Dynamic calibration of a CGE-model with a demographic application

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Abstract

The paper introduces a methodology developed to perform dynamic calibration of Statistics Denmark's dynamic computable general equilibrium model, DREAM. To illustrate the technique, the paper briefly discusses an example of an application, namely analyzing the economic consequences of the future development of the Danish population. An important aspect of this is the so-called aging problem. The analysis is carried out under different assumptions concerning financing schemes and announcement of policy responses. The outcomes differ markedly as to whether the policy responses stimulate the incentives to save and to provide for the old age, or not.

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1. Introduction

Analyzing an economy which - even as a rough approximation - is initially out of stationary state (or steady state) by means of a dynamic CGE-model, applying the usual static calibration procedure is of little use. Instead it is necessary to perform dynamic calibration of the model dealing specifically with the fact that the base (calibration) year is a temporary equilibrium on the path of temporary equilibria eventually converging to the final stationary state (or steady state).

The first part of this paper describes the methodology developed to perform dynamic calibration of Statistics Denmark's dynamic computable general equilibrium model, DREAM (Danish Rational Economic Agents Model). In short, the methodology implies using DREAM to generate the entire dynamic path of the endogenous variables subject to the fact that DREAM must reproduce the value of the endogenous variables in the base year exactly.

A non-stationary population is one important source of non-stationarity of the economy. To illustrate the methodology developed, the last part of the paper briefly discusses an example of an application, namely analyzing the economic consequences of the future developments of the Danish population. An important aspect of this is the so-called aging problem consisting of a rising future share of elderly people of the population. Although DREAM is still under development the model is well-suited for this kind of experiments because the population enters the model with a high level of realism, which makes it possible to utilize Statistic Denmark's official population forecast directly. The analysis is carried out under different assumptions concerning financing schemes and announcement of policy responses. The outcomes differ markedly as to whether the policy responses stimulate the incentives to save and to provide for the old age.

The paper is organized as follows: Section 2 briefly sketches the main features of DREAM. Section 3 deals with the dynamic calibration of DREAM while section 4 outlines the population forecast and briefly reports some simulation results.

2. The main features of DREAM

DREAM is an overlapping generations CGE-model of a small open economy. Agents in the model have perfect foresight. Production is divided into a private and a public production sector. Firms in the private sector face convex costs of installation. The government produces a public good, distributes age-dependent transfers, social pension and unemployment benefits. The activities of the government are financed through a range of different tax instruments. In this application, the government obeys its intertemporal budget constraint by running a balanced budget in each period.

The following subsections give an overview of the main features of DREAM, emphasizing the areas which are most important for applications studying the economic impacts of demographic changes as the example of section 4. In the first part of the present section, the demographic structure of the model is discussed. This is followed by a description of the household's decision problem, and a presentation of the central behavioral equations.

The behavior of the firms will be described shortly, and finally the model is concluded with a foreign and a public sector. This is only an overview - see Knudsen *et al.* (1998) for a complete derivation of the behavioral equations.[1]

2.1. The representative household

The population is transformed into generations of representative households that are the decision units of the model. The age of a generation of households is defined as the age of the woman of the representative household. Men and children are assigned to (the women of the) the households using a couple matrix (showing women-men correspondence) and a maternal matrix (showing women-children correspondence) respectively. These matrices are obtained from actual Danish register data.

The construction implies that the household as a decision unit consists of a couple with children, i.e. there are no singles in the model. Children belong to their mothers household until they reach the age of 18, where they form households of their own.

We assume that there is one representative household in each generation. The size of this household is given as

$$\text{household size} = \text{"number of women"} + \text{"number of men"} + \frac{\text{"number of children"}}{2}$$

We measure children in adult-equivalents. For simplicity this is taken to mean that children count as only half an adult to mirror that children do not require the same level of consumption as adults.

The planning horizon of a new household (a woman of age 18) is 60 years.[2] The household (as a decision unit) expects with certainty to survive until the end of the planning horizon. However, the *size* of the representative household is expected to be reduced over time according to the individual probability of death of the members of the household.[3] The fact, that children are born into the household and later leave home at the age of 18 (to form a household of their own), also affects the size of the parents' household.

The adults in the household that are younger than the average retirement age (currently 61 years) are active in the labor market; subsequently they retire and become pensioners.

When the end of the planning horizon is reached (i.e. when the woman turn 78 years) the household is dissolved, and a foreseen bequest is left to the children. Persons surviving the planning horizon of the household are assumed only to receive social security pensions, pay taxes and consume the rest, i.e. they do not save or own assets.

[1] It can be downloaded from our web-pages at http://www.dst.dk.

[2] This corresponds to the expected lifetime for a 18 year old Danish woman, which was 78.41 years in 1995.

[3] This is equivalent to an assumption that a surviving member of the household will always exist and that this member retains undivided possession of the estate, which is standard procedure according to Danish law and tradition.

2.2. Forecasting the household size

Figure 2.1 shows the absolute age distribution of the two genders in 1995.

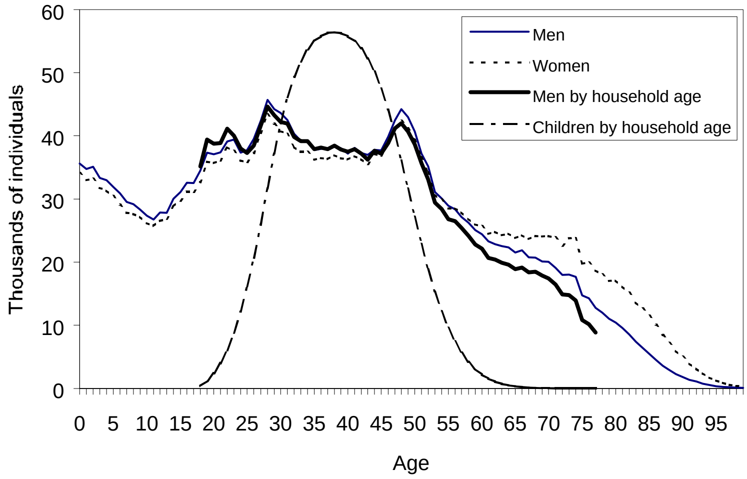


Figure 2.1: Age distribution of the population in 1995

The thin full line is the age distribution of men measured according to their own age, the dotted line is the age distribution of women, and the thick full line is the age distribution of men measured according to the age of their partner (the woman). This curve is constructed using the couple matrix (i.e. the age-specific couple pattern). Finally, the hump-shaped curve is the age distribution of children according to the age of their mother using the maternal matrix.

Figure 2.2 displays the composition of the population in the year 2040 predicted in the official demographic forecast of Statistics Denmark. The figure shows that the adopted assumption of a constant age dependent fertility rate from 2010 and onwards implies that the first part of the age distribution for both men and women becomes smooth. Further this assumption ensures the echo effects in the population (resulting from the huge generations right after World War II) to continue, which manifests themselves in the two humps in the smooth end of the curves. However, the mapping of the age distribution of men into the age distribution of their partners remains slightly uneven due to fluctuations in the age difference of couples through the life cycle. For the simulation exercises of section 4, the DREAM model group has prolonged the official forecast so the population arrives at its stationary state after about 250 years, assuming a continued positive net immigration.

2.3. Behavior of households

Households derive utility from consumption and incur disutility from time spent working. Each household maximizes the total lifetime utility of its members (as opposed to the

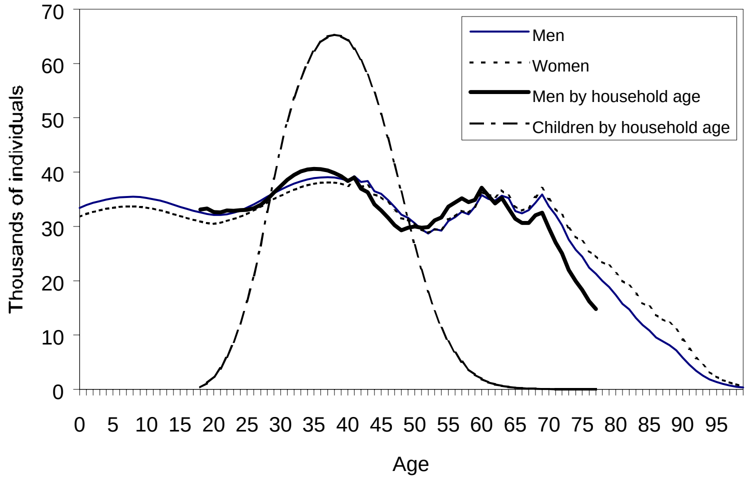


Figure 2.2: Age distribution of the population in 2040

average utility).[4] This leads to the following consumption function

$$Q_{b,t} = \left(\xi_b \frac{1 + r_t \left(1 - t_t^r\right)}{1 + \theta} \frac{\eta_{b-1,t-1}}{P_t} N_{b-1,t-1}^{EF} \right)^S \frac{a_{b-1,t-1} + H_{b-1,t-1}}{\eta_{b-1,t-1}} \ , \ 18 \leq b \leq 78 \qquad (2.1)$$

where $\xi_b = 1$ for $b = 18, ..., 77$, and $\xi_{78} = \xi$. The subscripts b, t indicate the generation that is b years old at time t. $Q_{b,t}$ is the consumption per adult equivalent net of disutility from work. The right hand side of equation (2.1) may be thought of as a standard CES demand relationship, where the specific feature is that aggregates are defined over time instead of over goods. S is the intertemporal elasticity of substitution, θ is the rate of pure time preference and ξ is the weight that generations associate with the bequest. The term $\left(\frac{\eta_{b-1,t-1}}{P_t / \left(1 + \tilde{r}_{b,t}\right)} \right)^S$ gives the slope of the standard CES demand curve. The fraction is the price index of future consumer prices divided by the consumer price in the previous period. P_t is the consumer price index at time t, r_t is the rate of interest, and t_t^r is the capital income tax rate. The denominator is therefore discounted back to time $t - 1$. $\eta_{b-1,t-1}$ is a CES price index of all future prices for the rest of the household's time horizon discounted back to time $t - 1$. Finally, $\frac{a_{b-1,t-1} + H_{b-1,t-1}}{\eta_{b-1,t-1}}$ defines the position of the CES demand curve, and is defined as the total (non-human and human) wealth of the representative household measured in units of the index of future prices. $a_{b-1,t-1}$ is the stock of non-human wealth, $H_{b-1,t-1}$ is the stock of human wealth net of disutility of work, i.e. the discounted value of future non-interest income minus the discounted value of future periods disutility of work.

The result in (2.1) states that individual consumption depends on total individual wealth

[4] Lifetime utility is defined as the sum of annual total household utilities in the remaining lifetime plus an additional term reflecting the utility from leaving a bequest - all discounted to the time of planning. The annual total household utility equals the instantaneous utility per adult-equivalent, times the number of adult-equivalents in the household at the given point in time.

(the sum of financial wealth and human capital) divided by the CES price index. The effect on consumption of high future prices can be seen through a high level of this price index. This reduces the total wealth evaluated at future prices (for a given value of the stock of human wealth, $H_{b-1,t-1}$), and thereby consumption deteriorates. On the other hand, an increase in future prices makes consumption today relatively cheaper, and hereby increases the propensity to consume today. If the intertemporal elasticity of substitution, S, is greater than 1 the latter effect will dominate, whereas a low value of S (< 1) will mean that an increase in future prices will be associated with a fall in today's consumption - then the income effect is dominating.

The fact that current consumption depends on the sum of financial and human wealth, is caused by the joint assumption of perfect foresight and perfect capital markets. This enables the consumer to transform any stream of income into another stream of income with the same discounted value. In other words we assume that the consumer does not face constraints at the capital market, as long as he does not violate his consolidated budget constraint. Therefore future income may readily be transformed into current consumption.

In each period, total consumption is distributed at the different consumption goods assuming a nested instantaneous CES utility function giving rise to standard static CES demand functions.

Current income of households arises from 8 main sources. The first is wages received for the hours worked, and the second is unemployment benefits for the hours members of the household spend as unemployed. By the construction of the model, work-sharing prevails such that everyone in the labor force is equally underemployed. The third source of income is age-dependent income transfers from the public sector, such as child-care transfers, education benefits, and sickness benefits. The fourth source of income is pension, which is given to all persons who are 61 years of age or older. The pension is identical for all individuals. The fifth source of income is a lump-sum transfer that the public sector might make in order to make the budget balance every period. The sixth type of income is the proceeds to the households from holding bonds and shares (interest income). The seventh source of net income is transfers from abroad. The final type of income arises from inheritance left by the parent household.

The demographic assumptions described in the preceding sections has several direct impacts on household savings. First, the model features life-cycle motives for saving as adult members of the household retire when they reach the age of 61 years. Social pensions are lower than wage income, therefore consumption smoothing requires accumulation of assets in the younger part of life. Second, as (2.1) reveals, the size of the household measured in adult equivalents, $N_{b,t}^{EF}$, directly affects consumption. Therefore the number of children also affects savings. Children expand household consumption lowering its savings (ceteris paribus). It is foreseen that children leave home and therefore spending may be higher in the periods before this event occurs. Analogously, prior to the period in which the maximum number of children is reached, assets are accumulated (ceteris paribus). Third households leaves a foreseen bequest to their (now adult) children, since a bequest has a positive effect on the donor's utility - reflecting a "joy of giving" motive.

2.3.1. The labor supply

The labor supply of the household is the sum of the labor supply of its members. Each adult below 61 years supplies labor. The labor supply is chosen such that the life-time utility of the household is maximized. The present version of the model embodies the simplifying assumption that the instantaneous[5] utility function is additively separable in utility of leisure. This assumption implies that the amount of work is chosen such that, at each moment of time the level of non-interest income net of disutility from work is maximized, since this expands the feasible value of the instantaneous utility index, $Q_{b,t}$. Therefore the labor supply decision is not subject to intertemporal speculation, but rather chosen as a sequence of atemporal optimization problems. Perhaps more importantly, the labor supply is independent of the wealth of the household.

To be able to calibrate the model to the actual level of unemployment in the Danish economy, we assume that persons who belong to the workforce and supply less labor to the market than an institutionally fixed maximum supply, are entitled to (supplementary) unemployment benefits. Thus the labor market equilibrium implies that agents voluntary reduce their individual labor supply such that underemployment and worksharing prevails. The calibrated level of unemployment in the model is thus by assumption entirely voluntary. The assumption of work sharing eliminates intractable aggregation problems by avoiding a distinction between fully employed and fully unemployed workers. So all adults younger than 61 years are active in the labor market.

The non-interest income associated with activity in the labor market is the sum of the salary net of taxes, and the unemployment benefit net of taxes. This amounts to

$$\left(1 - T_t^w\right) W_t \ell_{i,t}^J + \left(1 - T_t^b\right) b_t \left(\bar{\ell} - \ell_{i,t}^J\right) \qquad , J = F, M \tag{2.2}$$

where W_t is the wage rate at time t,[6] b_t is the level of unemployment benefits measured per hour, T_t^w and T_t^b are the tax rates of income from employment and unemployment benefits respectively. $\ell_{i,t}^J$ is the amount of time spend employed for gender J (F=female or M=male) of age i. $\bar{\ell}$ is an institutionally fixed maximum working time, such that e.g. $\left(\bar{\ell} - \ell_{b,t}^F\right)$ is the amount of hours that a woman of b years of age is unemployed (or more correctly underemployed) during period t.

The labor supply problem of the household can be reduced to a static maximization problem of weighing marginal net benefit from work (i.e. the difference between wage and unemployment benefit - both net of taxes) against the marginal disutility from work. The solution is a labor supply function, ℓ_t, that is uniform across gender and generations

$$\ell_t = \left(\frac{\left(1 - T_t^w\right) W_t - \left(1 - T_t^b\right) b_t}{\gamma_1 P_t} \right)^\gamma \tag{2.3}$$

where γ is the labor supply elasticity.

[5] The instantaneous utility function is the utility function for each period in the life time.

[6] Observe that the wage is assumed to be identical across generations and gender, implying that labor productivity is the same for all generations and both genders.

2.4. Behavior of firms

The fundamental behavioral assumption of the corporate sector is that firms strive to maximize the value of the outstanding stock of shares. It is assumed that the firm finances investments by an exogenous combination of debt and retained profits. (This is the so-called new view of dividend taxation). The value of shares is given by the discounted stream of future dividends. By assumption shares of domestic firms are owned by domestic citizens. Thus economic policy that affects future dividends and therefore the value of firms will generate a wealth effect of domestic households on impact, which again will affect the pattern of consumption.

The private sector endures convex costs of installation of capital, which implies that an individual firm wants to adjust its capital stock gradually towards a stationary capital stock, if the firm faces constant prices and demand. Investments are driven by the so-called marginal q-theory of investment. No turnover costs are present in the case of labor (and materials).

Figure 2.3 illustrates the assumed technology of the producers which applies to both private and governmental producers. The upper part of figure 2.3 outlines the assumed production function which is specified as two-factor CES (sub) production functions nested as indicated at the figure. At the top level materials are combined with value added to produce gross output. Materials are obtained by combining governmentally produced materials and privately produced materials. Finally, privately produced materials can either be of domestic or foreign origin, while no governmentally produced materials are imported.

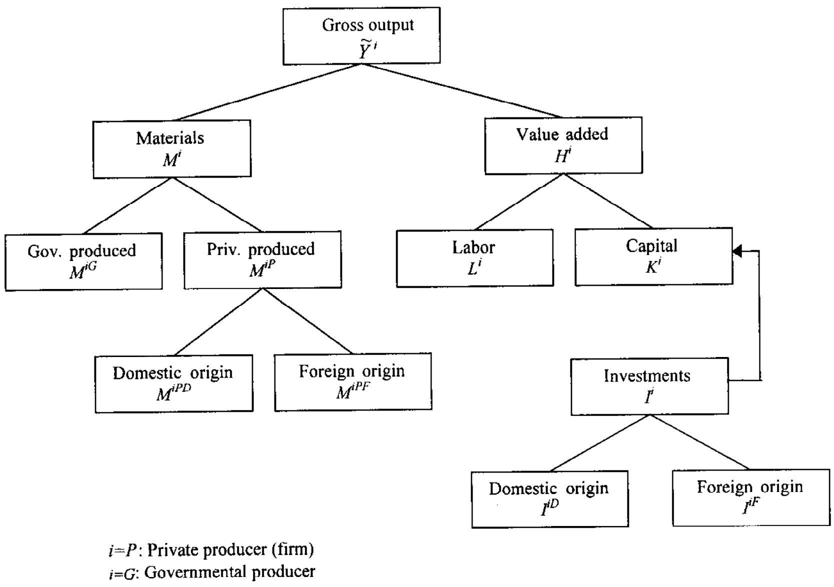


Figure 2.3: The production technology of the producers

Value added is an aggregate of labor and capital. The capital stock is changed by investments (and physical depreciation), and lower part of figure 2.3 indicates that the aggregate index of firms' investments is composed of investment goods from both domestic and foreign origin. There are no investment goods produced by the government sector (either domestic or foreign).

2.5. The foreign sector

The economy is integrated in the world economy through trade and capital flows. Perfect capital mobility, the presence of fixed exchange rates and capital income taxation according to the residence principle imply that domestic and foreign pre-tax interest rates are equalized. We assume that the foreign interest rate is fixed from the point of view of a small open economy.

Foreign goods are used as materials, investment goods, and consumption goods. We apply the standard Armington approach and assume that foreign goods are imperfect substitutes for the domestic production, implying that the terms of trade is endogenous.

For simplicity it is assumed that the export demand curve is isoelastic and that its position is fixed through time.

3. Calibration and Data

Calibration is the process where numerical values are assigned to the parameters of the model. Calibration of single country static CGE-models is fairly standard: they are calibrated to a data set for a specific year (for a specific country). This is typically done by constructing a Social Accounting Matrix, which matches the markets and the budget constraints of the agents in the theoretical model. The well-known problem that data for investments, savings, and interest payments have to be fitted into a theoretical model, where such variables are not defined, has to be dealt with by manipulating either the data set or the behavioral relations of the model. One way of dealing with the problem, is to assume that the static equilibrium is in fact a stationary (or a steady) state of an underlying dynamic model.[7]

To approach the direct use of non-manipulated data the formulation of a dynamic model, in which concepts like interest payments and debt/deposits are well defined, represents a first step. Traditionally dynamic models are calibrated subject to the assumption that the base year is a stationary state or a steady state.[8] In case of a stationary state assumption this typically (still) involves manipulating data such that (1) there are zero net-investments in the base-year, (2) the government budget balances, (3), the current account is in balance, (4) the size and composition of the population constant. The (manipulated) base data set might heroically be interpreted as depicting the stationary state which would have prevailed in the base year, "if the base year were a stationary

[7] Petersen (1997) gives an introduction to the calibration of static CGE models.

[8] For simplicity, we abstract from growth throughout this paper. Therefore we focus on stationary state rather than steady state. However, the dynamic calibration procedure (and DREAM itself) can easily cope with growth caused by exogenously expanding labor productivity.

state".[9]

As pointed out by Pereira and Shoven (1988, p. 423) this method can be questioned: "the model, not the modeler, should dictate the nature of the base-case path". It is clear that the assumption of a base year stationary state (or steady state) remains problematic, although it constitutes an improvement from the static models. However, it is also clear, that the extent to which this should cause concern depends on the issue that one tries to model, as well as how far away from stationary state the economy under consideration is.

One important reason for the Danish economy to be far from a stationary state (steady state) is that the size and composition of the population is changing significantly, cf. section 4. Static calibration of the dynamic model is of little use when analyzing the economic impacts of this.

Therefore we suggest a method for calibrating a dynamic model to a base year that is not a stationary state (steady state), but rather a temporary equilibrium on the path to the final stationary state. A major advantage is that the base dataset is not manipulated to represent a stationary state.

In the calibration 1995 is chosen as the base year. The base year data set consist mainly of National Accounts data, but other data sources are utilized as well. The core of the data set is a standard input output table in current prices.

3.1. Dynamic Calibration

In general, when solving the dynamic model it is assumed that the perfect foreseeing agents optimize just before the end of period zero making their plans for all their future periods. The model is solved at once for all periods from period one to the terminal period where a steady state eventually has been reached. Some forward looking variables (the value of the firms, the wealth of households and the shadow prices of capital) are solved for (the end of) period zero, too.

The principle of the dynamic calibrating procedure is to use the dynamic model to generate the entire dynamic path of the endogenous variables subject to the fact that the model solution for the first period exactly reproduces the values of the endogenous variables in the base year data set. Technically this procedure implies that base year observations of endogenous variable are added to the model as constraints. These additional constraints imply that a corresponding number of parameters is determined endogenously.

To be more specific, we set up a calibration variant of the model, which consists of the standard dynamic model equations enhanced with additional restrictions for period one. They take the form of either additional restrictions that exogenously fix some of the (normally) endogenous variables at their value in the base year set in period one, or of extra equations concerning variables in period one. These additional restrictions determine (normally) endogenous variables in period one which are already determined by existing,

[9] Another method, first used by Auerbach and Kotlikoff (1983, 1987) and known as *qualitative calibration*, is to choose the structural parameters exogenously such that the economy under consideration follows a "reasonable" path into the future.

standard equations. This allows for determining a number of parameters and additional variables (corresponding to the number of additional restrictions) endogenously in the dynamic calibration. From period 2 and onwards the dynamic calibration model is identical to the standard dynamic model. In conclusion, period one is regarded as a temporary equilibrium which is aligned to the actual base data set by additional restrictions valid for period one only.

The fact that the dynamic calibration is performed by solving the model for all periods at once implies that any change in the future value of an exogenous variable (e.g. the population) foreseen at the end of period zero affects the outcome of the calibration. This is a direct consequence of the assumption of perfect foresight. Future shocks that are not anticipated at the end of period zero will of course not affect the calibration.

Although manipulation of the base year data set is not necessary, data for some variables presently has to be imputed by using the period one solution of the model to generate the base year values. This is either because actual data for these variables do not exist (e.g. the stock of human capital) or the actual data are compiled so that they are a poor indicator of the model-relevant concepts (e.g. the value of firms).

The majority of parameters concern virtually atemporal equations and may be determined using single equation calibration similar to the standard technique used in static models. However, the parameters of genuine dynamic equations require a more intricate treatment. We now focus on important parameters where the dynamic calibration differs from the static calibration. They occur in two parts of the model: the production technology of firms and the intertemporal consumption/savings decision of households.

3.1.1. The production technology of the private sector firms

A main obstacle to calibrating the production technology of private sector firms is that installation costs are not directly observable. They are therefore imputed by using the model, i.e. by assuming that installation costs in the base year data set amounts to exactly that value which is compatible with a temporary equilibrium in the base calibration year (period one). Installation costs are assumed to be zero for the public sector.

In the following we introduce some definitions and specification of functions in the model which are necessary to define the calibration procedure:

The gross production function for the private sector giving gross output, $\tilde{Y}_t^P$, as a function of the inputs of the CES index of materials, M_t^P, capital, K_{t-1}^P, and labor, L_t^P, exhibits constant returns to scale, i.e. Euler's theorem yields

$$\begin{aligned}
\tilde{Y}_t^P &= F^P\left(M_t^P, K_{t-1}^P, L_t^P\right) \\
&= \frac{\partial F^P}{\partial M^P}\left(M_t^P, K_{t-1}^P, L_t^P\right) M_t^P + \\
&\quad \frac{\partial F^P}{\partial K^P}\left(M_t^P, K_{t-1}^P, L_t^P\right) K_{t-1}^P + \\
&\quad \frac{\partial F^P}{\partial L^P}\left(M_t^P, K_{t-1}^P, L_t^P\right) L_t^P
\end{aligned}$$

Utilizing the competitive first order conditions that the value marginal product of a production factor must equal its price, we obtain for period one ($t = t_1$)

$$p_{t_1}^P MPK_{t_1}^P K_{t_0}^P = p_{t_1}^P \tilde{Y}_{t_1}^P - (p_{t_1}^{PM} M_{t_1}^P + \left(1 + t_{t_1}^a\right) W_{t_1} L_{t_1}^P) \quad (3.1)$$

$$= \Pi_{NA,t_1}^P + p_{t_1}^P \Phi^P(I_{t_1}^P, K_{t_0}^P) \quad (3.2)$$

where p_t^P is the price index of output, MPK_t^P is the marginal product of capital, p_t^{PM} is the price index of materials, W_t is the wage rate, t_t^a is the payroll tax rate, $\Pi_{NA,t}^P$ is the gross operating surplus as defined in the national accounts and $\Phi^P(I_t^P, K_{t-1}^P)$ is the real installation costs given by

$$\Phi^P(I_t^P, K_{t-1}^P) = \phi^P \left(\frac{|I_t^P|}{K_{t-1}^P}\right)^{\iota} |I_t^P|, \ \iota > 0 \quad (3.3)$$

where I_t^P is private gross investments, and ι and ϕ^P are parameters. (3.2) states that the return to capital covers gross operating surplus as measured in the national accounts and the installation costs.[10]

Table 3.1: Calibration of installation costs and the marginal product of capital for the private sector (numbers in brackets)

1. Additional restrictions [4]
a. Additional exogenizing of normally endogenous variables
Gross investments in period one, I_1^P, [1]
b. Additional equations
Definition of installation costs of period one, (3.3), for $t = t_1$, [1]
Marginal value product of capital at period one, (3.2), [1]
Gross output of period one, (3.1), [1]

2. Parameters and variables to be calibrated [4]
Scale parameter in installation costs function, ϕ^P, [1]
Installation costs at period one, $\Phi^P(I_{t_1}^P, K_{t_0}^P)$, [1]
The marginal product of capital in period one, MPK_{t_1}, [1]
Gross output of period one, $\tilde{Y}_{t_1}^P$, [1].

Table 3.1. sketches the calibration of the production function of the private sector showing 4 parameters and variables to be calibrated by 4 additional restrictions in form of fixing exogenously otherwise endogenous variables in period one and additional calibration equations. The convexity parameter of the installation cost function (3.3) is conveniently fixed to 1. The scale parameter, ϕ^P, is essentially calibrated by the demand that (gross) investments in period one, $I_{t_1}^P$, equal the national account level. The capital stock at the beginning of period one, K_{t_0}, is an ordinary exogenous variable also aligned to the national accounts measure. Then the total installation costs of period one can be calculated

[10] (3.2) implies that net output = gross output net of installation costs, corresponds to the value of production in the national accounts.

by using (3.3)[11] for $t = t_1$. The variables $p_{t_1}^P$, $p_{t_1}^{PM}$, $M_{t_1}^P$, t_{t1}^a, W_{t_1}, $L_{t_1}^P$ and Π_{NA,t_1}^P are considered exogenous to this sub-problem (the prices are fixed by normalizing conventions, and the other items are aligned to National Accounts figures elsewhere in the calibration procedure). Then, from (3.2) we deduct the marginal product of capital at period one, MPK_{t_1}, and finally from (3.1) we infer the gross output of period one, $\tilde{Y}_{t_1}^P$.

Calibration of the CES-parameters of the firms' production function then proceed analogously to the static calibration.

3.1.2. The intertemporal decisions of households

As suggested above, the model solves for the value of the firms at the end of period zero depending on the present value of the future flows of dividends. The value of firms directly affects the wealth of households at the end of period zero, i.e. the total wealth of households at the end of period zero is also endogenously determined. From period one and onwards the distribution of wealth across age groups is endogenously determined. However, the distribution across age groups at the end of period zero is not determined, i.e. it has to be fixed exogenously. Ideally, the exogenous distribution at the end of period zero should be obtained from actual data. However, there is only scarce evidence on this. Therefore we prefer to determine it endogenously in the model from the distribution of total private consumption across age groups at period one. Information on this distribution across age groups is provided in the consumer expenditure survey.[12]

Table 3.2 counts the number of restrictions and the number of parameters to be calibrated in the sub-model of households' intertemporal decisions. The distribution of total private consumption across age groups in period one is assumed to be known from actual data. In addition, we assume to know the share of total household wealth belonging to the eldest age group at the end of period zero, i.e. the bequest share of total household assets at the end of period zero. Finally, the fact that the sum of wealth shares, α_{b,t_0}, of age groups must add up to one provides one restriction determining one of the shares. These 62 restrictions determines 62 parameters: 60 wealth shares and two of the three intertemporal parameters. We chose to fix the time rate of preference a priori, calibrating the intertemporal elasticity of substitution and the bequest preference.

[11] The equation (3.3) does not appear explicitly in the standard dynamic model, although it of course operates implicitly.

[12] It however turns out to be difficult to use the actual data from the consumer expenditure survey to establish the age related pattern of consumption in 1995, mainly because of a crude treatment of housing rents. Therefore it has presently been preferred to fix the consumption pattern of 1995 at the result from the static calibration.

Table 3.2: Joint calibration of households' wealth distribution and intertemporal parameters (numbers in brackets)

1. Additional restrictions [62]
a. Additional exogenizing of normally endogenous var. and new parameters
Consumption per adult equivalent in period 1, C_{b,t_1}, $b = 18, ..., 77$, [60]
Bequest share of all household wealth at the end of period zero, α_{78,t_0}, [1]
b. Equations
Age groups shares of wealth at the end of period zero must sum to one, [1]

2. Parameters to be calibrated [62]
Share of wealth of age group b of all wealth at the end of period zero, α_{b,t_0}, [60]
Intertemporal elasticity of substitution, S, [1]
Preference for bequest, ξ, [1]

Note: The time rate of preference, θ, is fixed a priori.

3.1.3. Result of the base case calibration

There are numerous parameters in the model, and we shall only report the most interesting. Table 3.3 shows some important parameters fixed *a priori*.

To save computer time, the model is solved for 5 year periods. Instead of multiplying all flows by 5 we adopt the convention of keeping all flows unchanged, dividing all stocks by 5 to keep the relation between flows and stocks correct. Correspondingly all rates (interest rates etc.) are multiplied by 5, i.e. the rate of time preference at 0.013 for a 5 year period corresponds to an annual rate of 1/4 percent.

Table 3.3: Important parameters fixed a priori

r	Real rate of interest	0.250
θ	Rate of time preference	0.013
α_{12,t_0}	Bequest share of assets at the end of period zero	0.050
γ	Elasticity of labor supply with respect to real reward	0.100
σ_{iY}	ES between value added and materials (both sectors)	0.250
σ_{iYH}	ES between capital and labor (both sectors)	0.600
σ_{PYMP}	ES between dom. and imp. materials in private sector	1.200
σ_{GYMP}	ES between dom. and imp. materials in gov. sector	1.100
σ_{PI}	ES between dom. and imp. investment goods in private sector	1.500
σ_{PI}	ES between dom. and imp. investment goods in gov. sector	1.300
σ_{CP}	ES between dom. and imp. consumer goods	1.500
ε	Numerical export demand elasticity	1.400
ι	Convexity of adjustment costs of investments	1.000
g	Corporate debt share	0.600

Note: ES is an abbreviation for elasticity of substitution.

Table 3.4 displays the results of the calibration of the most important parameters.

Table 3.4. Important parameters calibrated

S	Intertemporal elasticity of substitution	0.844
ξ	Preference for bequest	0.703
ϕ	Scale parameter of installation costs of investments	1.082

4. An Example of an application: The economic consequences of the future demographic evolution

4.1. The population forecast

Figure 4.1 illustrates aspects of the official population forecast of Statistics Denmark and the technical extension after 2040 by the DREAM modelling group performed such that the stipulated population gradually converges to a stationary state. The x-axis depicts 5 year periods from period one (the base period of 1995-99) to period 50 (2240-44).

Figure 4.1 shows that the population changes induce a series of cyclical shocks to the number of persons of working age (18-60 years old) around a declining trend. During the first three periods the number grows to a peak which is 1.5 percent larger than in period one. Then it drops to a trough in period 11 at 5.8 percent below the 1995 level. During dampened cycles it gradually settles to a level of 4.0 percent below the 1995 level. Although the number of persons of working age in fact continue to move until period 182, it is seen that the amplitude is already rather small around period 30 and almost negligible around period 50. While the number of people of age 18-60 years drops by 4.0 percent during the entire period, the size of the overall population grows by 7.2 percent. Behind this lies the so-called aging problem where a shrinking potential labor force has to provide for still more pensioners.

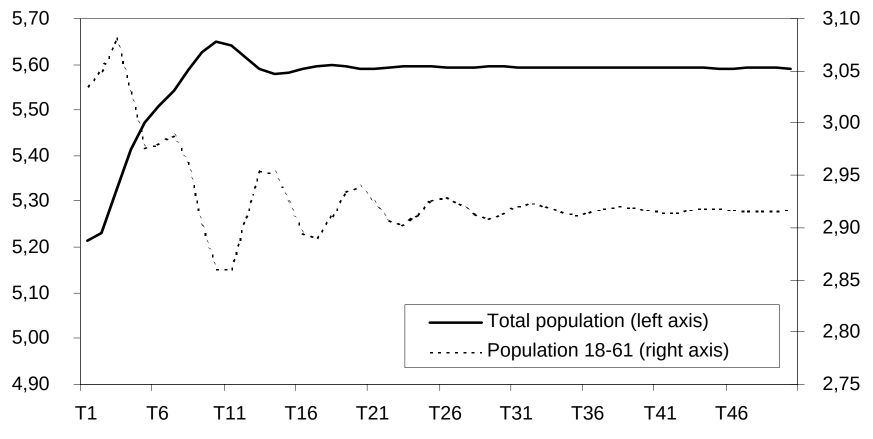


Figure 4.1: The demographic shocks

The resulting main shocks to the economy in the longer run may briefly be summarized as a shrinking potential labor force combined with an increased burden of provision which has to be financed somehow.

4.2. Overview of the simulation exercises

We present two scenarios which differ according to how the increased burden of provision is financed.

The base scenario is defined as a situation where the existing policy rule of indexing social pensions (and other social transfers) to the wage rate is maintained. Expenditures for public consumption and investments are indexed to real GDP. Further it is assumed that the government runs a balanced budget by parallel changes in the tax rates applied to different types of personal non-interest income i.e. tax rates on wage income, on unemployment benefits and on other social transfers.[13] Therefore the increased demographic burden is primarily borne by the part of population in the workforce. These policy measures are assumed to be common knowledge in the economy along with the demographic evolution.

Even though a similar evolution in the demographic structure is expected in most Western economies, it is assumed that the real rate of interest, the position of the export demand curve and the foreign price level are unaffected and remain fixed through out the time horizon.

In the alternative scenario the government introduces an unforeseen shift in the policy package. As of period 2 tax rates remain fixed at the initial level. Expenditures for social pensions are scaled such that the government runs a balanced budget in every period. Therefore the increased demographic burden is financed primarily by the retired generations.

4.3. Base scenario: Tax financing

The base scenario is simulated as a dynamic calibration scenario. As should be clear from the preceding section on calibration, the simulations are performed by initiating the economy at the end of period zero. Some few forward looking variables jump on impact, but the first year giving a solution for all variables is period one (1995-1999). The calibration assures that the data set for 1995 is exactly reproduced in period 1. From period two (2000-2004) and onwards the model simulates without extra constraints on endogenous variables for 200 5-year periods eventually converging to a stationary state.

Table 4.1 briefly overviews the results for the base case. Figure 4.2-4.5 depicts some key results. The bold lines on figure 4.2-4.4 show the results for the base case while the

[13] In period one the public budget balance is calibrated to the actual 1995 figure by means of balancing lump sum transfers to households compensating for not all revenue and expenditure items of the real world public budget being accurately determined in the model. In the subsequent periods the sum of lump sum transfers to households is fixed at the period one level, implying that the lump sum transfers per adult varies with the number of adults. In period one all tax rates assume their calibrated levels.

thin lines show the results for the alternative to be discussed below. All time series are measured per capita. The graphs depicts the first 50 five year periods (i.e. 250 years), while the new stationary state solutions are marked on the right hand ordinate. In most cases the figures are shown as indices with period one (1995-99) equal to 100.[14] Figure 4.5 depicts the developments of consumption and assets per adult equivalent over the life span to be discussed more fully below.

1000 Dkr. per capita	Calibration year (period 1`	2 periods	5 periods	10 periods	20 periods	Steady state
Private consumption	80	80	80	77	76	76
		(-0.2)	(-0.3)	(-3.8)	(-4.6)	(-5.2)
Real GDP	159	162	156	148	151	150
		(1.6)	(-1.7)	(-7.0)	(-5.3)	(-5.9)
Employment, index	100	100	98	93	93	95
		(-0.1)	(-2.2)	(-6.5)	(-6.5)	(-4.9)
Capital stock	528	540	537	517	525	524
		(2.2)	(1.6)	(-2.2)	(-0.6)	(-0.9)
Value of firms	260	258	241	227	231	230
		(-0.7)	(-7.3)	(-12.8)	(-11.1)	(-11.7)
Household assets	607	609	593	557	557	548
		(0.4)	(-2.3)	(-8.3)	(-8.2)	(-9.7)
Foreign assets	-51	-49	-39	-48	-53	-60
Average tax rate	0.427	0.447	0.463	0.479	0.491	0.486
(Absolut change)		(0.020)	(0.036)	(0.052)	(0.064)	(0.059)
Disposable pension per heac in 1000 Dkr.	68	67	69	70	68	68
		(-2.2)	(1.4)	(2.8)	(-0.2)	(-0.3)
Total pension expenditures net of taxes in bill. Dkr.	68	66	81	95	84	86
		(-2.4)	(19.8)	(40.6)	(24.2)	(26.8)

Note: The numbers in parantheses are the percentage change compared to the calibra

Table 4.1: Base case

A brief look at the figures reveals that the projected fluctuations in the number of workers are to some extent mirrored in the path of most variables. The initial rise in the number of workers implies that many variables initially show opposite movements than later on, when the number of workers declines, even though the burden of provision also increases during the first periods.

In general the movements on the path of temporary equilibria from period one to the final stationary state occur for several reasons:

[14] Exceptions are figure 5.3.c showing the rate of unemployment and figure 5.4.f showing the change in foreign assets as per cent of GDP.

1. The population is not stationary.
2. The capital stock increases during period one as there are positive net investments.
3. The foreign debt increases during period one due to a current account deficit.[15]
4. The public debt increases during period one due to a public budget deficit.
5. The distribution of private consumption across age groups in period one is not a stationary state pattern.

In the following we ignore the fluctuations and focus upon the general reduction of the population of workers relative to the rest of the population under the assumed financing rule.

4.3.1. Supply side effects of a relative reduction in the working population

In the competitive labor market, the labor supply curve moves inwards deriving an increase in the real product wage, cf. figure 4.3.f and a drop in the demand for labor and employment, cf. figure 4.3.b. This further diminishes the marginal product of capital, dampening private investments, cf. figure 4.2.c and the capital stock, cf. figure 4.3.a, although the capital labor ratio responds positively to the increased price of labor compared with capital. Production and income declines along with the inputs, cf. figure 4.4.c, depressing human capital, consumption, savings and wealth, cf. figure 4.4.e.

The domestic demand for the domestic good drops more than supply, cf. the next subsection. The excess production is supplied to the world market, implying an increase of exports and a drop in the domestic output price, cf. figure 4.4.a. However exports does not grow enough to prevent per capita exports from falling along with the expanded population, cf. figure 4.2.e.

4.3.2. Demand effects of a relative reduction in the working population

The decline of the work force relative to the rest of the population implies an increased burden of provision mainly due to aging. The Danish pension system is a mixture of a public tax-financed pay-as-you-go system and funded schemes in form of private pension funds of varying coverage for most groups at the labor market. In the present version of DREAM no distinction is made between households' "forced" savings in the private pension funds and their "voluntary" savings made on their own initiative.[16] Due to the assumptions of perfect foresight and a perfect capital market the households can always compensate for inoptimal "forced" savings by changing their "voluntary" savings. The

[15] The official figures states that the current account was in surplus in 1995, but that the foreign debt increased due to changes in currency valuations etc. In DREAM, this is summed up in a current account deficit.

[16] The minor semi-public funded pension supplement schemes (ATP and LD) are classified as private arrangements in DREAM.

public pay-as-you-go system in DREAM handed out 88 bill. DKK. in public pensions in 1995 (gross of taxes) corresponding to 10.6 percent of GDP at factor costs.[17]

At present the calculations do not deal with the potential inflating impacts on public consumption and investments per capita of the increased relative number of elderly, i.e. it is assumed that if such impacts show up (e.g. in form of more hospitals, rest homes etc.), compensated savings are implemented in other areas, till example because the public consumption and investments related to the workforce decline.

In sum, the major impact on the public budget of the increased burden of provision of the elderly is a marked increase in the amount of public pensions. Correspondingly, table 4.1 shows that the public pension expenditures net of received taxes increase significantly from 68 bill DKK in period 1 to a peak of 95 bill DKK in period 10 (i.e. in 2040-2044). The assumed continuation of the linking of the public pension to the wage rate by full indexation prevents the burden of provision to be eroded in real terms by increased income of the working people. This necessitates tax hikes to balance the public budget.

As can be seen from table 4.1 the tax rate applied to wages increases already in the first periods due to the increased burden of provision. In period 10 it has increased by 5.2 percentage points to 47.9 percent. Figure 4.3.e shows that the disposable wage rate drops because the tax hike more than counteracts the wage rise. This adds to the drop in human capital, savings and consumption.

4.3.3. Individual labor supply effects of a relative reduction in the working population

Although we assume an equal rise in terms of percentage points in the tax rates applied to wages and transfers, disposable wage income declines by more than transfer income in absolute terms, because wage income is larger. This has two implications: First, the labor supply contracts further along with the diminishing reward of working compared with receiving unemployment benefits, cf. the labor supply schedule (2.3). As the elasticity of labor supply with respect to this reward, γ, is only 0.1, this effect is minor. In the long run, it contributes by 1 percentage point to the overall drop in the labor supply and employment of almost 5 percent, cf. figure 4.3.b.

Second, it depresses savings of the younger to smooth consumption over the life cycle in light of the favorable development of disposable public pensions compared to disposable wage income. This diminishes the decline of consumption in the first periods, but the effect is small and in the longer run it is far from sufficient to counterweight the depressing impact on household wealth accumulation of the declining human capital.

[17] The public pension expenditures in DREAM include all kinds of ordinary pensions as well as early retirement pensions and health related early retirement pensions handed out to people at least 61 years old.

4.3.4. Generational effects of a relative reduction in the working population

In sum, the impact on employment, income, consumption and wealth accumulation is depressing. The impacts on generations appears at figure 4.5.a. showing the consumption profile over the life cycle (i.e. panel data) of chosen generations compared with the consumption profile across different age groups at period one. Figure 4.5.b. displays the same for households' asset profile compared with the calibrated asset profile across age groups at the beginning of period one. "Generation 0" enters the economy at the end of period zero, i.e. it is 18-22 years old in period one. "Generation -10" is 68-72 years old in period 1, while "generation 5" is 18-22 years old at period 6 and so forth. It is seen that the consumption profile of households changes markedly with the steepest decline in consumption of the eldest. This mainly reflects movements away from the assumed temporary equilibrium profile of period one which is obviously far away from the new stationary state profile. The steepening of the consumption profile implies that temporarily some generations obtain an increase in consumption per adult equivalent in the youngest years, but in the long run (generation 100) this is limited to a slight increase in consumption of age group 18-22 years. All elder age groups face a loss implying that the generation clearly looses seen over the entire life span.

The fact that the amplitude of the demographic shocks can be more or less recognized in the solution path of most variables reveals sluggishness in the adjustment process in spite of the assumption of perfect foresight. First, the households' utility function is assumed to be additive in consumption and disutility of work implying that wealth effects are absent in the labor supply function, cf. (2.3), so the workers do not seek to compensate (partly) for real wage reductions by expanding their labor supply. Second, due to installations costs the capital stock only adapts gradually. Third, the Armington assumption of imperfect competition in foreign trade implies that domestic prices can vary compared with the fixed foreign price. The anticipated future variation in the domestic consumer prices affects the actual consumption of households, cf. the consumption function (2.1), where actual consumption depends on discounted future consumer prices relative to the actual consumer price.

4.4. Alternative scenario: pension rate financing as an unanticipated shock

In the alternative simulation, it is assumed that the government at the end of period one unexpectedly announces a change of policy regime: The impacts of the population changes on the public budget from period two and onwards will now be neutralized by a reduction in the amount of pension per pensioner so that the public budget is still balanced from period two and onwards. As the policy change is unanticipated, the solution for period one as well as the calibration is unaffected.

The results are reported in table 4.2 as well as figure 4.2-4.4 (thin lines), with the generational profiles of consumption and wealth depicted at figure 4.6.a and 4.6.b respectively.

1000 Dkr. per capita	Calibration year (year 1)	2 periods	5 periods	10 periods	20 periods	Steady state
Private consumption	80	78	79	80	83	84
		(-2.8)	(-0.8)	(-0.2)	(3.6)	(4.9)
Real GDP	159	161	157	150	154	153
		(1.3)	(-1.2)	(-5.6)	(-3.3)	(-3.6)
Employment, index	100	101	99	95	95	97
		(0.5)	(-1.3)	(-4.6)	(-4.6)	(-2.9)
Capital stock	528	541	542	529	540	540
		(2.5)	(2.6)	(0.0)	(2.1)	(2.1)
Value of firms	258	259	244	234	242	243
		(-0.4)	(-5.2)	(-9.2)	(-6.0)	(-5.7)
Household assets	604	614	622	621	657	665
		(1.6)	(2.9)	(2.8)	(8.7)	(10.1)
Foreign assets	-51	-42	-15	3	24	30
Average tax rate	0.427	0.427	0.427	0.427	0.427	0.427
(Absolut change)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Disposable pension per head in 1000 Dkr.	68	51	54	54	65	66
		(-25.5)	(-21.1)	(-21.6)	(-5.1)	(-3.3)
Total pension expenditures net of taxes in bill. Dkr.	68	50	63	73	73	83
		(-25.4)	(-6.8)	(7.2)	(18.0)	(23.0)

Note: The numbers in parantheses are the percentage change compared to the calibration year

Table 4.2: Alternative financing

The main impact of the policy change is that savings of people of working age are now stimulated significantly by the marked decrease of pensions. Younger households now find that they have to save much more to provide for their old age and smooth consumption over their life span. This implies that consumption initially declines faster, cf. figure 4.2.a. However, in the longer run the increased incentive to save has profound impacts on the economy. As the larger savings accumulate in the wealth of households, cf. figure 4.4.e, private consumption surpasses the base case level and even end up at a higher level than in period one, cf. figure 4.2.a. The positive impacts on the public budget even leaves room for increasing the pension rate again compensating a major part of the earlier decline in the very long run. Table 4.2 shows that although disposable pension drops by 21.6 percent after 10 periods, it gradually picks up and has only declined by 5.1 percent after 20 periods. As consumer prices in the long run increase by almost 1 percent, cf. figure 4.4.b, this implies a drop of real disposable pensions per head of around 6 percent. The beneficial impacts on the real reward of working shaves around 1 percentage point off the demographically conditioned drop in the labor supply, so employment ends up by falling only 2.9 percent, cf. figure 4.3.b.

The generational pattern of consumption also changes, cf. figure 4.6.a. In the long run (”generation 100”) all households up to those being 58-62 years old see an increase in consumption per adult equivalent, while all pure pensioner households see a drop in consumption per adult equivalent. Comparing figure 4.6.a and figure 4.5.a. it is seen that the consumption profile in the long run (generation 100) has the same slope, reflecting that the slope of the Keynes-Ramsey rule coincides in the two simulations when consumer prices have settled at a stable level. This also confirms that the changes in the slope of the consumption profile (as opposed to level) compared with period one merely reflects how far the assumed cross-section profile of period one is away from the new stationary state.

4.5. Comparison

In sum, the tax financing of the burden of provision implies that after 20 periods (100 years) real private consumption drops by 4.6 percent, real GDP drops by 5.3 percent, employment drops by 6.5 percent and households' assets drop by 8.2 percent all measured per capita. In the alternative financing scheme, where the pension amount is reduced, private consumption expands by 3.6 percent, real GDP drops by 3.3 percent, employment drops by 4.6 percent and households' assets increase by 8.7 percent after 20 periods. The more favorable outcome for till example private consumption of the last alternative is first materialized in period 6 (i.e. after 30 years) where it shows up for all age groups including the pensioners. The profound difference between the two alternative financing rules stems primarily from the different impact on the incentive to save and to a minor extent from the different stimulus to the labor supply.

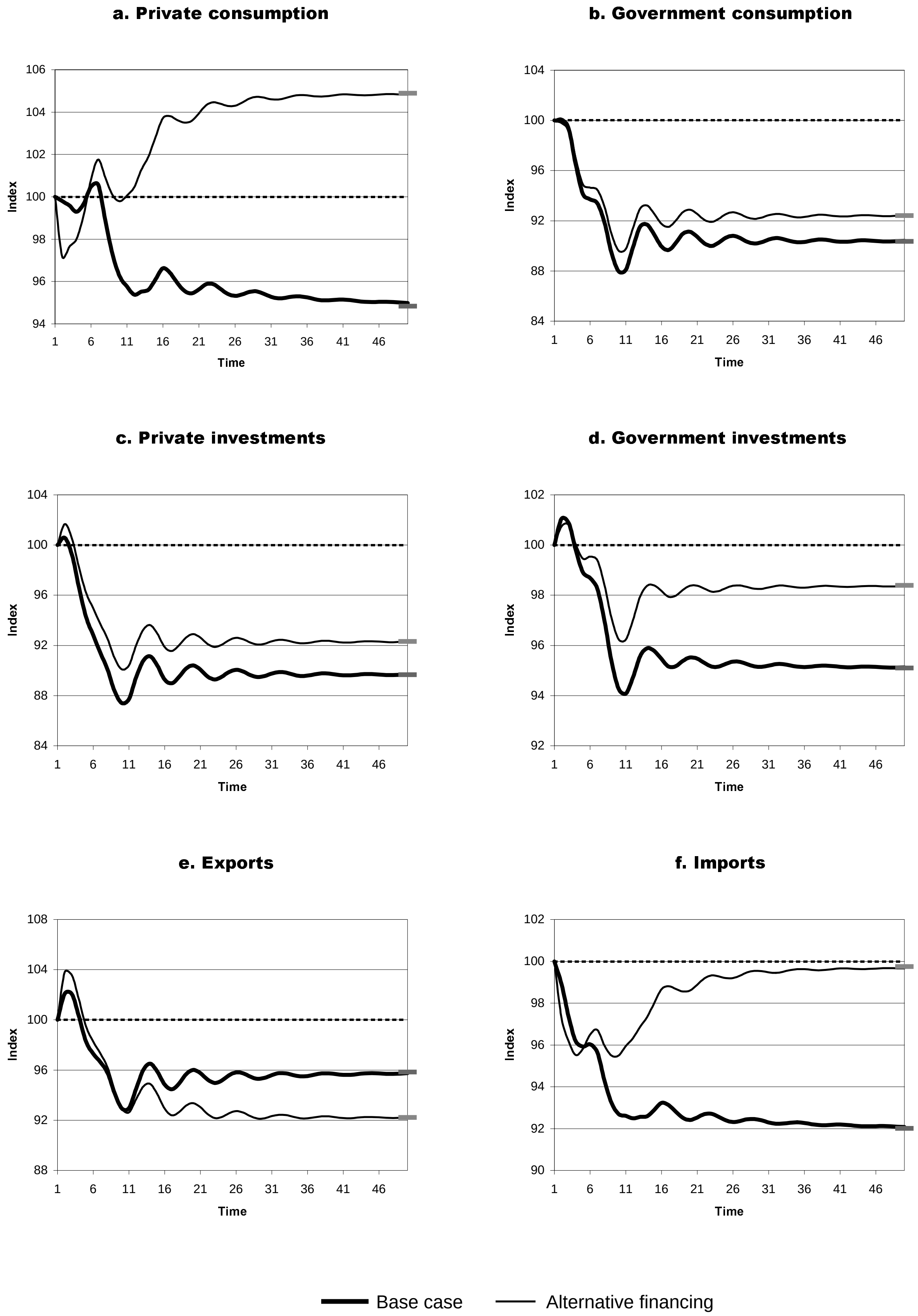


Figure 4.2: Macroeconomic effects

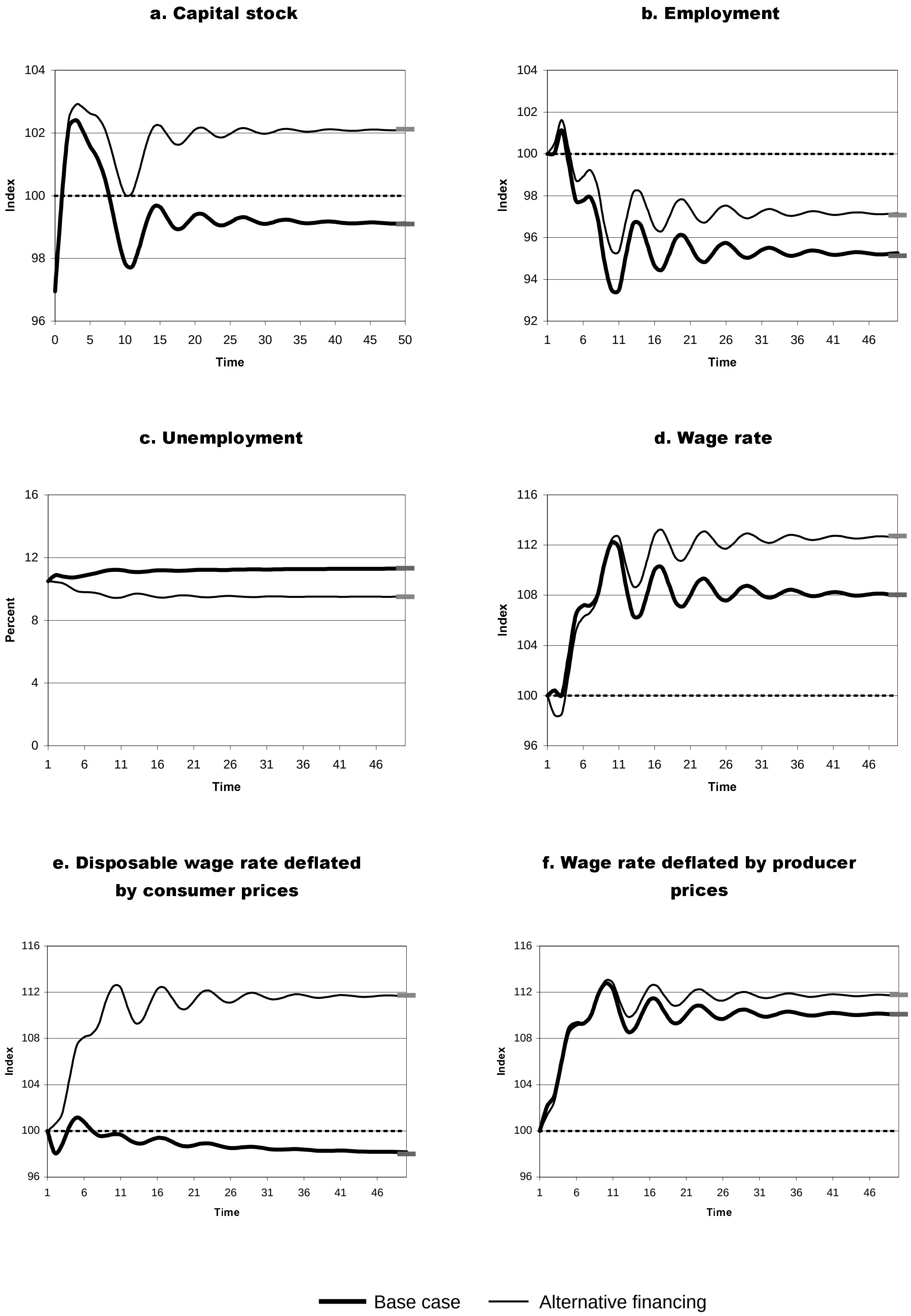


Figure 4.3: Macroeconomic effects

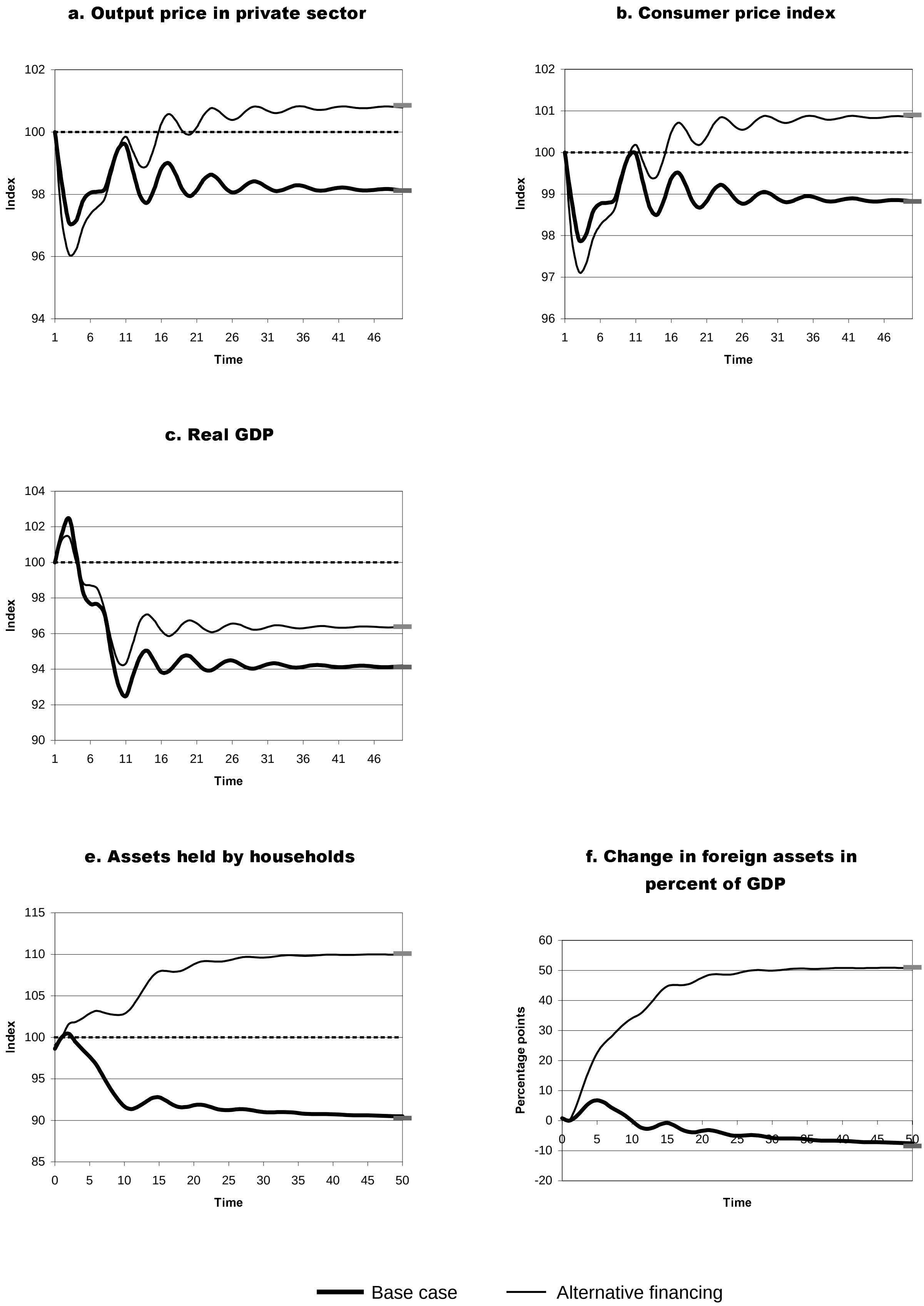
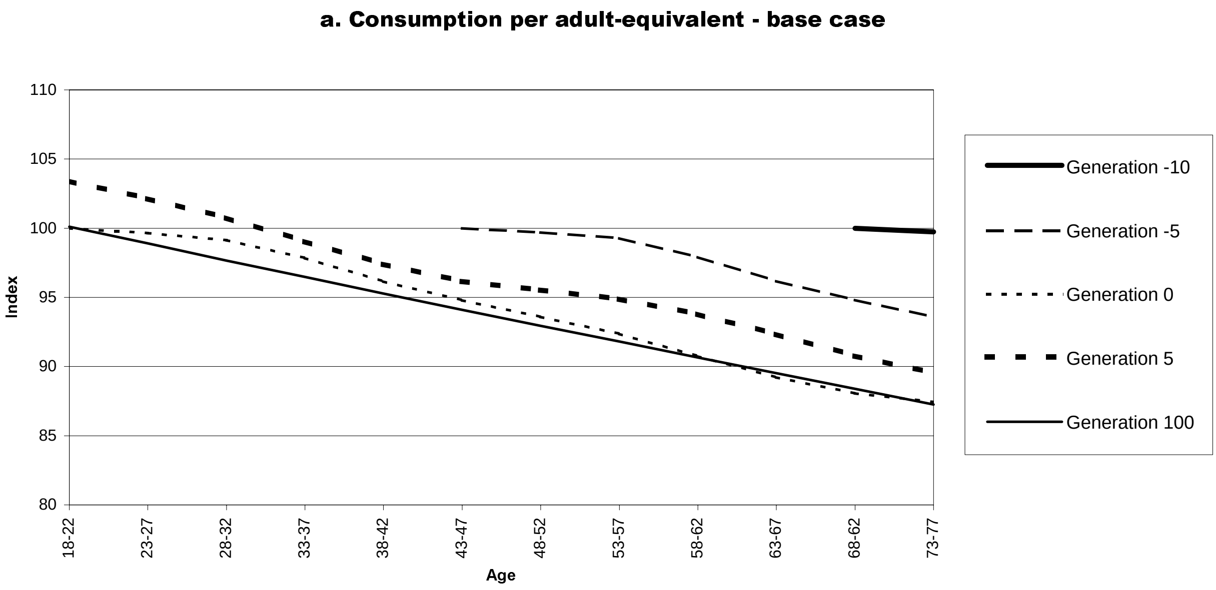


Figure 4.4: Macroeconomic effects



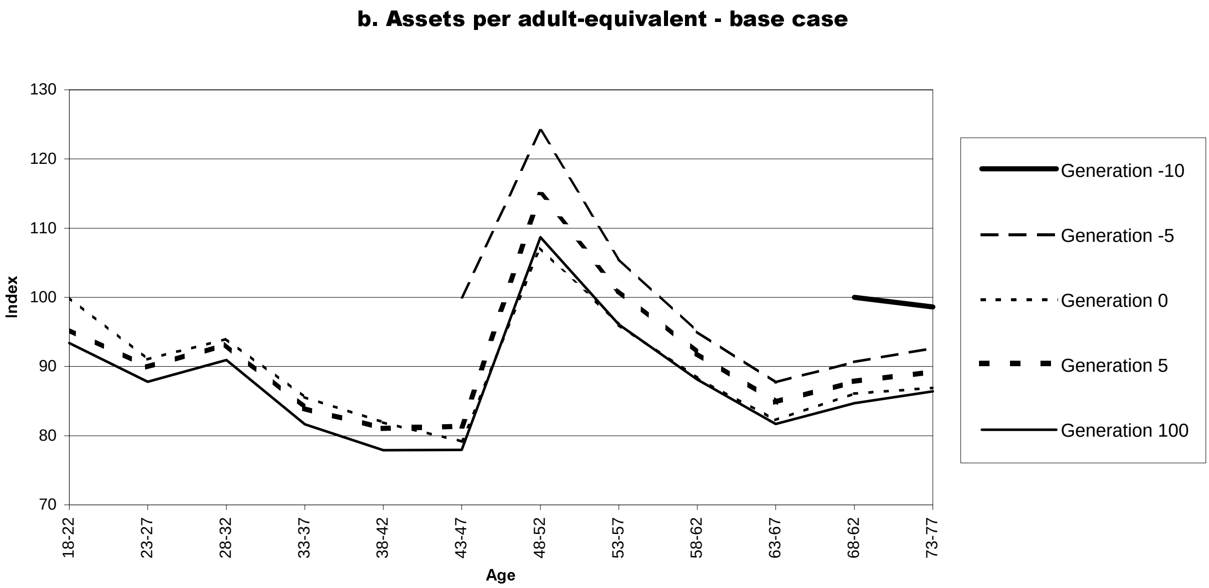


Figure 4.5: Generational effects

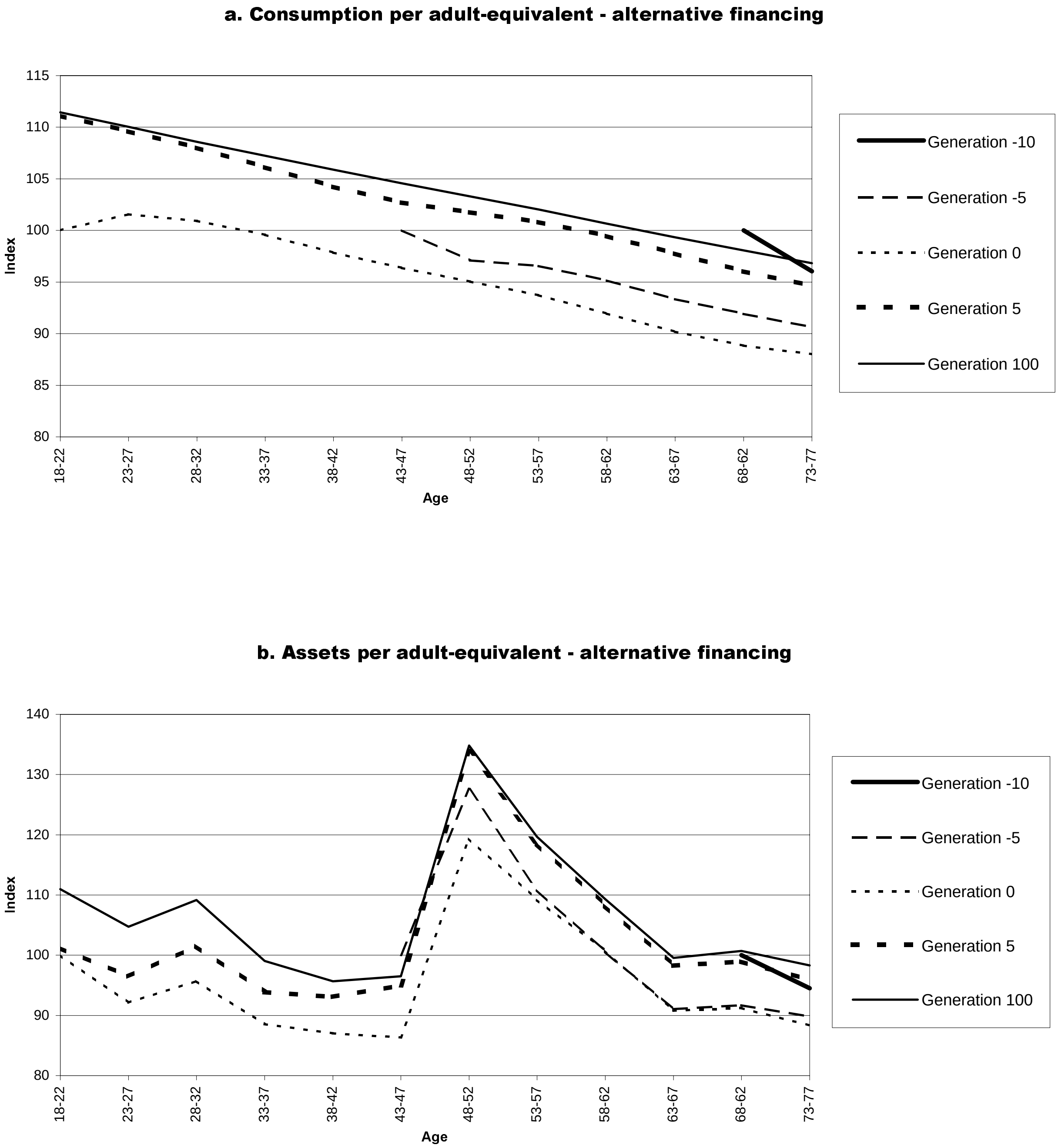


Figure 4.6: Generational effects

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