

Indivisible Labor and the Welfare Effects of Labor Income Tax Reform^{*}

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Abstract

This paper investigates the importance of the usual assumption of divisibility. In the labor market a finite set of choices is introduced: between working full or part-time or not to work at all. To add realism and to ensure smooth aggregate behavior the option of limited overtime for individuals working full time is introduced.

The simulations show that indeed indivisibilities matter - the results obtained in each of the two models are markedly different. The impact of the policy experiment (a move from progressive to proportional taxation of labor income) is much larger in the case where the labor supply is continuous; the welfare gains of the switch from progressive to proportional taxation is almost 150 percent larger with continuous labor supply. The sensitivity analysis shows that this result depends on how the indivisibilities are specified, but in almost all cases are the welfare gains from the tax reform more than twice as large in the continuous model.

JEL classification: C68, D58, D91, H2.

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1 Introduction

One key assumption behind most macroeconomic models - and most economic models in general - is that of divisibility: it is assumed that the representative consumer can choose consumption and other goods in a continuous fashion. In some markets, however, this assumption of continuity is at variance with empirical facts. A good example of this is the labor market, where a worker typically cannot choose to work any number of hours he wants, but has a finite set of choices. Typically the choice is between working full or part-time or not to work at all - all the in-between options, that are assumed in the macroeconomic model, are missing in real life.

In reality some groups on the labor market (or some types of jobs) are subject to discrete labor supply, and some are not. Although there are exceptions to every rule, it is easy to think of job types where one either has to work full time (plus perhaps some overtime) or not at all. And it is not just professors, military personnel or other individuals who hold positions that require extensive education or training that *de facto* cannot work less than full time. Even workers at an assembly line in a factory cannot choose their working hours as they please - production plans etc. are often based on 8-hour shifts. For these individuals the flexibility may be the choice of how many days a week to work - but each working day must be an 8-hour day.

That indivisibility is an empirically important matter is illustrated in Figure 1 on the next page that shows the distribution of weekly working hours in Denmark in 1996.¹ Notice how the choice of number of hours per week cluster around full-time, with few individuals choosing part-time, and almost nobody choosing to work less than 20 hours per week.

An obvious question is if - and how - this discreteness affects policy analysis - in other words: is the assumption of divisibility innocent? If agents are heterogeneous (for instance with respect to age, preferences or along some other dimension) a policy change may cause some agents to change their labor supply (for instance switch from working full to part-time), and some to have unchanged labor supply. Some agents may already be at a corner solution, i.e. work full time and are therefore not able to perform intertemporal substitution and work more hours. Intuition tells that this will influence the equilibrium outcome as well as any welfare calculations. The fact that this is a quantitative rather than a qualitative issue does not mean that it is unimportant and can be ignored.

¹Data from Graversen and Smith (1998).

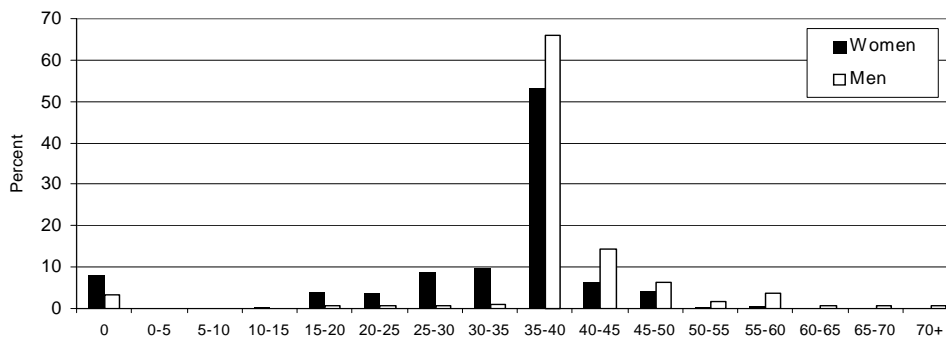


Figure 1. Hours worked per week by gender, Danish data 1996.

The assumption of divisibility usually is defended on the ground that we consider a representative agent that represents a large number of consumers: therefore the representative agent's labor supply can be continuous even if the individual labor supplies (over which this representative agent is formed) is in fact discrete. However, there is also a second more practical reason: all the nice mathematical results and methods do not work with non-differentiable functions and even to a lesser degree with functions that are not continuous.

The cause of the observed indivisibilities in the labor market have convincingly been explained by the presence of fixed costs of labor supply by Cogan (1981), and Grilli and Rogerson (1987) have shown that fixed costs of labor supply implies indivisibilities. The indivisibilities may have changed though history, in the same way as the number of working hours per week, and hence in the long run be variable. However, as a simplification they will in this paper be considered institutionally exogenous - an approach similar to Bhattacharai and Whalley (1997).

Using numerical simulations we can quantify the importance of divisibility and find out if it matters, and this paper uses numerical simulation studies to investigate the effects of indivisible labor supply. This quantitative approach is somewhat similar to the business cycle literature, where the effects of indivisibilities in the labor supply have been studied extensively, starting with the contributions of Hansen (1985) and Rogerson (1988).² In this paper the quantitative implications are investigated by performing pol-

²The standard results in the business cycle literature are that the fluctuations are larger with indivisibilities, and hence the welfare loss due to the business cycle uncertainty is also larger with indivisible labor.

icy experiments in a deterministic Computable General Equilibrium (CGE) model to examine if - and how - tax policy works differently in the presence of indivisibilities. In this case a set of simulations are carried out where the initial progressive taxation of labor income is replaced by a proportional tax - and the results from a continuous model are compared to the results from a similar model where the labor supply is discrete.

The basic model is the well-known Auerbach and Kotlikoff (1987) model - with the slight modification that labor supply may be restricted to a discrete number of possibilities. The work is somewhat along the lines of Bhattarai and Whalley (1997), but where they use a static CGE-model and some ad-hoc specifications of preferences to achieve heterogeneity, the present paper uses the standard life-cycle model. The OLG-model has the advantage of having the built-in heterogeneity (among consumers with different ages) that is needed in order to converge to an equilibrium, as well as the added realism of an intertemporal savings driven model. Compared to the work of Mulligan (1999) that is of theoretical nature, the simulation studies presented here are less general, but can give significant insights into areas that cannot be explored by analytical methods.

The results show a large difference between models with and without indivisibilities. The effect on capital accumulation is around 250 percent larger without indivisibilities, and the effect on output and 125 percent larger in the situation without indivisibilities. There is also a noticeable difference between utility in the two situations: an increase of 3.31 percent compared to 1.36 percent in the presence of indivisibilities - in other words the effect is 143 percent larger when no indivisibilities are present. These effects are similar to the findings in Bhattarai and Whalley (1997) although smaller: comparing utilities they find a difference of between factor 3 and factor 5 (depending on the set-up) between models with and without indivisibilities.³

This paper proceeds as follows: section two briefly describes the problem of indivisibility in a simple one period model. Section three describes the general equilibrium model used, section four describes the calibration of the model, and section five describes how the model is solved. Section six presents the results of the investigation, and section seven a sensitivity analysis to some of the key assumptions. Finally section eight summarizes

³Although it should be noted that Bhattarai and Whalley (1997) with a particular redistribution scheme can obtain reverse results (such that the model where the labor supply is discrete reacts more than the continuous model) - see footnote 10.

the lessons learned from the exercise, and suggests some directions for further research.

2 Models of Discrete and Continuous Labor Supply

In the model presented in this paper it is assumed that labor is not perfectly divisible. The effects of this is most easily illustrated in a single period model where the consumer faces a choice between consumption and leisure (and there is no savings decision)⁴. This is illustrated in the figure below.

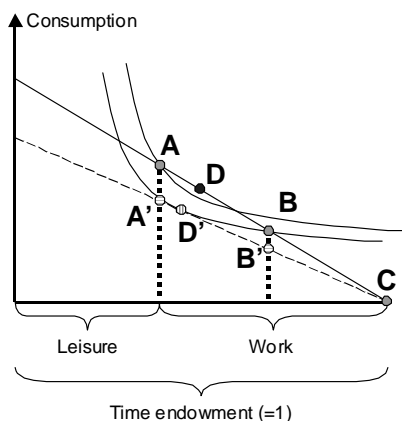


Figure 2. The discrete labor supply choices.

The consumer in Figure 2 has preferences for consumption and leisure as indicated by the indifference curves, and faces three labor supply options: Full time (A), part-time (B) and being unemployed (C). When presented with these three options the consumer prefers bundle A: working full time (which means less leisure) and more consumption.⁵ If labor supply had been a continuous variable, the consumer would have chosen to work a bit less than full time - point D in the figure would have chosen.

⁴The basic idea is the same in the life-cycle model, but the higher number of dimensions makes it less convenient to illustrate. In the life cycle model the consumer has another continuous choice variable: net savings.

⁵As indicated by the indifference curves both consumption and leisure are normal goods.

Next observe what happens if the after-tax wage declines: in this case the slope of the budget line decreases. If labor supply was continuous the optimal choice of leisure and consumption would respond in a continuous fashion to the decrease in wages: it would move from D to D' which in this case means working more hours and consuming less. In other words the income effect dominates the substitution effect. But with divisibilities the results are different: the choice is now between continuing to work full time or switch to part-time (or stop working at all). In the figure this corresponds to moving to A' or B' (or C that is unchanged). The optimal choice under the new lower wage is A' which means that all the response to the lower wage is reflected in lower consumption - the labor supply is unchanged. Had the indifference curves had a different shape the optimal response may have been to switch to part-time work: to enjoy more leisure and decrease consumption.

2.1 Labor supply

The optimal labor supplies derived from the previous Figure 2 is illustrated in the figure below:

	<i>base case</i>		<i>subst.ela. btw. leisure and consumption</i>				<i>intertemporal elasticity of substitution</i>			
			Low (0.3)		High (1.5)		Low (0.1)		High (0.5)	
	Cont.	Discrete	Cont.	Discrete	Cont.	Discrete	Cont.	Discrete	Cont.	Discrete
National income	4,026	1,793	1,278	-0,097	9,224	4,431	3,088	1,834	3,059	1,464
Wealth	5,401	1,558	2,056	0,256	8,917	2,863	3,061	1,238	1,980	1,414
Labor (hours)	3,173	1,494	0,706	-0,474	8,844	4,244	2,718	1,685	3,229	1,177
Labor (eff)	3,571	1,872	1,019	-0,215	9,327	4,959	3,097	2,034	3,421	1,480
Utility	3,312	1,362	2,071	-0,113	4,848	2,268	7,174	3,182	0,813	0,360

Figure 3. Continuous and discrete labor supply.

The figures to the left and in the middle show the labor supply under continuous and discrete labor supply (the figure to the right will be explained below). Notice that when labor is indivisible the labor supply jumps when the wage passes certain threshold values. At these threshold values the consumer is in fact indifferent between two options: for instance between working full and part-time.

2.2 Achieving aggregate continuity

Clearly the solution to the consumers problem as sketched above is not continuous - it is what is known as a bang-bang solution. If there was only one representative consumer we could end up in a situation with no equilibrium at all, because the aggregate labor supply response would be discontinuous. One solution to this problem is to introduce a large number of consumers that are different along some dimension - for instance have different elasticities of substitution or different preferences for leisure. Even if each agent has only a discrete set of choices, the aggregate labor supply response would become more smooth (the more consumers the less jagged would the aggregate labor supply be). This way of ensuring that aggregate behavior is close to continuous was chosen by Bhattarai and Whalley (1997): they used 100 consumers (that can either work full time or not at all) with preferences that follow a uniform distribution.

This paper uses an alternative way of achieving continuity in the aggregate labor supply. First of all the life-cycle model is used: this means that there already are multiple consumers (differing by age). With a 55-period model (where consumers can either work full or part-time or not at all) this means that the aggregate labor response is almost smooth. Secondly, the possibility of overtime is introduced for people working full time. Apart from being convenient from a smoothness perspective, it is also rather realistic from an empirical perspective. Figure 4 below shows what the consumer's choice set looks like under discrete labor supply with the possibility for overtime for individuals working full time.

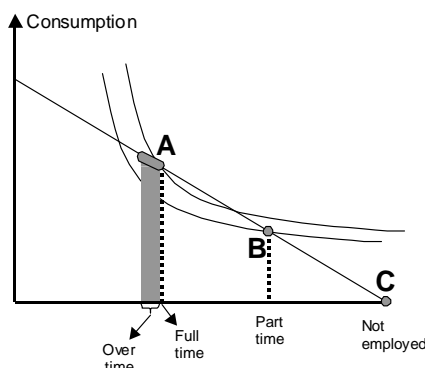


Figure 4. The discrete labor supply choices with overtime.

In the figure above the possibility of overtime will not alter the optimum

- the consumer will still prefer to participate full time in the labor market (actually working undertime is preferred). But if the wages (or preferences) had been different it might have been optimal for the consumer to work overtime. The labor supply function for this individual is shown to the right in Figure 3. Notice that the top of the step-function becomes continuous (but not necessarily differentiable) at the threshold where working overtime starts being optimal.

How much overtime is allowed and whether overtime should be allowed for individuals working part-time is of course important for the results. If we allow for a lot of over- and undertime we end up with a continuous model. The effect and importance of this overtime assumption is investigated in the sensitivity analysis.

3 A General Equilibrium Model

As mentioned previously the model described below is similar to that of Auerbach and Kotlikoff (1987), except for indivisible labor supply. To keep the model simple only the stationary state of the model is analyzed, which means that all time indices are removed in the formulae below.

3.1 The consumer's problem

The closed economy is populated with overlapping generations of consumers. Consumers live for 55 periods, and face no lifetime uncertainty. All individuals of a specific generation are identical - this means that we can describe the aggregate behavior for a generation by the behavior of a single member. The representative consumer has a CES life-time utility function

$$U = \frac{1}{1 - \frac{1}{\gamma}} \sum_{i=1}^{55} (1 + \theta)^{-(i-1)} u_i^{(1-1/\gamma)} \quad (1)$$

where γ is the household's intertemporal elasticity of substitution, θ is discount rate, and u_i is an annual utility function. The annual utility function over consumption and leisure is defined by the CES index

$$u_i = \left[c_i^{(1-1/\rho)} + \alpha l_i^{(1-1/\rho)} \right]^{1/(1-1/\rho)} \quad (2)$$

where c_i is consumption in period i , l_i is the leisure enjoyed in period i (and hence $1 - l_i$ is labor supply), and where α represents the household's

preferences for leisure relative to consumption, and ρ being the intratemporal elasticity of substitution between leisure and consumption.

The budget constraint for the consumer is just the discounted stream of future income after taxes minus consumption, which can be written as

$$\sum_{i=1}^{55} (1+r)^{-i} [e_i w (1-l_i) (1-\tau) - c_i] = 0 \quad (3)$$

where r is the (constant) interest rate and w is the standardized wage rate, and e_i is the age-dependent productivity profile⁶, and τ is the average tax on labor income. When the labor income tax is progressive it is assumed that the marginal tax rate, $\hat{\tau}$, takes the form:

$$\hat{\tau} = \underline{\tau} + \psi B \quad (4)$$

where $\underline{\tau}$ is the marginal tax applicable at zero income (the "intercept"), B is the taxable amount (the "base"), and ψ is the progressivity parameter (the "slope"). With the income B this yields an average tax, τ , of

$$\tau = \underline{\tau} + \frac{\psi B}{2}$$

Notice that if $\psi = 0$ then we have a proportional tax system.⁷

Finally let \mathbb{L} denote the set of admissible values for the labor supply, \mathbb{L} . In the standard AK model $\mathbb{L} = [0, 1]$. If we want to impose the restriction that individuals only can choose between working full or half time, or not to work at all (and we let \varkappa represent full-time work), then we use $\mathbb{L} = \{0, \frac{\varkappa}{2}, \varkappa\}$. This is a flexible formulation that encompasses the standard AK model as a special case. Suppose we want to allow individuals who work full time the opportunity to work overtime - and want to model this as a continuous decision: in this case we use $\mathbb{L} = \{0, \frac{\varkappa}{2}\} \cup [\varkappa, 1]$.

The solution to the consumers problem (equation (1) subject to the budget constraint (3) and possibly the divisibility constraint $(1-l_i) \in \mathbb{L}$) is denoted

⁶This profile is hump-shaped over the life-cycle. See section 4.

⁷Notice that when $\psi > 0$ the tax system has a constantly increasing marginal tax. Clearly this represents a simplification of real-life tax systems, where the marginal tax usually is constant within an income bracket. However, the choice of tax system in this paper is not based on realism: the formulation above is convenient because of its simplicity, since it only contains two parameters. Given the other simplifications in this stylized model, this assumption does not appear too critical.

the consumption level c_j^* , the choice of leisure l_j^* , and the optimal end-of-period asset-holdings a_j^* .⁸

The size of each of the overlapping generations has a total measure of unity. Since there is no mortality in the set-up used here, this means that the total (normalized) number of agents at any point in time is 55. To calculate the total labor supply, aggregate consumption and aggregate savings we simply sum over all agents:

$$\text{Aggregate labor supply} : L = \sum_{i=1}^{55} e_i (1 - l_i^*) \quad (5)$$

$$\text{Aggregate assets} : A = \sum_{i=1}^{55} a_i^* \quad (6)$$

$$\text{Aggregate consumption} : C = \sum_{i=1}^{55} c_i^* \quad (7)$$

3.2 The rest of the economy

The production side is identical to Auerbach and Kotlikoff (1987). There is a single good, that is produced using capital and labor subject to a constant-returns-to-scale technology. Labor across ages differs in efficiency, and we calculate the total effective labor supply by equation (5). Production takes place using the CES production function:

$$Y(K, L) = \Lambda \left[\epsilon K^{(1-1/\sigma)} + (1 - \epsilon) L^{(1-1/\sigma)} \right]^{1/(1-1/\sigma)} \quad (8)$$

where K and L are capital and labor in the period, Y is output, Λ is a scaling constant, ϵ is a capital-intensity parameter and σ is the elasticity of substitution between K and L .

Since we assume no adjustment costs in K or L , we have the standard result that the gross wages must equal the marginal revenue product of labor (both measured in efficiency units):

$$w = (1 - \epsilon) \Lambda \left[\epsilon K^{(1-1/\sigma)} + (1 - \epsilon) L^{(1-1/\sigma)} \right]^{1/(1-1/\sigma)} L^{-1/\sigma} \quad (9)$$

⁸Notice that we have not defined the end-of-period asset-holdings here: it is implicitly defined in the budget constraint (equation 3) and is given by the equation of motion:

$$a_j = (1 + r) a_{j-1} + e_j w_j (1 - l_j) (1 - \tau) - c_j$$

and the interest rate (in the closed economy) equals the marginal revenue product of capital:

$$r = \epsilon \Lambda \left[\epsilon K^{(1-1/\sigma)} + (1 - \epsilon) L^{(1-1/\sigma)} \right]^{1/(1-1/\sigma)} K^{-1/\sigma} \quad (10)$$

Notice that the output price is numéraire ($p = 1$) and there is no depreciation.

The government sector is kept very simple. Government revenue is raised by taxation of labor income, and the tax revenue from the labor income tax is:

$$G = \sum_{i=1}^{55} e_i w (1 - l_i) \tau$$

where i is the index over generations, and τ is the average tax on labor income.

As in the Auerbach and Kotlikoff (1987) model, government revenue is consumed and not recycled - and this will also be assumed in the base case simulations here⁹. In other words we do not use the assumption that is popular in the public finance literature that the revenue is redistributed lump-sum back to the consumers.¹⁰

4 Calibration

This section describes how the model is calibrated, and how the steady-state is calculated. To get baseline results that are close to Auerbach and

⁹Notice that utility from government consumption does not enter the utility function directly. However it can be thought of as a component that is additively separable and kept constant (and therefore not modelled explicitly).

¹⁰If the revenue had been recycled, the life cycle model used here prevents the unsatisfactory result in the Bhattarai and Whalley (1997) model, namely that the results are influenced by how the revenue is redistributed. Since their model contains a heterogeneous population (100 agents with different preferences), some agents in the model end up receiving a net transfer from the government sector if revenue is redistributed on a per head basis. But if revenues are returned to those who pay the tax then no interagent redistribution takes place. Depending on how the revenue is redistributed reverse results can be obtained.

In the the life-cycle model used here agents differ only by the stage of their life-cycle. In this case the timing of any net transfer from the government has no impact (as long as the present value of the transfer is the same it does not matter at what age it is paid, since there are no borrowing constraints). The present approach eliminates the results' dependence on the redistribution scheme.

Kotlikoff most of their parameters are chosen. The only major differences to the AK set-up is (i) that only labor income is taxed in the benchmark and (ii) the proportional labor income tax rate used is 30 percent, where as the base case income tax in Auerbach and Kotlikoff is 15 percent.

For the age-dependent productivity we use the same equation for productivity over the life-cycle as Auerbach and Kotlikoff (1987).¹¹ This hump-shaped profile gives an earnings profile that peaks at age 30, (corresponding to an actual age of 50) at wages that are 45 percent higher than at age 1 (corresponding to 21 years). For the household's intertemporal elasticity of substitution, γ , we use $\gamma = 0.25$, and the one-period discount factor, $\beta = \frac{1}{1.015} = 0.98522$. For the taste parameter reflecting the joy of leisure, α , we use Auerbach and Kotlikoff's value of $\alpha = 1.5$, and the elasticity of substitution between leisure and consumption, ρ , is set to 0.8.

In the continuous model (with proportional taxation) the labor supply varies over the life-cycle: when entering the labor market the labor supply is 46.8/100, after some years it peaks at 47.1/100 and starts decreasing towards 0 (more details below). As Auerbach and Kotlikoff (1987) we choose to interpret the endowment of hours as 100 hours per week.

The important question is what discrete labor supply possibilities should be used. As mentioned previously we use 3 discrete options: not employed (0/100), part-time (20/100) and full time (40/100) which corresponds to working 0, 20 and 40 hours a week. In the base scenario we also allow individuals working full time to work up to 10 percent overtime. Formulated in terms of the set \mathbb{L} this means that labor supply must be chosen from the set $\mathbb{L} = \{0, \frac{20}{100}\} \cup [\frac{40}{100}, \frac{44}{100}]$. Obviously this choice of divisibility influences the results, and is therefore subject to sensitivity analysis in section 7.

Since the production side is identical to Auerbach and Kotlikoff (except for the absence of installation costs), we use the same parameters as them: the elasticity of substitution: $\sigma = 1.0$ (Cobb-Douglas), the capital intensity parameter: $\epsilon = 0.25$, and the production function constant: $\Lambda = 0.8927$.

The tax on labor income is initially the only tax. In all simulations we set the revenue requirement to the level that a 30 percent labor income tax would yield under proportional taxation (this particular normalization is discussed below).

¹¹These data in turn originate from a cross-sectional regression study by Welch (1979). This profile represents a simplification, and ignores more recent evidence (e.g. Levy and Murnane (1992) or Katz and Murphy (1992)), but is nevertheless a useful point of departure given the other abstractions in the model.

5 Solving the Model in Stationary State

Solving the model consists of two parts: doing the dynamic programming necessary to solve the consumer's problem, and performing iterations to compute the general equilibrium. Notice that the simulations only consider stationary state.

Consider the consumer's problem in a simple case with no overtime: in this case he can in each period choose between full employment, part-time or being unemployed, which is a total of 3 states. With 55 periods this gives $3^{55} = 1.744E26$ potential paths that need to be examined - a number that is beyond computation even with a supercomputer. Fortunately the problem can be simplified using dynamic programming as described in Stokey and Lucas (1989) and Bertsekas (1995) - the methodology is also described in Petersen (2001b). The solution procedure can be summarized as follows:

1. First a guess is made on the aggregate values for K and L . The associated real wage and interest rate is computed using equations (9) and (10). These guesses for w and r are used when solving the dynamic program.
2. Solving the consumer's problem we start off by discretizing the state- and control-variables. In the base case (with indivisibilities) assets which is both a control and a state variable are discretized on a 1001-point mesh between 0 and 5 (where 5 represents an upper bound that is never chosen by any agent). Labor supply consists of the 3 "indivisible values" 0, 0.2 and 0.4 and the continuous overtime interval $[0.40, 0.44]$ is discretized in 201 points.
3. Aggregate K and L are computed based on the solution to the consumer's dynamic program using equations (5) and (6). A convex combination of these and the old values are used to calculate a new guesses for K and L . If these new guesses are within some tolerance from the old guesses then convergence is achieved and we have a solution. This step is similar to the iterative technique used in Auerbach and Kotlikoff (1987).
4. If convergence is not achieved then the old solution for the dynamic program is used to narrow the interval for the grid (on which the state- and control variables live). Likewise is the mesh enlarged in case any agent choose assets too close to the boundary for the mesh.

In this way the discrete grid for assets is successively refined in each iteration, and when convergence is achieved the grid-points in the mesh are 0.0005 units apart which corresponds to 0.02 percent of the average asset holdings. For the labor supply the grid-points are 0.00005 units apart, corresponding to 0.015 percent of the average labor supply. Refining these grids further does not alter the results significantly but influences the computation time dramatically ¹².

6 Results

This section illustrates the consequences of indivisibility by comparing the effect of switching from progressive to proportional taxation of labor income (under an equal yield requirement) in a model with continuous labor supply with a model with discrete labor supply.

Before we start comparing the effect of economic policy in models with continuous and discrete labor supply it is important to realize that the benchmark equilibria we are comparing are different. Table 1 below summarizes some important characteristics of the two models we are comparing:

	Continuous	Discrete	%-Difference
National income	24.622	24.118	-2.048
Wealth	91.056	87.661	-3.728
Labor (hours)	14.298	14.038	-1.819
Labor (eff)	18.525	18.250	-1.482
Revenue	5.540	5.427	-2.043
Utility *)	100.000	99.172	-0.828

*) Utility is normalized to 100 in the continuous case

Table 1. Equilibria under continuous and discrete labor supply.

Not surprisingly utility as well as output are lower in the presence of indivisibilities. Labor supply is lower - both when measured in hours and in efficiency units. Since the size of the national income is different in the two models, the benchmark revenue requirement is not a fixed nominal value (relative to the numéraire) but is measured as a ratio to the national in-

¹²The practical computations are carried out in Microsoft Visual C++ 5.0. Please visit the author's website to download the source code for this paper at <http://www.econ.ku.dk/twp/indivisible>

come¹³.

Under the progressive system we need to specify two parameters: the level and the progression, corresponding to τ and ψ in equation (4). Since the function has two parameters we must determine one exogenously, and let the other be determined by the model (using the equal yield requirement). Here we chose to specify the level of progression exogenously (in the benchmark $\psi = 0.5$ is used) and determine the "intercept" (the marginal tax at zero income) endogenously.¹⁴

6.1 Aggregate effects

The results in the continuous model are similar to the findings in chapter 8 of Auerbach and Kotlikoff (1987); the main differences can be attributed to the different set-up (labor income taxation is the only tax and the size of the government sector is larger). The list below summarizes the main findings:

- Progressive taxation induces intertemporal speculation in labor supply. With progressive taxation the marginal taxes are higher in the highly productive years in the life-cycle, - and to avoid these high marginal taxes agents choose to work less in the middle ages, and more when old.
- Removing progressivity causes an increase in welfare.
- There is an increase in labor supply, capital stock and production.

In the present study similar qualitative effects are found in both models, and the table below shows the percentage change in various variables from the policy experiment:

¹³More precisely we have normalized the proportional labor income tax to 30 percent, which implies that the size of the government sector is between 22 and 23 percent of national income in the various models (remember that there is no capital income taxation).

¹⁴Auerbach and Kotlikoff (1987) chose to specify the the marginal tax at zero income as $\frac{2}{3}$ of the tax rate in the proportional case, and let the level of progression be determined endogenously by an equal yield requirement.

	Continuous	Discrete
National income	4.026	1.793
Wealth	5.401	1.558
Labor (hours)	3.173	1.494
Labor (eff)	3.571	1.872
Utility	3.312	1.362

Note: Numbers show percentage increase in variable when switching from progressive to proportional labor income taxation

Table 2. Effects of switching from progressive to proportional labor income taxation.

However, notice the significant difference between the two models: all variables react much more to the policy change in the continuous model. First of all the increase in welfare associated with the switch from progressive to proportional labor income taxation is 143 percent larger with continuous labor supply: under continuous labor supply the increase in utility of a newborn is 3.31 percent whereas it only increases 1.36 percent when labor supply is discrete. Secondly the positive effect on capital accumulation is 246 percent larger when labor supply is continuous: with continuous labor supply capital accumulation increases 5.4 percent whereas it only increases 1.6 percent when labor supply is discrete. The increase in output doubles with continuous labor supply. The difference between these two models is very remarkable, and shows that indeed divisibility matters - in fact so much that it should give rise to concern to anyone using these kind of models for policy evaluation.

Table 3 below gives some idea of how the marginal and average taxes are affected by the change of tax system:

	Average	Marginal
5	31.30%	37.64%
25	31.78%	38.61%
45	26.92%	28.90%

Table 3. Average and marginal tax rates under progressive taxation in the continuous case for the agents that are 5, 25 and 45 (model) years.

These numbers should be compared to the proportional case where the tax is 30 percent. Notice the significant difference in the marginal tax: for

consumers with high income the marginal tax switches from 38.61 percent to 30 percent when the tax regime is switched to proportional taxation.

6.2 Life-cycle behavior

In both cases the move from progressive to proportional taxation induces the agents to work more when highly productive and less when old. Figure 5 below shows the labor supply over the life cycle in the two models - the continuous case is the figure to the left and the discrete case to the right:

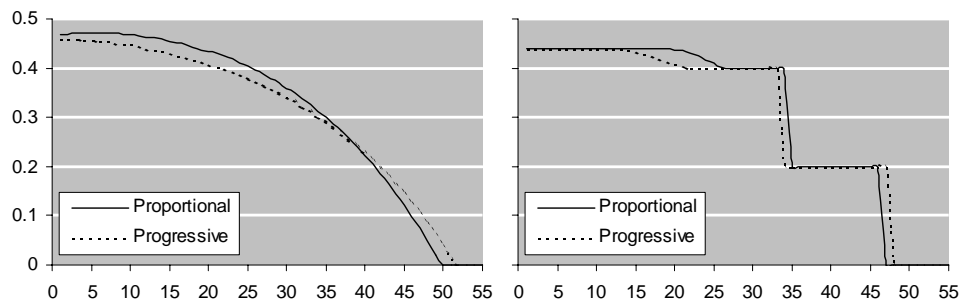


Figure 5. Labor supply.

In the discrete case we observe that under progressive taxation individuals start working full time plus overtime for the first 11 years (corresponding to the age of 31)¹⁵, and then gradually reduce overtime (remember that overtime is a continuous choice variable). After 20 years on the labor market (corresponding to the age of 40) agents choose to stop working overtime and only work full time. In the following years the agents work full time - they would actually have liked to work less than full time, but the indivisibility constraint prevents them from adjusting their labor downwards - and it is still not optimal to switch to part-time. After 32 years in the labor market (corresponding to the age of 52) agents switch to working part-time, and they keep working part-time until their 46th year in the labor market (corresponding to the age of 66) after which they retire.

In the continuous model we observe that progressive taxation induces the expected intertemporal speculation in labor supply, and that individuals

¹⁵As Auerbach and Kotlikoff (1987) we assume that entry in the labor market takes place at age 20. Therefore we can think of an individual who has participated in the labor market for 11 years as being 31 years old.

choose to work less in the middle ages and more when old. Removing progression causes individuals to work more when young (when highly productive) and less when old (and less productive). But with indivisibilities in place agents cannot continuously adjust their labor supply, and are constrained from working a couple of hours more a week when young (and a couple of hours less when old). Instead they choose to work overtime for 7 more years, as well as postpone their move to part-time with one year, in return for retiring one year earlier.

Clearly the choice allowing maximum 10 percent overtime influences the results - under proportional taxation the consumers would like to work more when highly productive but are prevented from doing so by this constraint. In other words the shadow price on this constraint is higher under proportional taxation. To investigate the importance further the sensitivity analysis contains experiments with no maximum-overtime constraint, as well as with other choices for full and part-time.

Since utility is derived from both consumption and leisure, and these to some degree can substitute each other, we should not expect the consumption under indivisible labor supply to smooth over the life-cycle since leisure is not. Rather we should, as pointed out by Ghez and Becker (1975), expect consumption to jump downwards whenever leisure jumps up since these two inputs substitute each other. Figure 6 below shows that this conjecture is correct:

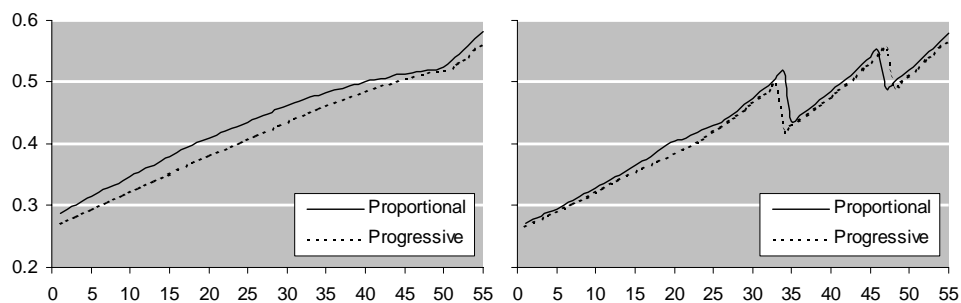


Figure 6. Consumption.

When the consumer switches from full to part-time (around the real age 52-53) and from part-time to retirement (around the real age of 66-67) the consumption curve is kinked: it decreases drastically.

Finally Figure 7 shows the end-of-period asset holdings for the consumer:

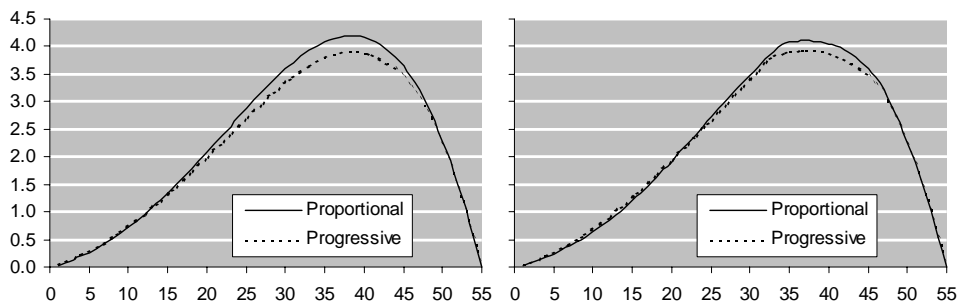


Figure 7. Assets.

These two figures are quite similar when compared to the previous life-cycle figures. It is interesting that the existence of indivisibilities in the labor market does not mean that the asset curve becomes kinked - it is in fact very smooth. This means that indivisibilities does not affect smoothing between periods.

7 Sensitivity Analysis

This section looks at some alternative formulations of the important assumptions in the simulations presented in the previous section. We will look at two important changes a) different choices of discretization when the labor market is discrete, and b) different choices for the elasticities of substitution (γ and ρ).

7.1 Alternative discrete choice sets

We will look closer at the following assumptions: 1) The choice of discretization - alternative values for part and full time work, 2) part-time work: multiple opportunities and not allowed at all and 3) overtime - the alternative situations where there is no upper limit on overtime, and where overtime is not allowed. These alternatives are only a subset of all the conceivable ways labor supply can be made discrete, but present a wide range of alternative formulations that can be used to test the robustness of the results.

In the base scenario above we defined part and full time labor supply as $\frac{20}{100}$ and $\frac{40}{100}$ of the available time endowment. In the continuous model the consumer would have chosen a labor supply larger than $\frac{40}{100}$ for the first 26

years at the labor market, and it is interesting to see what would happen if full time was defined as $\frac{45}{100}$ or $\frac{50}{100}$ instead. Part-time will still be defined as 50 percent of full time. Thus we look at two situations: first we let $\mathbb{L}_{1.1} = \{0, \frac{22.5}{100}\} \cup [\frac{45}{100}, \frac{49.5}{100}]$ and second let $\mathbb{L}_{1.2} = \{0, \frac{25}{100}\} \cup [\frac{50}{100}, \frac{55}{100}]$.

Another potential objection to the model is that it would be more realistic with more than one opportunity to work part-time: therefore we perform alternative simulations with more options for part-time. The more options there are for working part time the closer the model comes to the continuous case. If we think of full time as being 40 hours a week (or $(1-l) = \frac{40}{100}$) then we can imagine that the indivisibility instead lies in how many working days the consumer chooses to work. In other words the choice stands between working 0, 1, 2, 3, 4 or 5 days of 8 hours. In this case $\mathbb{L}_{2.1} = \{0, \frac{8}{100}, \frac{16}{100}, \frac{24}{100}, \frac{32}{100}\} \cup [\frac{40}{100}, \frac{44}{100}]$. Another extreme would be to assume no possibilities for working part-time: In this case we would have $\mathbb{L}_{2.2} = \{0\} \cup [\frac{40}{100}, \frac{44}{100}]$.

Finally it would be interesting to examine how the concept of overtime influences the results. To shed some light on this issue we perform simulations where overtime is limited¹⁶ $\mathbb{L}_{3.1} = \{0, \frac{20}{100}\} \cup [\frac{40}{100}, \frac{41}{100}]$ and where there is no upper limit on overtime, i.e. $\mathbb{L}_{3.2} = \{0, \frac{20}{100}\} \cup [\frac{40}{100}, \frac{100}{100}]$.

7.1.1 Results from the alternative discrete choice sets

Table 4 below shows the effects of switching tax system in situations where the labor supply is constrained to the choice sets discussed above ($\mathbb{L}_{1.1}$ – $\mathbb{L}_{3.2}$):

	Continuous	Discrete	<i>Other discrete choices</i>		<i>Multiple/no part time</i>		<i>No/unlimited overtime</i>	
			Case 1.1	Case 1.2	Case 2.1	Case 2.2	Case 3.1	Case 3.2
National income	4.026	1.793	2.315	3.297	2.285	1.015	0.440	3.298
Wealth	5.401	1.558	3.750	5.268	2.067	-0.215	-0.198	4.936
Labor (hours)	3.173	1.494	1.662	2.205	1.924	1.309	0.391	2.325
Labor (eff)	3.571	1.872	1.842	2.647	2.358	1.428	0.653	2.757
Utility	3.312	1.362	1.933	2.533	1.795	0.158	0.213	2.702

Note: Numbers show percentage increase in variable when switching from progressive to proportional labor income taxation

Table 4. Sensitivity analysis for alternative discrete choice sets.

¹⁶Note that overtime is not eliminated - this is due to the previously mentioned convergence improving properties of overtime. Without overtime it may not be possible to compute an equilibrium.

First notice that the outcomes under discrete labor supply varies quite a lot. Apart from the variability in the overall effects it is interesting to note that on average the welfare gain from switching tax system in these discrete cases is less than 50 percent of the gain in the continuous case.

In case 1.1 and 1.2 where the discrete choices for full and part-time work was set higher than initially, we see that the overall effects come closer to the continuous case - but the welfare effects are still 42 and 24 percent below the continuous case.

In case 2.1 where the part option had 4 choices the results come closer to the continuous case, but utility is still 46 percent lower. In case 2.2 where part-time is not allowed we see a drastic change in the welfare effect: here individuals work full time until they turn 59 whereupon they retire (compared to the base case where they switched to part-time at age 52, and retired completely at age 66). It is remarkable that without the part-time option the welfare gain is as low as 5 percent of the continuous case.

Finally we see from cases 3.1 and 3.2 that the overtime assumption is important: When overtime is very limited (case 3.1) the response to the tax change is small: the increase in output is only 10 percent of the continuous case and the welfare gain is only 6.4 percent of the continuous case. In this case the capital accumulation actually decreases slightly.

In the situation with no upper bound on overtime (case 3.2) we notice that both the increase in output and the welfare gain is approximately 80 percent of the continuous case. Thus the remaining 20 percent is accounted for by the indivisibility imposed by the restriction that agents cannot choose continuously when working less than full time, but in this case only can choose between 20 or 0 hours a week. It is interesting that this large indivisibility (large in an hour sense: 2 "blocks" of 20 hours) account for such a relatively small part of the overall welfare effect, compared to the continuous choice in the interval 0.40 to 0.47 (which is the interval in which agents actually choose their labor supplies) that account for 80 percent. This shows the importance of the overtime assumption - without it the consumers can engage in a lot of intertemporal labor supply speculation, which in turn increases their utility.

7.2 Alternative elasticities of substitution

The traditional type of sensitivity analysis in models of the Auerbach and Kotlikoff type consists of altering the important parameters in the model. Here we will look at alternative specifications of the household's intertemporal elasticity of substitution, γ , that in the previous simulations was 0.25. Clearly this is an important parameter, since it is intertemporal speculation in the labor supply that drives the results in the model. The second elasticity that will be subject to sensitivity analysis is the elasticity of substitution between leisure and consumption, ρ , that so far was equal to 0.8. This parameter is also important, since the degree of substitutability affects labor supply.

Compared to the sensitivity analysis in the previous chapter, we must in these simulations compare the policy change (a switch from progressive to proportional labor income taxation) in a standard continuous model, with a model with indivisibilities in the labor market. However, both models must use the same alternative specification of the two elasticities of substitution.

Similarly to Auerbach and Kotlikoff (1987) we select a low and a high value for each of the elasticities. The elasticity of substitution between leisure and consumption, ρ , was 0.8 in the standard model, and we will look at the consequences of lowering this to 0.3, and of increasing it to 1.5. Secondly we will look at the household's intertemporal elasticity of substitution, γ , that initially was 0.25 - we will look at the consequences for the analysis when the value is decreased to 0.1 and increased to 0.5. Clearly these choices are arbitrary (but similar to Auerbach and Kotlikoff (1987)), and many other combinations could have been chosen. However, these two parameters are both important for the model's properties, and altering these gives a good idea whether the previously obtained results are robust.

Table 5 below shows the percentage increase in a variable when switching from progressive to proportional labor income taxation for the 4 alternative specifications of the elasticities - both for the continuous model and the model with discrete labor supply.

	<i>base case</i>		<i>subst.ela. btw. leisure and consumption</i>				<i>intertemporal elasticity of substitution</i>			
			Low (0.3)		High (1.5)		Low (0.1)		High (0.5)	
	Cont.	Discrete	Cont.	Discrete	Cont.	Discrete	Cont.	Discrete	Cont.	Discrete
National income	4,026	1,793	1,278	-0,097	9,224	4,431	3,088	1,834	3,059	1,464
Wealth	5,401	1,558	2,056	0,256	8,917	2,863	3,061	1,238	1,980	1,414
Labor (hours)	3,173	1,494	0,706	-0,474	8,844	4,244	2,718	1,685	3,229	1,177
Labor (eff)	3,571	1,872	1,019	-0,215	9,327	4,959	3,097	2,034	3,421	1,480
Utility	3,312	1,362	2,071	-0,113	4,848	2,268	7,174	3,182	0,813	0,360

Table 5. Sensitivity analysis with alternative elasticities of substitution.

With the alternative specifications of the intertemporal elasticity of substitution and the elasticity of substitution between consumption and leisure, we observe differences in the absolute gains from the policy reform - in most cases the qualitative effects are the same. With very low substitutability between consumption and leisure ($\rho = 0.3$), the welfare effect changes sign; this is a result of a combination of indivisibility and a low level of substitutability between consumption and leisure which makes the large changes in labor supply over the life cycle detrimental to welfare. The general conclusion from before holds; there are in all cases large differences between the results from a model with and without indivisibilities - the continuous models react significantly more to the policy change. In the presence of indivisibilities the welfare change from the policy change is a lot smaller than in the continuous case.

8 Summary

This paper has examined the importance of the divisibility assumption in a standard macroeconomic model with overlapping generations of consumers. An indivisibility was introduced at the labor market: the consumer had to choose between working full time, part-time or not to work at all - all other options were ruled out. The option of overtime for individuals working full time was introduced, both to add realism and to ensure smooth aggregate behavior. Numerical simulations were carried out in order to compare a model with continuous labor supply to a model with discrete labor supply. Both models were used to analyze a similar differential incidence policy experiment: a move from progressive to proportional taxation of labor income.

The question posed in the introduction was: does indivisibility matter? Unfortunately it turned out that the answer is yes. The results obtained in the two situations were markedly different - the impact of the policy experiment

was much larger in the case where the labor supply was continuous. The main cause for this was that many workers were at a corner solution, i.e. working full time, and therefore did not perform any intertemporal substitution in their labor supply in response to the change in policy. In fact the welfare gains, as well as the increase in output, of the a switch from progressive to proportional taxation was 143 percent larger with continuous labor supply. The sensitivity analysis showed that these results were heavily influenced by how the indivisibilities were specified. When the specification of part-time and full-time was changed or the possibility of part-time or overtime was removed or expanded the policy conclusions did change. But on average the welfare gain from switching tax system was still less than 50 percent of the gain in the continuous case. Even in the case without any limit on overtime (where agents were allowed to work multiple jobs - for instance full and part time), the indivisibility caused a remarkable decrease in the welfare gain when compared to the standard continuous case.

Clearly the present analysis with discrete labor supply represents the other extreme from the standard assumption of continuous labor supply. The ideal model - that will be left to future research - would have a part of the population with labor supply that is divisibility constrained, for instance along the lines used in this paper, and a part of the population who can choose their labor supply continuously. The ratio of individuals with divisibility constrained labor supply could then be varied between 0 and 1. This would present the two extremes: with no indivisibility constrained agents the results would be those of the standard model, and in the situation where everyone faced a discrete labor choice the results would be those presented in this paper. The realistic case is likely to lie somewhere in-between. Another extension would be to include fixed costs in labor supply à la Cogan (1981); this is likely to make observed indivisibilities in the labor supply arise endogenously as a result of optimizing behavior (Grilli and Rogerson, 1987).

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