^{K,H,B,Gain} \left((1-\delta) K_{a-1,t-1}^{H,B} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} + I_{a,t}^{H,B,NoShock}\right)
\end{aligned} \tag{5.38}$$

which gives us period disposable income net of disutility of work per adult-equivalent $Y_{a,t}^{H,Disp,Net}$, household average yield on financial assets net of changes in household size and the sum of wealth changes associated with surprise shocks.

We can then rewrite the asset accumulation equation by adding and subtracting $P_t^{C,H} Z_{a,t}$ from the right hand side of (5.35) and use (5.1), (5.36), (5.37) and (5.38) to get

$$\begin{aligned} A_{a,t}^{H,Ind} &= \left(1 + i_t^{H,Net}\right) A_{a-1,t-1}^{H,Ind} + Y_{a,t}^{H,Disp,Net} - P_t^{C,H} u_{a,t} \\ &+ k_t^{User} \sum_{j\in\{D\}} \left(P_{j,t-1}^{K,H,B} K_{a-1,t-1}^{H,B} + P_{j,t-1}^{K,H,L} K_{a-1,t-1}^{H,L,Ind}\right) \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} \\ &+ Gain_{a,t}^{Shock}. \end{aligned} \tag{5.39}$$

It is clear that any optimal solution to the problem of the households must involve a choice of labour supply that maximizes wage income net of disutility of work $Y_{a,t}^{H,Disp,Net}$ in every period. A higher value of $Y_{a,t}^{H,Disp,Net}$ expands the feasible set of period utility $u_{a,t}$, so it must be optimal to maximize $Y_{a,t}^{H,Disp,Net}$ w.r.t. labour supply. We will describe labour supply in details in section 5.6, but in the following section we focus on the consumption/saving decision, taking as given the self-contained labour supply decision and hence non-capital income.

5.5 Consumption and saving

5.5.1 Consolidated budget

Based on (5.5) and (5.39), the strategy will be to state the intertemporal consumption/saving problem as one of maximizing a simple standard CES function subject to a fixed budget with general solution given in the appendix on CES functions.

In order to state the problem in this simple form, we first need to derive a fixed life-time budget. To this end we rewrite the asset accumulation (5.39) to obtain the consolidated budget or life-time budget constraint facing a given household when it plans consumption over its remaining time horizon.

First define the following discounting factor which discounts from period i to period a:

$$R_{a,i} \equiv \prod_{s=a+1}^{i} \frac{1}{1 + i_{s,t-a+s-1}^{H,Net}} \text{ for } i > a \;\; and \quad R_{a,a} \equiv 1. \tag{5.40}$$

I.e. the value of household wealth in period i discounted to period $a-1$ is given by

$$R_{a-1,i} A_{i,t-a+i}^{H,Ind}. \tag{5.41}$$

We can split the value of household wealth in period i discounted to period $a-1$ into two components: The value of household wealth in period $a-1$ plus the present discounted flow of income net of consumption and disutility of work between period $a-1$ and period i. Using (5.39),

$$\begin{aligned} R_{a-1,i}A^{H,Ind}_{i,t-a+i} &= A^{H,Ind}_{a-1,t-1} \\ &+\sum_{q=a}^{i} R_{a-1,q}\Big[\quad Gain^{Shock}_{q,t-a+q} + Y^{H,Disp,Net}_{q,t-a+q} \\ &+k^{User}_{t-a+q}\sum_{j\in\{D\}}\left(P^{K,H,B}_{j,t-a+q-1}K^{H,B}_{q-1,t-a+q-1} + P^{K,H,L}_{j,t-a+q-1}K^{H,L,Ind}_{q-1,t-a+q-1}\right) \\ &\times\frac{N^{AdultEq}_{q-1,t-a+q-1}}{N^{AdultEq}_{a,t-a+q}} - P^{C,H}_{t-a+q}u_{q,t-a+q} \quad \Big]. \end{aligned} \tag{5.42}$$

Now using the terminal condition that household wealth must be zero at the end of the period before the household turns 78, we get

$$\begin{aligned} R_{a-1,77}A^{H,Ind}_{77,t-a+77} &= A^{H,Ind}_{a-1,t-1} \\ &+\sum_{i=a}^{77} R_{a-1,i}\Big[\quad Gain^{Shock}_{i,t-a+i} + Y^{H,Disp,Net}_{i,t-a+i} \\ &+k^{User}_{t-a+i}\sum_{j\in\{D\}}\left(P^{K,H,B}_{j,t-a+i-1}K^{H,B}_{i-1,t-a+i-1} + P^{K,H,L}_{j,t-a+i-1}K^{H,L,Ind}_{i-1,t-a+i-1}\right) \\ &\times\frac{N^{AdultEq}_{i-1,t-a+i-1}}{N^{AdultEq}_{a,t-a+i}} - P^{C,H}_{t-a+i}u_{i,t-a+i} \quad \Big] \\ &= 0. \end{aligned} \tag{5.43}$$

Denoting the present value of disposable income, capital gains on buildings, taxation of surprise capital gains and the National Account calibration term as respectively

$$\begin{aligned} NPV^{Disp}_{a-1,t-1} &= \sum_{i=a}^{77} R_{a-1,i}Y^{H,Disp,Net}_{i,t-a+i}, \\ NPV^{Gain}_{a-1,t-1} &= \sum_{i=a}^{77} R_{a-1,i}Gain^{Shock}_{i,t-a+i}, \end{aligned}$$

$$NPV^{kUser}_{a-1,t-1} = \sum_{i=a}^{77} R_{a-1,i}k^{User}_{t-a+i}\left(P^{K,H,B}_{j,t-a+i-1}K^{H,B}_{i-1,t-a+i-1} + P^{K,H,L}_{j,t-a+i-1}K^{H,L,Ind}_{i-1,t-a+i-1}\right)\frac{N^{AdultEq}_{i-1,t-a+i-1}}{N^{AdultEq}_{a,t-a+i}} \quad , j\in$$

we can write the consolidated budget constraint as

$$\sum_{i=a}^{77} R_{a-1,i}P^{C,H}_{t-a+i}u_{i,t-a+i} = A^{H,Ind}_{a-1,t-1} + NPV^{Disp}_{a-1,t-1} + NPV^{Gain}_{a-1,t-1} + NPV^{kUser}_{a-1,t-1}. \tag{5.44}$$

Recall that we are looking to specify the consumption/saving choice as maximization of a CES function subject to a given budget. Here the budget is given by the right-hand side of (5.44), and we note that initial assets $A^{H,Ind}_{a-1,t-1}$ and human capital net of disutility of work

$NPV^{Disp}_{a-1,t-1}$ are both exogenous to the optimal consumption decision. Furthermore, although the present discounted value of unforeseen capital gains net of taxes $NPV^{Gain}_{a-1,t-1}$ does depend on the consumption/saving choices made by the households, it is by definition unforeseen and therefore has an ex ante value of zero.

However, the NA calibration term $NPV^{kUser}_{a-1,t-1}$ cannot be said to be neither unforeseen nor independent of the choice of an optimal consumption path, since it is related to the households' ownership of residential buildings and land, which is modelled as part of household consumption. It is therefore assumed that households, when optimizing with respect to consumption, ignore this effect of its consumption choice on the life-time budget.

5.5.2 The Keynes-Ramsey rule

We can now state the maximization problem of the household

$$\max_{\{u_{i,t-a+i}\}_{i=a}^{77}} U_{a-1,t-1} \equiv \left[\sum_{i=a}^{77} \left(u_{i,t-a+i}\right)^{\frac{\nu-1}{\nu}} \upsilon_{a-1,i} N^{AdultEq}_{i,t-a+i} \right]^{\frac{\nu}{\nu-1}} \tag{5.45}$$

$$s.t. \quad \sum_{i=a}^{77} R_{a-1,i} P^{C,H}_{t-a+i} u_{i,t-a+i} = A^{H,Ind}_{a-1,t-1} + NPV^{Disp}_{a-1,t-1} + NPV^{kUser}_{a-1,t-1}. \tag{5.46}$$

Thus we have now rewritten the problem as one of maximizing a CES function over the sequence of utilities $\{u_{i,t-a+i}\}_{i=a}^{77}$ with prices $\left\{R_{a-1,i} P^{C,H}_{t-a+i}\right\}_{i=a}^{77}$ and distribution parameters $\left\{\upsilon_{a-1,i} N^{AdultEq}_{i,t-a+i}\right\}_{i=a}^{77}$ subject to a given lifetime budget $\sum_{i=a}^{77} R_{a-1,i} P^{C,H}_{t-a+i} u_{i,t-a+i} = A^{H,Ind}_{a-1,t-1} + NPV^{Disp}_{a-1,t-1} + NPV^{kUser}_{a-1,t-1}$. The advantage of this formulation is that we can now use the general solution to a standard constrained CES maximization problem to solve the intertemporal decision problem of households. Thus referring to the appendix on maximizing CES functions the solution is easily found as

$$u_{i,t-a+i} = \left(\upsilon_{a-1,i} N^{AdultEq}_{i,t-a+i}\right)^{\nu} \left(\frac{R_{a-1,i} P^{C,H}_{t-a+i}}{P^{Life}_{a-1,t-1}}\right)^{-\nu} \frac{A^{H,Ind}_{a-1,t-1} + NPV^{Disp}_{a-1,t-1} + NPV^{kUser}_{a-1,t-1}}{P^{Life}_{a-1,t-1}}. \tag{5.47}$$

$P^{Life}_{a-1,t-1}$ is the standard CES price index which in our context is related to lifetime consumption and given by (referring to the appendix again)

$$P^{Life}_{a-1,t-1} = \left[\sum_{i=a}^{77} \left(\upsilon_{a-1,i} N^{AdultEq}_{i,t-a+i}\right)^{\nu} \left(R_{a-1,i} P^{C,H}_{t-a+i}\right)^{1-\nu} \right]^{\frac{1}{1-\nu}}. \tag{5.48}$$

Given the expression in (5.47) for period utility we can derive the Keynes-Ramsey equation representing the optimal path over time of period utility. Consider leading (5.47) one period and dividing the result with (5.47):

$$\frac{u_{i+1,t-a+i+1}}{u_{i,t-a+i}} = \left(\frac{\upsilon_{a-1,i+1}}{\upsilon_{a-1,i}} \frac{N^{AdultEq}_{i+1,t-a+i+1}}{N^{AdultEq}_{i,t-a+i}}\right)^{\nu} \left(\frac{R_{a-1,i+1}}{R_{a-1,i}} \frac{P^{C,H}_{t-a+i+1}}{P^{C,H}_{t-a+i}}\right)^{-\nu}. \tag{5.49}$$

Note first that

$$\frac{R_{a-1,i+1}}{R_{a-1,i}} = \frac{1}{1+i^{H,Net}_{i+1,t-a+i+1}} = \frac{1}{\left(1+i^{H}_{i+1,t-a+i+1}\right)} \frac{N^{AdultEq}_{i+1,t-a+i+1}}{N^{AdultEq}_{i,t-a+i}},$$

and

$$\frac{\upsilon_{a-1,i+1}}{\upsilon_{a-1,i}} = \frac{\xi_{i+1}}{1+\theta}.$$

Inserting this in (5.49) gives

$$\begin{aligned} \frac{u_{i+1,t-a+i+1}}{u_{i,t-a+i}} &= \left(\frac{\xi_{i+1}}{1+\theta} \frac{N^{AdultEq}_{i+1,t-a+i+1}}{N^{AdultEq}_{i,t-a+i}}\right)^{\nu} \\ &\times \left(\frac{1}{\left(1+i^{H}_{i+1,t-a+i+1}\right)} \frac{N^{AdultEq}_{i+1,t-a+i+1}}{N^{AdultEq}_{i,t-a+i}} \frac{P^{C,H}_{t-a+i+1}}{P^{C,H}_{t-a+i}}\right)^{-\nu} \Leftrightarrow \\ \frac{u_{i+1,t-a+i+1}}{u_{i,t-a+i}} &= \left(\frac{\xi_{i+1}}{1+\theta} \left(1+i^{H}_{t-a+i+1}\right) \frac{P^{C,H}_{t-a+i}}{P^{C,H}_{t-a+i+1}}\right)^{\nu}. \end{aligned} \tag{5.50}$$

The above equation is the Keynes- Ramsey rule. Note in (5.50) the presence of the parameter ξ_{i+1} which reflects the preference for leaving a bequest. As stated in (5.6), ξ equals 1 for households that are younger than 76 and therefore not leaving a bequest. In this case, for household a at time t, the Keynes-Ramsey rule looks like the following:

$$\frac{u_{a+1,t+1}}{u_{a,t}} = \left(\frac{1+i^{H}_{t+1}}{1+\theta} \frac{P^{C,H}_{t}}{P^{C,H}_{t+1}}\right)^{\nu}, a = 17, ...75. \tag{5.51}$$

The Keynes-Ramsey rule shows the standard relationship where a high value of the interest rate and low impatience leads to an increasing consumption profile for given relative prices, while an increasing profile of relative prices tends to lower future consumption. The strength of these effects is determined by the willingness to substitute intertemporally as represented by the intertemporal elasticity of substitution ν.

5.5.3 The Bequest decision

As explained in relation to (5.5), the modelling of the bequest decision is based on the assumption that parent households obtain utility from leaving a bequest as if it was an extra year's consumption. This means that we find the optimal size of the bequest simply by letting the Keynes-Ramsey rule run for one period after the household has ceased to exist as a planning household. Hence, the equation that determines the size of the optimal bequest in the model looks similar to (5.51).

$$\frac{\left(1+i^{H}_{t+1}\right) A^{H,Beq}_{i,t}}{P^{C,H}_{t+1} u_{a,t}} = \left(\frac{1}{1+\theta} \left(1+i^{H}_{t+1}\right) \xi \frac{P^{C,H}_{C,t}}{P^{C,H}_{C,t+1}}\right)^{\nu}, a = 76. \tag{5.52}$$

Comparing (5.51) and (5.52), note first that we have now included the preference for bequest ξ from (5.6). Furthermore, the left-hand side of the equation reflects the fact that although the parent household obtains utility from the bequest in the period after it has turned 77, it leaves the bequest at the end of the preceding period. Therefore the value of the bequest in terms of utility in the following period is the size of the bequest left in the previous period $A^{Beq}_{a,t}$ plus the average yield it earns between the two periods $i^{H}_{a+1,t+1}A^{H,Beq}_{a,t}$ divided by the price index $P^{C,H}_{C,t+1}$. Thus $\frac{(1+i^{H}_{a+1,t+1})A^{H,Beq}_{i,t}}{P^{C,H}_{C,t+1}}$ is the period $a+1$ value in terms of utility of the bequest.

We have now described the intertemporal decision problem of the households in the model. Equations (5.50) and (5.1) together give the optimal intertemporal path of the overall utility from consumption per adult-equivalent $C^{H}_{a,t}$. However, we need to determine the household's disutility of work per adult-equivalent $Z_{a,t}$, which enters (5.1). Hence, in the following section we specify labour supply.

5.6 Labour supply

5.6.1 Union model

Effective labour supply in DREAM is assumed to be affected by the presence of a trade union which has the power to ration labour supply. This gives rise to aggregate unemployment in the model.

Labour supply in DREAM is based on the assumption of worksharing or underemployment, which means that workers take equal part in aggregate unemployment. Due to the fact that people of the same origin, sex and age are identical, the lack of uncertainty in the model requires that people of the same origin, sex and age work the same number of hours and are unemployed for the same number of hours. Unemployment for a worker of origin o, sex s and age a is the difference between the worker's actual labour supply $L^{S}_{o,s,a,t}$ and the value $L^{Max}_{o,s,a,t}$ taken to be the maximum number of working hours the worker would supply if the union did not exist. Unemployment data is matched at the economy-wide level and at the level of groups of individuals of the same origin, sex and age.

In the following we will formally derive the equations related to labour supply. First of all we focus on the determinaton of $L^{Max}_{o,s,a,t}$. We define full time employment as the number of hours which an employed individual would optimally choose to work if no union was present when maximizing $Y^{H,Disp,Net}_{a,t}$ facing a given after-tax real wage and disutility of work of the form (5.2). Because labour supply is disaggregated and differs according to origin, sex and age, we maximize individual disaggregated non-capital income net of disutility of work

$Y^{H,Disp,Net}_{o,s,a,t}$ (which is equivalent to maximizing $Y^{H,Disp,Net}_{a,t}$ since it is just a weighted average over $Y^{H,Disp,Net}_{o,s,a,t}$). Furthermore, all other income but wage earnings is taken as given, and contributions to various pension schemes $q^{ZF}_{a,t}, q^{ZP}_{a,t}, q^{ATP}_{a,t}$ and $q^{SP}_{a,t}$ are not taken into account by the worker (it should be remembered that they represent savings which are given back to the average worker). Note further that this decision is a marginal hours consideration by an individual who is already employed and active in the labour force. Hence, adj^{Hours} and $r^{LabFull}_{o,s,a,t}$ will not affect the choice of hours to supply and only marginal tax considerations are relevant.

This means that we can specify the problem in the following simple form:

$$\begin{aligned}
\left(1+adj^{LS}_t\right) L^{Max}_{o,s,a,t} &= \arg\max_{L^S_{o,s,a,t}} \Big[\left(1 - t^{Marg}_t + r^{EITC}_t t^{EITC}_t \left(t^{Cou}_t + t^{Mun}_t + t^{Chu}_t\right)\right)\left(1 - t^{Payroll}_t\right) \\
&\quad \times \rho_{o,s,a,t} W_t \left(1+adj^{LS}_t\right) L^S_{o,s,a,t} \\
&\qquad - P^{C,H}_t \left(\left(\eta_{o,s,a,t}\right)^{-\frac{1}{\gamma^{LMax}}}\right)\left(\frac{\gamma^{LMax}}{1+\gamma^{LMax}}\right)\left(1+adj^{LS}_t\right)\left(L^S_{o,s,a,t}\right)^{1\cdot}
\end{aligned}$$

When deriving the marginal tax t^{Marg}_t for each individual, which is used when determining labour supply, we use the differentiable functions for $TR^{Mid,Ind}_{o,s,a,t}$ and $TR^{Top,Ind}_{o,s,a,t}$ defined in (5.8) and (5.9). The marginal tax rate t^{Marg} consequently is equal to the sum of the municipal, county, church and bottom-bracket tax rates and an expression equal to the differential quotient of (5.8) and (5.9) with respect to $Y^{H,Pers}_{o,s,a,t}$:

$$\begin{aligned}
t^{Marg}_{o,s,a,t} &= t^{Bot}_t + t^{Cou}_t + t^{Mun}_t + t^{Chu}_t \\
&\quad + t^{Mid}_t \Big[6k^{Mid6}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^5 \\
&\qquad + 5k^{Mid5}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid})^4 \\
&\qquad + 4k^{Mid4}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^3 \\
&\qquad + 3k^{Mid3}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid})^2 \\
&\qquad + 2k^{Mid2}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t) \\
&\qquad + k^{Mid1}_{a,t}\Big] \\
&\quad + t^{Top}_t \Big[6k^{Top6}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top})^5 \\
&\qquad + 5k^{Top5}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^4 \\
&\qquad + 4k^{Top4}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^3 \\
&\qquad + 3k^{Top3}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^2 \\
&\qquad + 2k^{Top2}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t) \\
&\qquad + k^{Top1}_{a,t}\Big].
\end{aligned} \tag{5.54}$$

Note that the parameters for disutility of work used in (5.53) have different superscripts than those used in (5.2). Hence, although disutility of work is still of the same functional form, we allow for the parameters to take on different values than those given in (5.2). See text below (5.57).

Differentiating (5.53) with respect to $L^S_{o,s,a,t}$ and solving the first-order condition, we obtain the maximum number of hours a worker would supply facing a given after-tax real wage:

$$\begin{aligned} L^{Max}_{o,s,a,t} &= \eta_{o,s,a,t} \Bigg[\frac{1}{P^{C,H}_{C,t}} \left(1 - t^{Marg}_{o,s,a,t} + r^{EITC}_t t^{EITC}_t \left(t^{Cou}_t + t^{Mun}_t + t^{Chu}_t\right)\right) \\ &\quad \times \left(1 - t^{Payroll}_t\right) \rho_{o,s,a,t} W_t \Bigg]^{\gamma^{LMax}} . \end{aligned} \tag{5.55}$$

I.e. optimal maximum labour supply is a function of the after-tax real wage where $\eta_{o,s,a}$ is a level parameter in disutility of work and γ^{LMax} determines the sensitivity of labour supply with respect to the after-tax real wage. $L^{Max}_{o,s,a,t}$ is the optimal number of hours that an individual of origin o and sex s would optimally supply facing a given after-tax real wage. Hence, we interpret any supply of labour below $L^{Max}_{o,s,a,t}$ to give rise to $L^{Max}_{o,s,a,t} - L^S_{o,s,a,t}$ hours of unemployment.

The reason that actual labor supply $L^S_{o,s,a,t}$ turns out to be lower than our definition of full time employment $L^{Max}_{o,s,a,t}$ is the presence of a trade union which has the power to ration labour supply below $L^{Max}_{o,s,a,t}$. Due to the availability requirements of the Danish unemployment benefit scheme, an individual worker has no power to ration his labour supply and demand unemployment benefits for the time during which he is unemployed. The union, however, has the power to set a lower rate of employment, still taking into account that its members receive unemployment benefits. We assume that the union maximizes the sum of its members' utilities and that all members enter with the same weight. This specificaton implies that the union simply chooses individual labour supply to maximize individual disposable income net of disutility of work. Hence, we can state the problem as in (5.53) noting that the union takes into account that any labour supply below $L^{Max}_{o,s,a,t}$ earns $L^{Max}_{o,s,a,t} - L^S_{o,s,a,t}$ hours of unemployment benefits. This affects the marginal comparison of benefits and costs of one hour's extra work and results in a labor supply lower than $L^{Max}_{o,s,a,t}$.

Formally we have:

$$\left(1+adj_t^{LS}\right) \quad L_{o,s,a,t}^S = \arg\max_{L_{o,s,a,t}^S} \Bigg[\; \left(1 - t_{o,s,a,t}^{Marg} + r_t^{EITC} t_t^{EITC}\left(t_t^{Cou} + t_t^{Mun} + t_t^{Chu}\right)\right) \tag{5.56}$$
$$\times \left(1 - t^{Payroll}\right) \rho_{o,s,a,t} W_t \left(1+adj_t^{LS}\right) L_{o,s,a,t}^S$$
$$-P_t^{C,H}\left(\frac{\gamma}{1+\gamma}\right)\zeta_{o,s,a}^{-\frac{1}{\gamma}}\left(1+adj_t^{LS}\right)\left(L_{o,s,a,t}^S\right)^{\frac{1+\gamma}{\gamma}}$$
$$+\left(1-t_{o,s,a,t}^{Marg}\right) o_t^{G,H,Unemp}\left(L_{o,s,a,t}^{Max} - L_{o,s,a,t}^S\right) \; \Bigg],$$

which is (5.53) augmented to include income from unemployment benefits, where $o_t^{G,H,Unemp}$ is the rate of unemployment benefits. Note that the disutility parameters used by the union are different from the ones used in (5.53) by the individual worker.

Differentiating with respect to $L_{o,s,a,t}^S$ and solving the first order condition yields the labor supply equation in DREAM:

$$\begin{aligned}\left(1+adj_t^{LS}\right) L_{o,s,a,t}^S \;=\;& \zeta_{o,s,a,t}\Bigg[\; \frac{1}{P_t^{C,H}}\left(1 - t_{o,s,a,t}^{Marg} + r_t^{EITC} t_t^{EITC}\left(t_t^{Cou} + t_t^{Mun} + t_t^{Chu}\right)\right) \\ & \times\left(1 - t_t^{Payroll}\right)\rho_{o,s,a,t} W_t - \frac{1}{P_t^{C,H}}\left(1-t_{o,s,a,t}^{Marg}\right) o_t^{G,H,Unemp} \;\Bigg]^{\gamma} \\ =\;& \zeta_{o,s,a,t}\left[\frac{1}{P_t^{C,H}} L_{o,s,a,t}^{S,Numerator}\right]^{\gamma},\end{aligned} \tag{5.57}$$

where

$$\begin{aligned} L_{o,s,a,t}^{S,Numerator} \equiv\;& \left(1 - t_{o,s,a,t}^{Marg} + r_t^{EITC} t_t^{EITC}\left(t_t^{Cou} + t_t^{Mun} + t_t^{Chu}\right)\right)\left(1 - t_t^{Payroll}\right)\rho_{o,s,a,t} W_t \\ & -\left(1-t_{o,s,a,t}^{Marg}\right) o_t^{G,H,Unemp}.\end{aligned}$$

γ^{LMax} and γ are set to the following values: $\gamma^{LMax} = 0.08$ and $\gamma = 0.1$. This means that actual labour supply $L_{o,s,a,t}^S$ is more sensitive to changes in the after-tax real wage than is the case for the maximum hours of labour supply $L_{o,s,a,t}^{Max}$. The result is that the model features decreasing unemployment as a response to an increasing real wage.

The level parameters in disutility of work $\eta_{o,s,a}$ for $L_{o,s,a,t}^{Max}$ and $\zeta_{o,s,a}$ for $L_{o,s,a,t}^S$ are calibrated to match Danish unemployment data.

5.7 Non-planning households

The above exposition is related to the planning households of the model meaning households that have not yet turned 77 years. Once a household is 77 years old, it no longer makes consumption and savings decisions in an intertemporal manner. Rather, in each period a non-planning household simply consumes all of its after-tax income (pension receipts and transfers) according to an intratemporal utility function which is identical to the one specified for planning households presented in the following section. Recall from section 5.2 that a household leaves all of its assets as a bequest (including residential assets) to its heirs at the end of the period before it turns 77 years and becomes a non-planning household. This formulation is made in order to avoid very large assets holdings per adult-equivalent of very old households with few members. It means that the household acts completely as if the last planning period is the last period during which it exists at all. But immediately after having been dissolved as an economic entity, the household realizes that it will exist as a non-planning household during the following period. At this point the household invests in next year's dwelling by borrowing and hence ends its last planning period with negative financial assets equivalent to the value of the real estate purchased. This means that at the beginning of the first period as a non-planning household, the household carries negative financial assets from last period which it must finance, along with its other consumption, out of the sum of current period income of pensions and transfers and the ultimo realization value of its real estate. Hence, at the end of the period the household sells its dwelling and brings its net financial assets to 0 as if this was the last period it existed (again). This cycle continues every period until the household ceases to exist at the end of the year in which its members are 101 years old.

Consider the asset accumulation equation derived earlier in (5.35). The above description implies that for a non-planning household total assets carried from last period $A^{H,Ind}_{a-1,t-1}$ are 0 (the negative financial assets equal the value of real estate) just as ultimo period assets $A^{H,Ind}_{a,t}$ are 0. Then, as long as no unexpected shocks occur, we can rewrite (5.35) in the form applicable to non-planning households as

$$P^{C,H}_{t} C^{H}_{a,t} = Y^{H,Disp}_{a,t} \frac{N^{Adult}_{a,t}}{N^{AdultEq}_{a,t}} + k^{User}_{t} \sum_{j \in \{D\}} \left(P^{K,H,B}_{j,t-1} K^{H,B}_{a-1,t-1} + P^{K,H,L}_{j,t-1} K^{H,L,Ind}_{a-1,t-1} \right) \frac{N^{AdultEq}_{a-1,t-1}}{N^{AdultEq}_{a,t}}. \tag{5.58}$$

I.e. the household simply consumes all its income in each period. $Y^{H,Disp}_{a,t}$ includes pensions and transfers received by the household, cf. (5.7), (5.10) and (5.13), while the last term represents supplementary rental income from owning residential estate as described earlier.

In the case of a surprise shock, the above description is slightly modified. In this case, the non-planning household experiences a (possibly negative) capital gain on its residential assets and consequently generally ends up with non-zero total assets in the corresponding period. It is then assumed that the household during the following period consumes this capital gain as well as the ordinary income of that period. Note that non-planning generations do not receive a capital gain from their holdings of shares - these are solely allocated between planning

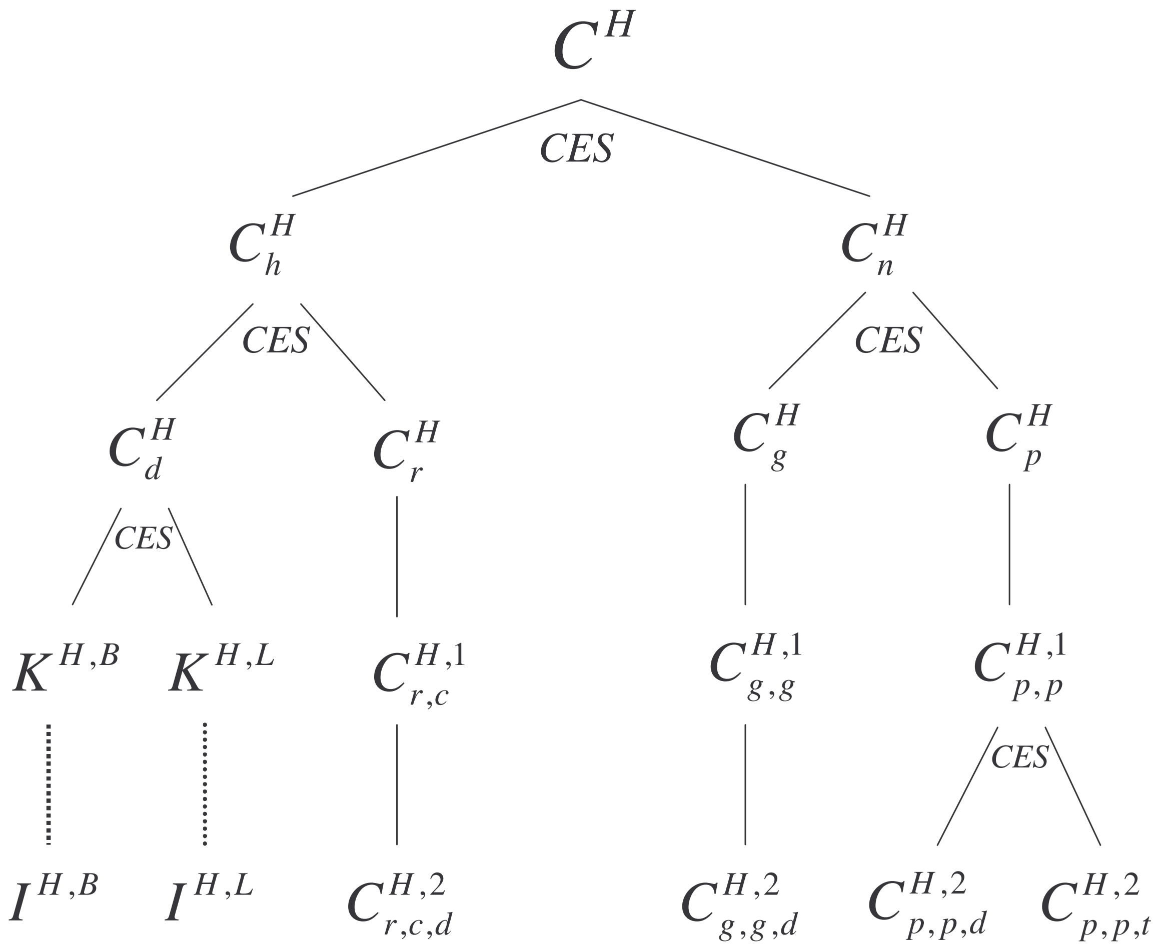

Figure 5.2:

generations.

5.8 Consumption split

This section describes the intratemporal decision problem of households, which is about dividing total consumption per adult-equivalent into its components in any given period. This is done by minimizing the cost of obtaining a given level of utility from consumption $C_{a,t}^H$ subject to the households' CES preferences and the prices on individual goods available for consumption. The figure below shows the CES nest representing households' intratemporal preferences for individual components of consumption. The reader may want to go back and read the material on page 80, on which we shall draw heavily in the following.

We will now formally state the optimization problem of a single representative household a in period t. At the top nest the household chooses between the aggregates housing and non-housing each period to minimize the cost of obtaining the level of total household consumption per adult-equivalent $C_{C,a,t}^H$. Hence, the consumers face the following problem where consumption is per adult-equivalent unless stated otherwise:

$$\min_{C^H_{e,a,t}} \sum_{e\in\{H,N\}} P^{C,H}_{e,t} C^H_{e,a,t} \tag{5.59}$$

$$s.t. \quad C^H_{C,a,t} = \left[\sum_{e\in\{H,N\}} \mu^{CH}_e (C^H_{e,a,t})^{\frac{\sigma^C-1}{\sigma^C}} \right]^{\frac{\sigma^C}{\sigma^C-1}}, \tag{5.60}$$

where $C^H_{e,a,t}$ is household a's demand for the housing aggregate and non-housing aggregate with price indexes $P^{C,H}_{e,t}$. μ^H_e are distribution parameters in the CES function while σ^C is the elasticity of substitution between the housing and non-housing aggregates and $C^H_{C,a,t}$ is considered given.

As shown in an appendix, the solution to the above problem results in the following demands for housing and non-housing respectively:

$$C^H_{e,a,t} = (\mu^{CH}_e)^{\sigma^C} \left(\frac{P^{C,H}_{e,t}}{P^{C,H}_t} \right)^{-\sigma^C} C^H_{a,t} \quad , \; e = \{H, N\}, \tag{5.61}$$

and the overall consumption price index is given by:

$$P^{C,H}_t = \left[\sum_{e\in\{H,N\}} (\mu^{CH}_e)^{\sigma^C} (P^{C,H}_{e,t})^{1-\sigma^C} \right]^{\frac{1}{1-\sigma^C}}. \tag{5.62}$$

Now as mentioned above, the housing and non-housing aggregates are further decomposed through additional cost minimization problems. Looking at the composition of the non-housing aggregate first, we have the following problem of minimizing the cost of obtaining $C^H_{e,a,t}$ by choosing between consumption of private goods and government goods $C^H_{e,a,t}$ according to:

$$\min_{(C^H_{e,a,t})} \sum_{e\in\{P,G\}} P^{C,H}_{e,t} C^H_{e,a,t} \tag{5.63}$$

$$s.t. \; C^H_{N,a,t} = \left[\sum_{e\in\{P,G\}} \mu^{CH}_e (C^H_{e,a,t})^{\frac{\sigma^C_N-1}{\sigma^C_N}} \right]^{\frac{\sigma^C_N}{\sigma^C_N-1}}, \tag{5.64}$$

where $C^H_{H,a,t}$ is given from above and $P^{C,H}_{e,t}$, $e \in \{P, G\}$ are the price indexes for private and government intermediate goods respectively.

This gives rise to the following demands:

$$C^H_{e,a,t} = (\mu^{CH}_e)^{\sigma^C_H} \left(\frac{P^{C,H}_{e,t}}{P^{C,H}_{N,t}} \right)^{-\sigma^C_N} C^H_{N,a,t} \;\; h \in \{P, G\}, \tag{5.65}$$

and the price index for non-housing consumption:

$$P_{N,t}^{C,H} = \left[\sum_{e \in \{P,G\}} \left(\mu_e^{CH}\right)^{\sigma_N^C} \left(P_{e,t}^{C,H}\right)^{1-\sigma_N^C} \right]^{\frac{1}{1-\sigma_{e\in N}^C}} . \tag{5.66}$$

Similarly, we disaggregate the housing aggregate found in (5.61) into its two components dwelling consumption and housing repair by solving the following problem:

$$\min_{C_{e,a,t}^H} \sum_{e \in \{D,R\}} P_{e,t}^{C,H} C_{e,a,t}^H \tag{5.67}$$

$$s.t.\ C_{H,a,t}^H = \left[\sum_{e \in \{D,R\}} \mu_e^{CH} (C_{e,a,t}^H)^{\frac{\sigma_H^C - 1}{\sigma_H^C}} \right]^{\frac{\sigma_H^C}{\sigma_H^C - 1}} , \tag{5.68}$$

where $C_{H,a,t}^H$ is the housing aggregate, and $C_{e,a,t}^H$ $e \in \{D, R\}$ represents consumption of dwellings and housing repair. Again using the appendix we find the solution to the above problem:

$$C_{e,a,t}^H = \left(\mu_e^{CH}\right)^{\sigma_H^C} \left(\frac{P_{e,t}^{C,H}}{P_t^{C,H}} \right)^{-\sigma_H^C} C_{H,a,t}^H, e = \{D, R\} , \tag{5.69}$$

for dwelling consumption and housing repair respectively and with housing price index given by:

$$P_{H,t}^{C,H} = \left[\sum_{e \in \{D,R\}} \left(\mu_e^{CH}\right)^{\sigma_H^C} \left(P_{e,t}^{C,H}\right)^{1-\sigma_H^C} \right]^{\frac{1}{1-\sigma_H^C}} .$$

Thus we have derived household a's demand per adult-equivalent for the 4 types of consumption $C_{e,a,t}^H$, $e \in \{D, R, G, P\}$, and we now want to aggregate this across households to obtain the total demand. At the same time we will now introduce the index $k \in \{C, P, G, D\}$ representing the delivering production sectors: construction, private, government and dwelling respectively. In this presentation there will be a one to one correspondance between the sets E and K (i.e.between consumption type and delivering sector) such that in what follows, consumption variables are defined only for the pairs $(e, k) = (P, P), (G, G), (D, D), (R, C)$. However, DREAM can be adapted to a multisector version where a given value of the index e may be associated with more than one value of the index k indicating more than one delivering sector.

Before proceeding with the aggregation we note that the price indices $P_{R,t}^{C,H}$ used above are equivalent to the prices derived in (5.76) below for $e = \{R, P, G\}, k = \{C, P, G\}$

$$P_{e,t}^{C,H} = P_{e,k,t}^{C,H,1} \mu_{e,k}^{CH,1} . \tag{5.70}$$

Note in relation to (5.70) above and (5.72) below that the one to one correspondance between the sets e and k is captured in the distribution parameter $\mu_{e,k}^{C,1}$, which takes on the value 1 or 0 in accordance with the previously defined (e, k) pairs.

For estate consumption delivered by the dwelling sector $(e,k)=(D,D)$, the price index $P^{C,H}_{D,t}$is given as the dwelling sector price index

$$P^{CH,1}_{D,t} = P^{Y}_{D,D,t}. \tag{5.71}$$

where $P^{Y}_{D,D,t}$ is composed of usercosts on buildings and land as can be seen from (5.88).

We now aggregate over households to obtain the total household demand for intermediate goods. Omitting dwelling consumption for the moment, we get:

$$C^{H,1}_{e,k,t} = \sum_{a} N^{AdultEq}_{a,t} \mu^{CH,1}_{e,k} C^{H}_{e,a,t}, \quad e \in \{R,P,G\}, k \in \{C,P,G\}. \tag{5.72}$$

Having obtained total demand for intermediate goods we are ready to derive the total household consumption demand for final goods, which involves splitting intermediate goods into domestic and foreign goods. For this reason we introduce the index $c \in \{D,F\}$ to distinguish between domestic and foreign goods. We get

$$\min_{\left(C^{H,2}_{e,k,c,t}\right)} \sum_{c \in \{D,F\}} P^{C,H,2}_{e,k,c,t} C^{H,2}_{e,k,c,t} \quad h \in \{H,G,P\} \;\; j \in \{C,P,G\} \tag{5.73}$$

$$s.t.\; C^{H,1}_{h,j,t} = \left[\sum_{c \in \{D,F\}} \mu^{CH,2}_{h,j,c} \left(C^{H,2}_{e,k,c,t}\right)^{\frac{\sigma^{C1}_{h,j}-1}{\sigma^{C1}_{h,j}}} \right]^{\frac{\sigma^{C1}_{h,j}}{\sigma^{C1}_{h,j}-1}} \quad h \in \{H,G,P\}, j \in \{C,G,P\}, \tag{5.74}$$

where $C^{H,2}_{e,k,c,t}$ are consumption of foreign and domestic goods respectively with prices $P^{H,C,2}_{e,k,c,t}$.

The solution gives rise to the following total demands for domestically and foreign produced goods:

$$C^{H,2}_{e,k,c,t} = \left(\mu^{CH,2}_{h,j,c}\right)^{\sigma^{C1}_{h,j}} \left(\frac{P^{H,C,2}_{h,j,t}}{P^{H,C,1}_{h,j,t}} \right)^{-\sigma^{C1}_{h,j}} C^{H,1}_{h,j,t}, \; h \in \{H,G,P\} \quad j \in \{C,G,P\} \;\; c \in \{D,F\}, \tag{5.75}$$

with corresponding price index for private manufactoring, private construction and government intermediate goods:

$$P^{C,H,1}_{h,j,t} = \left[\sum_{c \in \{D,F\}} \left(\mu^{CH,2}_{h,j,c}\right)^{\sigma^{C1}_{h,j}} \left(P^{C,H,2}_{e,k,c,t}\right)^{1-\sigma^{C1}_{h,j}} \right]^{\frac{1}{1-\sigma^{C1}_{h,j}}} \quad h \in \{H,G,P\} \quad j \in \{C,G,P\}. \tag{5.76}$$

However, as mentioned earlier the governmental good and the goods supplied by the construction sector are assumed to be produced domestically only and hence a dummy ensures that if $(h,j)=(G,G)$ *or* $(h,j)=(H,C)$ then (5.75) only applies for $c=\{D\}$ with

$$\mu^{C,2}_{h,j,D} = 1, \tag{5.77}$$

$$P^{C,H,2}_{h,j,D,t} = P^{C,H,1}_{h,j,t}, \tag{5.78}$$

implying

$$C^{H,2}_{h,j,D,t} = C^{H,1}_{h,j,t}. \tag{5.79}$$

The domestic consumer good price $P^{C,H,2}_{e,k,c,t}$ is related to the producer output price for $h \in \{H,G,P\}, j \in \{C,G,P\},\ c \in \{D\}$ as specified in the following equation:

$$P^{C,H,2}_{e,k,c,t} = \left(1 - s^{G,H,Dwe}_{e,t} + t^{H,Reg}_{e,t} + t^{H,VAT}_{e,t} + t^{H,DutyV}_{e,t} + t^{H,DutyQ}_{e,k,c,t} - s^{G,H,Spe}_{e,t} - s^{EU,H,Spe}_{e,t}\right) P^{Y}_{k,t}, \tag{5.80}$$

where $s^{G,H,Dwe}_{e,t}, s^{G,H,Spe}_{e,t} and\ s^{EU,H,Spe}_{e,t}$ are subsidies for household consumption, respectively subsidies for dwelling purposes, product-specific subsidies from the government and product-specific subsidies from the EU.

$t^{H,Reg}_{e,t}, t^{H,VAT}_{e,t}, t^{H,DutyV}_{e,t}, t^{H,DutyQ}_{e,k,c,t}$ denote respectively the registration tax rate on household vehicles, the value added tax rate on household demand, the product-specific value tax rate on household demand and finally the product-specific quantity tax rate on household demand, cf. chapter 7.

A similar relationship applies between the foreign producer price and domestic consumer price except for an additional term representing the customs tax rate on household demand for foreign goods. Hence for $k = P$, $e = P$, $c = F$ we have:

$$\begin{aligned} P^{C,H,2}_{e,k,c,t} &= \left(1 - s^{G,H,Dwe}_{e,t} + t^{H,Reg}_{e,t} + t^{H,VAT}_{e,t} + t^{H,DutyV}_{e,t} + t^{H,DutyQ}_{e,k,c,t} - s^{G,H,Spe}_{e,t} - s^{EU,H,Spe}_{e,t}\right) \\ &\quad \times \left(1 + t^{H,Cus}_{e,t}\right) P^{F}_{t}. \end{aligned} \tag{5.81}$$

Household investments in building and land

When aggregating over households in (5.72), we omit dwelling consumption, which is a capital good of the household and treated somewhat differently. Therefore the remaining part of the description of intratemporal consumption will deal with the disaggregation of household demand for dwellings found in (5.69) with the purpose of deriving household investment in buildings and land.

To begin with, we will now relabel utility from dwelling consumption per adult-equivalent to $K^H_{a,t}$ (as it has been denoted in previous sections), since in effect this is a capital good of the household. Specifically, we have the following relationship in the model:

$$K^H_{a,t} = C^H_{D,a+1,t+1}\left(\mu^{CH,1}_{D}\frac{N^{AdultEq}_{a+1,t+1}}{N^{AdultEq}_{a,t}}\right). \tag{5.82}$$

Since stock variables that are active in period $t+1$ are updated at the end of period t, the demanded dwelling capital stock in any given period is in place at the end of the previous period. The fraction $\frac{N^{Adult}_{a,t}}{N^{AdultEq}_{a,t}}$ is a correction factor taking into account that $K^H_{a,t}$ and $C^H_{D,a+1,t+1}$are given per adult-equivalent and thus corrects for changes in household size.

$K^{H}_{a,t}$ consists of capital in the form of buildings and capital in the form of land, and the household minimizes costs over these two capital types to obtain the index $K^{H}_{a,t}$:

$$\min_{K^{H,B}_{a,t},K^{H,L,Ind}_{a,t}} P^{K,H,B,User}_{D,t+1} K^{H,B}_{a,t} + P^{K,H,L,User}_{D,t+1} K^{H,L,Ind}_{a,t} \tag{5.83}$$

$$s.t.\ K^{H}_{a,t} = \left(\mu^{H,B}(K^{H,B}_{a,t})^{\frac{\sigma^{KH}-1}{\sigma^{KH}}} + \mu^{H,L}(K^{H,L,Ind}_{a,t})^{\frac{\sigma^{KH}-1}{\sigma^{KH}}}\right)^{\frac{\sigma^{KH}}{\sigma^{KH}-1}}, \tag{5.84}$$

$$K^{H}_{a,t} = C^{H}_{D,a+1,t+1}\left(\frac{N^{AdultEq}_{a,t+1}}{N^{AdultEq}_{a,t}}\right), \tag{5.85}$$

giving rise to the following demands for residential building capital and land capital respectively:

$$K^{H,B}_{a,t} = \sum_{k\in\{D\}} (\mu^{K,H,B})^{\sigma^{KH}} (\frac{P^{K,H,B,User}_{k,t+1}}{P^{Y}_{k,t+1}})^{-\sigma^{KH}} K^{H}_{a,t}, \tag{5.86}$$

$$K^{H,L,Ind}_{a,t} = \sum_{k\in\{D\}} (\mu^{K,H,L})^{\sigma^{KH}} (\frac{P^{K,H,L,User}_{k,t+1}}{P^{Y}_{k,t+1}})^{-\sigma^{KH}} K^{H}_{a,t}, \tag{5.87}$$

with the price index of dwelling services given by:

$$P^{Y}_{D,t} = ((\mu^{H,B})^{\sigma^{KH}} (P^{K,H,B,User}_{D,t})^{1-\sigma^{KH}} + (\mu^{H,L})^{\sigma^{KH}} (P^{K,H,L,User}_{D,t})^{1-\sigma^{KH}})^{\frac{1}{1-\sigma^{KH}}}, \tag{5.88}$$

where $P^{K,H,B,User}_{k,t}$ and $P^{K,H,L,User}_{k,t}$ are user-costs involved in possessing each type of residential capital as defined in (5.21) and (5.22).

Now given the above demands for residential housing capital in the form of buildings and land we can derive the residual investments in buildings and land capital per adult-equivalent:

$$K^{H,B}_{a,t} = \sum_{k\in\{D\}} \left(1-\delta^{PB}_{k,t}\right) K^{H,B}_{a-1,t-1} \frac{N^{AdultEq}_{a-1,t-1}}{N^{AdultEq}_{a,t}} + I^{H,B}_{a,t}, \tag{5.89}$$

$$K^{H,L,Ind}_{a,t} = K^{H,L,Ind}_{a-1,t-1} \frac{N^{AdultEq}_{a-1,t-1}}{N^{AdultEq}_{a,t}} + I^{H,L}_{a,t}. \tag{5.90}$$

Investments per adult-equivalent $I^{H,B}_{a,t}$ and $I^{H,L,Ind}_{a,t}$ are given by the difference between the demanded capital stock and the existing stock net of depreciation and change in the number of adult-equivalents in the household since capital stocks are given per adult-equivalent.

Finally, we can now aggregate over households to find the total household gross investment in buildings

$$I^{P,B}_{D,t} = \sum_{a} I^{H,B}_{a,t} N^{AdultEq}_{a,t}. \tag{5.91}$$

5.9 Appendix: Equations describing household behaviour in growth- and inflation-corrected terms

In this section, the equations from the preceding sections which are actually used in the computer version of the model are written in growth- and inflation-corrected terms. (5.2) inserted into (5.1) yields

$$u_{a,t} = C^{H}_{C,a,t} - \sum_{o}\sum_{s\in\{m,f\}} N^{Ind}_{o,s,a,t} adj^{Hours}_t r^{LabFull}_{o,s,a,t} \left(\zeta_{o,s,a,t}\right)^{-\frac{1}{\gamma}} \left(\frac{\gamma}{1+\gamma}\right)\left(1+adj^{LS}_t\right)\left(L^{S}_{o,s,a,t}\right)^{\frac{1+\gamma}{\gamma}} \frac{1}{N^{AdultEq}_{a,t}}$$

for $17 \leq a < 77$.

Equations concerning non-capital income are (5.7), (5.8), (5.9), (5.10) and (5.13):

$$\begin{aligned}
Y^{H,Pers}_{o,s,a,t} = & \left(1 - t^{Payroll}_t - q^{SP}_{a,t}\right) \\
& \times adj^{Hours}_t r^{LabFull}_{o,s,a,t} \left(1+adj^{LS}_t\right) L^{S}_{o,s,a,t} \left[\rho_{o,s,a,t} W_t \left(1 - q^{ZF}_{o,s,a,t} - q^{ZP}_{s,a,t}\right) - q^{ATP}_{a,t}\right] \\
& + adj^{Hours}_t r^{LabFull}_{o,s,a,t} \left(o^{G,H,Unemp}_{o,s,a,t} - w^{q,ATP,Unemp}_t q^{ATP}_{a,t}\right)\left(L^{Max}_{o,s,a,t} - \left(1+adj^{LS}_t\right) L^{S}_{o,s,a,t}\right) \\
& + \left(r^{S}_{o,s,a,t} + r^{LabS}_{o,s,a,t}\right) o^{G,H,S}_t \\
& + r^{LA}_{o,s,a,t} o^{G,H,LA}_t \\
& + r^{MB}_{o,s,a,t} o^{G,H,MB}_t \\
& + \left(r^{SB}_{o,s,a,t} + r^{Lab,SB}_{o,s,a,t}\right) o^{G,H,SB}_t \\
& + r^{BB}_{o,s,a,t} o^{G,H,BB}_t \\
& + r^{PEW}_{o,s,a,t} o^{G,H,PEW}_t \\
& + \left(r^{AP}_{o,s,a,t} + r^{Lab,AP}_{o,s,a,t}\right) o^{G,H,AP}_t \\
& + \left(r^{OAP}_{o,s,a,t} + r^{Lab,OAP}_{o,s,a,t}\right) o^{G,H,OAP}_{s,a,t} \\
& + r^{CA}_{o,s,a,t} o^{G,H,CA}_t \\
& + r^{AB}_{o,s,a,t} o^{G,H,AB}_t \\
& + r^{IB}_{o,s,a,t} o^{G,H,IB}_t \\
& + o^{G,H,AgeDepTax}_{s,a,t} \\
& + b^{ATP}_{s,a,t} + b^{R}_{o,s,a,t} + b^{D}_{o,s,a,t} + b^{S}_{o,s,a,t} \\
& + \frac{N^{Able}_{s,a,t}\left(b^{SP}_{s,a,t} + o^{G,H,CS}_{a,t}\right)}{\sum_o N^{Ind}_{o,s,a,t}},
\end{aligned}$$

$$\begin{aligned}
TR^{Mid,Ind}_{o,s,a,t} = & t^{Mid}_t \Big[k^{Mid6}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^6 + k^{Mid5}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^5 \\
& + k^{Mid4}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^4 + k^{Mid3}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^3 \\
& + k^{Mid2}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^2 + k^{Mid1}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^1 + k^{Mid0}_{a,t} \Big],
\end{aligned}$$

$$
\begin{aligned}
TR^{Top,Ind}_{o,s,a,t} &= t^{Top}_t \Big[\; k^{Top6}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^6 + k^{Top5}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^5 \\
&\quad + k^{Top4}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^4 + k^{Top3}_{a,t}(Y^{H,Pers}_{o,s,a,t} - kt^{Top}_t)^3 \\
&\quad + k^{Top2}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^2 + k^{Top1}_{a,t}(Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^1 + k^{Top0}_{a,t} \; \Big],
\end{aligned}
$$

$$
\begin{aligned}
Y^{H,DispGender}_{s,a,t} &= \frac{1}{N^{Adult}_{a,t}} \Bigg[\quad \sum_o N^{Ind}_{o,s,a,t} \Big[\quad Y^{H,Pers}_{o,s,a,t} \\
&\quad - r^{PEWq}_{s,a,t} k^{PEW}_t w^{PEW}_t Unemp^{Max}_t - r^{Unempq}_{s,a,t} k^{Unemp}_t w^{Unemp}_t Unemp^{Max}_t \\
&\quad - \left(t^{Cou}_t + t^{Mun}_t + t^{Chu}_t\right) \Big(\; Y^{H,Pers}_{o,s,a,t} - k^{Source}_t - allow^{Pers}_t - allow^{Assess}_t \\
&\qquad - r^{PEWq}_{s,a,t} k^{PEW}_t w^{PEW}_t Unemp^{Max}_t - r^{Unempq}_{s,a,t} k^{Unemp}_t w^{Unemp}_t Unemp^{Max}_t \; \Big) \\
&\quad - t^{EITC,Eff}_t \cdot adj^{Hours}_t r^{LabFull}_{o,s,a,t} \left(1 + adj^{LS}_t\right) L^S_{o,s,a,t} \left(\rho_{o,s,a,t} W_t \left(1 - q^{ZF}_{o,s,a,t} - q^{ZP}_{s,a,t}\right) - q^{ATP}_{a,t}\right) \\
&\quad - t^{Bot}_t \Big(\; Y^{H,Pers}_{o,s,a,t} - allow^{Pers}_t - k^{Source}_t \; \Big) \; \Big] \\
&\quad - \sum_o N^{Ind}_{o,s,a,t} \left(TR^{Mid,Ind}_{o,s,a,t} + TR^{Top,Ind}_{o,s,a,t}\right) \\
&\quad + \sum_o N^{Ind}_{o,s,a,t} \left(o^{G,H,AgeDepNoTax}_{s,a,t} + o^{G,H,NonAgeDep}_t\right) \\
&\quad + N^{Able}_{s,a,t} \left(b^{ZPR}_{a,t} + b^{LD}_{a,t}\right) \left(1 - t^{CapPen}_t\right) \quad \Bigg] \\
&\quad + \left(1 - t^{Beq}_t\right) \left(1 + i^H_t\right) \frac{A^{H,Beq}_{t-1}}{\left(1 + g_t\right)\left(1 + g^P_t\right)} \left(\frac{N^{AdultEq}_{apu,t-1}}{N^{Adult}_{a,t}}\right) Dist^{Heirs}_{s,a,aplast,t},
\end{aligned}
$$

$$
\begin{aligned}
Y^{H,Disp}_{a,t} &= \sum_{s\in\{m,f\}} Y^{H,DispGender}_{s,a,t} \\
&\quad + o^{F,H}_t + o^{G,H,LumpSustain}_t - o^{H,G,LumpRev}_t + o^{G,H,LumpExp}_t \\
&\quad - \frac{t^{H,Weight}_t P^{C,H}_{C,t} N^{AdultEq}_{a,t} C^H_{C,t}}{N^{Adult}_{a,t}} \\
&\quad - o^{H,G,Soc}_t - o^{H,G,SocOpt}_t - o^{H,G}_t - o^{H,G,Cap}_t \\
&\quad + o^{G,H}_t + o^{G,H,Cap}_t \\
&\quad - \frac{1}{N^{Adult}_{a,t}} \left(\sum_o \sum_{s\in\{m,f\}} o^{H,G,LumpPayroll}_{o,s,a,t} + o^{H,G,LumpDwe}_{a,t} + \sum_{s\in\{m,f\}} o^{H,SP,LumpSP}_{s,a,t}\right) \\
&\quad + o^{G,H,LumpInt}_t + o^{H,G,LumpGrOS}_t + o^{G,H,LumpGI}_t .
\end{aligned}
$$

Equations concerning capital income, asset accumulation and the optimal intertemporal allocation for households (the Keynes-Ramsey rule), equations (5.18), (??), (??), (??), (??), (5.51) and (5.52):

$$
\begin{aligned}
i_t^H &= \frac{w^{Assets}}{\sum_{j\in\{C,P\}} V_{j,t-1} \ / \ (1+g_t)\left(1+g_t^P\right)} \\
&\quad \times \sum_{j\in\{C,P\}} \Bigg[\quad \left(1-t_t^{H,Gain}\right)\left(V_{j,t} - \frac{V_{j,t-1}}{(1+g_t)\left(1+g_t^P\right)}\right) \\
&\quad + \left(1-t_t^{H,Div}\right) DIV_{j,t} \quad \Bigg] \\
&\quad + \left(1-w^{Assets}\right)\left(1-t_t^{H,Int}\right) i_t, \\
A_{a,t}^{H,Ind} &= A_{a,t}^{H,Fin,Ind} + P_t^{K,H} K_{a,t}^H \\
&= A_{a,t}^{H,Fin,Ind} + \sum_{j\in\{D\}} \left(P_{j,t}^{KHB} K_{a,t}^{HB} + p_{j,t}^{PHL} K_{a,t}^{HL,Ind}\right), \\
P_{j,t}^{K,H,B} &= P_{j,t}^{I,P,B} + P_{j,t}^{K,H,B,Gain}, \qquad j = D,
\end{aligned}
$$

$$
\begin{aligned}
P_{j,t}^{K,H,B,User} &= \left(t_t^{Dwe} + k_t^{User} + i_t^H\right) \frac{P_{j,t-1}^{K,H,B}}{1+g_t^P} + \delta_{j,t}^{PB} P_{j,t}^{K,H,B} - \left(P_{j,t}^{K,H,B} - \frac{P_{j,t-1}^{K,H,B}}{1+g_t^P}\right) \\
,\, j &= D, \\
P_{j,t}^{K,H,L,User} &= \left(t_t^{Dwe} + t_t^{H,Land} + k_t^{User} + i_t^H\right) \frac{P_{j,t-1}^{K,H,L}}{(1+g_t)\left(1+g_t^P\right)} - \left(P_{j,t}^{K,H,L} - \frac{P_{j,t-1}^{K,H,L}}{(1+g_t)\left(1+g_t^P\right)}\right) \\
,\, j &= D,
\end{aligned}
$$

$$
\begin{aligned}
\frac{u_{a+1,t+1}}{u_{a,t} \ / \left(1+g_{t+1}^P\right)} &= \left(\frac{1+i_{t+1}^H}{1+\theta} \frac{P_{C,t}^{C,H} \ / \ \left(1+g_{t+1}^P\right)}{P_{C,t+1}^{C,H}}\right)^{\nu}, a = 17, ...75, \\
\frac{\left(1+i_{t+1}^H\right) A_{i,t}^{H,Beq} \ / \ \left(1+g_{t+1}^P\right)}{P_{C,t+1}^{C,H} \ u_{a,t}} &= \left(\left(1+i_{t+1}^H\right)\xi \frac{1}{1+\theta} \frac{P_{C,t}^{C,H} \ / \ \left(1+g_{t+1}^P\right)}{P_{C,t+1}^{C,H}}\right)^{\nu}, a = 76.
\end{aligned}
$$

Equations describing labour supply are (5.54), (5.55) and (5.57):

$$
\begin{aligned}
t^{Marg}_{o,s,a,t} = \; & t^{Bot}_t + t^{Cou}_t + t^{Mun}_t + t^{Chu}_t \\
& + t^{Mid}_t \Big[6k^{Mid6}_{a,t} (Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^5 \\
& \quad + 5k^{Mid5}_{a,t} (Y^{H,Pers}_{o,s,a,t} - k^{Mid})^4 \\
& \quad + 4k^{Mid4}_{a,t} (Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t)^3 \\
& \quad + 3k^{Mid3}_{a,t} (Y^{H,Pers}_{o,s,a,t} - k^{Mid})^2 \\
& \quad + 2k^{Mid2}_{a,t} (Y^{H,Pers}_{o,s,a,t} - k^{Mid}_t) \\
& \quad + k^{Mid1}_{a,t} \Big] \\
& + t^{Top}_t \Big[6k^{Top6}_{a,t} (Y^{H,Pers}_{o,s,a,t} - k^{Top})^5 \\
& \quad + 5k^{Top5}_{a,t} (Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^4 \\
& \quad + 4k^{Top4}_{a,t} (Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^3 \\
& \quad + 3k^{Top3}_{a,t} (Y^{H,Pers}_{o,s,a,t} - k^{Top}_t)^2 \\
& \quad + 2k^{Top2}_{a,t} (Y^{H,Pers}_{o,s,a,t} - k^{Top}_t) \\
& \quad + k^{Top1}_{a,t} \Big],
\end{aligned}
$$

$$
\begin{aligned}
L^{Max}_{o,s,a,t} &= \eta_{o,s,a,t} \Big[\frac{1}{P^{C,H}_{C,t}} \left(1 - t^{Marg}_{o,s,a,t} + r^{EITC}_t t^{EITC}_t \left(t^{Cou}_t + t^{Mun}_t + t^{Chu}_t\right)\right) \\
& \qquad \times \left(1 - t^{Payroll}_t\right) \rho_{o,s,a,t} W_t \Big]^{\gamma^{L^{Max}}}, \\
\left(1 + adj^{LS}_t\right) L^S_{o,s,a,t} &= \zeta_{o,s,a,t} \left[\frac{1}{P^{C,H}_{C,t}} L^{S,Numerator}_{o,s,a,t} \right]^{\gamma},
\end{aligned}
$$

where

$$
\begin{aligned}
L^{S,Numerator}_{o,s,a,t} = \; & \left(1 - t^{Marg}_{o,s,a,t} + r^{EITC}_t t^{EITC}_t \left(t^{Cou}_t + t^{Mun}_t + t^{Chu}_t\right)\right) \left(1 - t^{Payroll}_t\right) \rho_{o,s,a,t} W_t \\
& - \left(1 - t^{Marg}_{o,s,a,t}\right) o^{G,H,Unemp}_t.
\end{aligned}
$$

Equations describing the optimal composition of the consumption bundle are (5.61), (5.62), (5.70), (5.71), (5.72), (5.75) and (5.76):

$$
\begin{aligned}
C^{H}_{d,e,a,t} &= \left(\mu^{CH}_{e}\right)^{\sigma^{C}} \left(\frac{P^{C,H}_{e,t}}{P^{C,H}_{d,t}}\right)^{-\sigma^{C}_{d}} C^{H}_{d,a,t} \quad , \; d \in \{C,H,N\},\; e=\{H,N\}\,, \\
P^{C,H}_{d,t} &= \left[\sum_{e\in\{H,N\}} (\mu^{CH}_{e})^{\sigma^{C}_{d}} (P^{C,H}_{e,t})^{1-\sigma^{C}_{d}}\right]^{\frac{1}{1-\sigma^{C}_{d}}} \quad , \; d \in \{C,H,N\} \\
P^{C,H}_{e,t} &= \sum_{k\in\{P,C,D,G\}} P^{C,H,1}_{e,k,t}\,\mu^{CH,1}_{e,k}, \\
P^{CH1}_{D,D,t} &= P^{Y}_{D,t}\;, \\
C^{H,1}_{e,k,t} &= \mu^{CH,1}_{e,k} \sum_{a} N^{AdultEq}_{a,t} C^{H}_{e,a,t}, \quad e=\{DR,P,G\}\,, k=\{C,P,G\}\,, \\
C^{H,2}_{e,k,c,t} &= (\mu^{CH,2}_{h,j,c})^{\sigma^{C1}_{h,j}} \left(\frac{P^{H,C,2}_{h,j,t}}{P^{H,C,1}_{h,j,t}}\right)^{-\sigma^{C1}_{h,j}} C^{H,1}_{h,j,t},\; h=\{H,G,P\} \quad j=\{C,G,P\} \;\; c\in\{D,F\}\,, \\
P^{C,H,1}_{h,j,t} &= \left[\sum_{c\in\{D,F\}} \left(\mu^{CH,2}_{h,j,c}\right)^{\sigma^{C1}_{h,j}} \left(P^{C,H,2}_{e,k,c,t}\right)^{1-\sigma^{C1}_{h,j}}\right]^{\frac{1}{1-\sigma^{C1}_{h,j}}} h=\{H,G,P\} \quad j=\{C,G,P\}\,.
\end{aligned}
$$

In the computer code, (5.80) and (5.81) together form one equation

$$
\begin{aligned}
P^{C,H,2}_{e,k,c,t} &= \left(1 - s^{G,H,Dwe}_{e,t} + t^{H,Reg}_{e,t} + t^{H,VAT}_{e,t} + t^{H,DutyV}_{e,t} + t^{H,DutyQ}_{e,k,c,t} - s^{G,H,Spe}_{e,t} - s^{EU,H,Spe}_{e,t}\right) P^{Y}_{k,t}, c = D \\
P^{C,H,2}_{e,k,c,t} &= \left(1 - s^{G,H,Dwe}_{e,t} + t^{H,Reg}_{e,t} + t^{H,VAT}_{e,t} + t^{H,DutyV}_{e,t} + t^{H,DutyQ}_{e,k,c,t} - s^{G,H,Spe}_{e,t} - s^{EU,H,Spe}_{e,t}\right) \\
&\quad \times \left(1 + t^{H,Cus}_{e,t}\right) P^{F}_{t}, \qquad c = F.
\end{aligned}
$$

The equations governing housing demand and residential investments are given by (5.82), (5.86), (5.87), (5.88), (5.89) , (5.90) and (5.91):

$$
\begin{aligned}
\frac{K^{H}_{a,t}}{1+g_{t+1}} &= C^{H}_{D,a+1,t+1}\; \mu^{CH,1}_{D} \frac{N^{AdultEq}_{a+1,t+1}}{N^{AdultEq}_{a,t}}, \\
K^{H,B}_{a,t} &= \sum_{k\in\{D\}} \left(\mu^{K,H,B}\right)^{\sigma^{KH}} \left(\frac{P^{K,H,B,User}_{k,t+1}}{P^{Y}_{k,t+1}}\right)^{-\sigma^{KH}} K^{H}_{a,t}, \\
K^{H,L,Ind}_{a,t} &= \sum_{k\in\{D\}} \left(\mu^{K,H,L}\right)^{\sigma^{KH}} \left(\frac{P^{K,H,L,User}_{k,t+1}}{P^{Y}_{k,t+1}}\right)^{-\sigma^{KH}} \frac{K^{H}_{a,t}}{1+g_t}, \\
P^{Y}_{D,t} &= \left[\left(\mu^{K,H,B}\right)^{\sigma^{KH}} \left(P^{K,H,B,User}_{D,t}\right)^{1-\sigma^{KH}} + \left(\mu^{K,H,L}\right)^{\sigma^{KH}} \left(P^{K,H,L,User}_{D,t}\right)^{1-\sigma^{KH}}\right]^{\frac{1}{1-\sigma^{KH}}}, \\
K^{H,B}_{a,t} &= \sum_{k\in\{D\}} \left(1-\delta^{PB}_{k,t}\right) \frac{K^{H,B}_{a-1,t-1}}{1+g_t} \frac{N^{AdultEq}_{a-1,t-1}}{N^{AdultEq}_{a,t}} + I^{H,B}_{a,t}, \\
K^{H,L,Ind}_{a,t} &= K^{H,L,Ind}_{a-1,t-1} \frac{N^{AdultEq}_{a-1,t-1}}{N^{AdultEq}_{a,t}} + I^{H,L}_{a,t}, \\
I^{P,B}_{D,t} &= \sum_{a} I^{H,B}_{a,t} N^{AdultEq}_{a,t}.
\end{aligned}
$$

Finally the household asset accumulation equation is given by (5.35):

$$
\begin{aligned}
A_{a,t}^{H,Ind} &= \left(1+i_t^H\right) A_{a-1,t-1}^{H,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} \frac{1}{\left(1+g_t\right)\left(1+g_t^P\right)} \\
&-J_{a,t}^{GainTax} \\
&+P_{D,t}^{K,H,B,Gain} I_{a,t}^{H,B} \\
&+Y_{a,t}^{H,Disp} \frac{N_{a,t}^{Adult}}{N_{a,t}^{AdultEq}} \\
&+k_t^{User} \sum_{j\in\{D\}} \left(P_{j,t-1}^{K,H,B} K_{a-1,t-1}^{H,B} + P_{j,t-1}^{K,H,L} K_{a-1,t-1}^{H,L,Ind}\right) \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} \frac{1}{\left(1+g_t\right)\left(1+g_t^P\right)} \\
&-P_{C,t}^{C,H} C_{C,a,t}^H, \qquad \text{if not last planning generation,}
\end{aligned}
$$

$$
\begin{aligned}
A_{a,t}^{H,Beq} &= \left(1+i_t^H\right) A_{a-1,t-1}^{H,Ind} \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} \frac{1}{\left(1+g_t\right)\left(1+g_t^P\right)} \\
&-J_{a,t}^{GainTax} \\
&+P_{D,t}^{K,H,B,Gain} I_{a,t}^{H,B} \\
&+Y_{a,t}^{H,Disp} \frac{N_{a,t}^{Adult}}{N_{a,t}^{AdultEq}} \\
&+k_t^{User} \sum_{j\in\{D\}} \left(P_{j,t-1}^{K,H,B} K_{a-1,t-1}^{H,B} + P_{j,t-1}^{K,H,L} K_{a-1,t-1}^{H,L,Ind}\right) \frac{N_{a-1,t-1}^{AdultEq}}{N_{a,t}^{AdultEq}} \frac{1}{\left(1+g_t\right)\left(1+g_t^P\right)} \\
&-P_{C,t}^{C,H} C_{C,a,t}^H, \qquad \text{if last planning generation,}
\end{aligned}
$$

where

$$
\begin{aligned}
&J_{a,t}^{GainTax} = d^{TGainIni} \\
&\times \left[\frac{\left(\sum_a N_{a-1,t-1}^{AdultEq} A_{a-1}^{H,FinIndNoshock} + \sum_{a\in ax0pu} N_{a,t-1}^{AdultEq} A^{H,BeqNoShock}\right)}{N_{a,t}^{AdultEq} \sum_{a\in apx0} \left(N_{a-1,t-1}^{AdultEq} A_{a-1}^{H,FinIndNoShock}\right)} \right] \\
&\times \left[t_{t-1}^{H,Gain} \sum_{j\in\{C,P\}} \frac{V_{j,t-1} - V_j^{NoShock}}{\left(1+g_t\right)\left(1+g_t^P\right)} w^{Assets} N_{a-1,t-1}^{AdultEq} \frac{A_{a-1}^{H,FinIndNoshock}}{\sum_{j\in\{C,P\}} V_{j,t-1}^{NoShock}} \right] \ \textit{if 1st shock period.}
\end{aligned}
$$