The Waste Sector in the Green Reform Model: A Tentative Static Partial Equilibrium Model *

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1 Introduction

This paper describes a tentative static version of the waste sector module to the Green Reform model. We expect the main structure described below to carry through to the final version of the module, while the level of detail increases.

The final waste module of the Green Reform model will differ from the present model in at least three important aspects. First, the model presented below is static, and thus, describes the waste sector in a particular year. Capital is fully adjustable within this period. The final Green Reform model will be dynamic, and capital will adjust endogenously over time. Second, the final Green Reform model will be able to project economy-wide waste generation over the coming decades. Thus, the final Green Reform model will feature endogenous waste generation processes. Meanwhile, the present model takes the waste generated by the economy as given. Finally, the final Green Reform model will feature international waste trading, while the present model abstracts from this.

2 Model

2.1 Overview

The economy generates waste from production and consumption. The waste is sorted by households and firms before collection and treatment.

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The present model takes waste generation as given, while waste collection and treatment are explicitly modelled. These activities require labor, capital, and energy input. Waste can be burned, transformed into new raw materials, or end up as landfill. The first activity is not explicitly modelled. Instead, firms handling waste treatment sell waste as a fuel input to the electricity and heating sector.

The chosen waste treatment for each waste fraction depends on economic trade-offs and regulation. Regulation requires that hazardous waste is deposited as landfill or undergoes special treatment. The waste hierarchy dictates that recycling is preferable to energy utilization, thereby placing a constraint on firms in the treatment sector. Finally, the government impose a non-profit constraint on firms within the waste sector. This constraint implies that firms in the sector obtain zero profits after paying the factors of production (including capital).

2.2 Economic sectors, waste fractions, and waste groups

There is a large number of economic sectors generating waste. These sectors are indexed by j. The produced waste mass has been sorted into waste fractions at the source.

The waste fractions are indexed $i = 1, 2, ..., N$. All sectors may in principle produce any of these waste fractions, but in practice, there are sectors that do not produce all waste fractions.

The waste from the sectors is divided into waste groups. The idea is to bundle waste from sectors that produce similar qualities of waste. The waste groups are indexed $g = 1, 2, ..., G$. Here $g = 1$ is the household sector, and $g = 2, .., G$ are production sectors. Note that the introduction of waste groups do not reduce the dimensionality of the model. On the contrary, a waste mass is characterized by its fraction (e.g. plastics) and waste group (e.g. the household sector).

The waste groups approximate the quality of the waste masses given the waste fractions. The waste qualities become important when assessing the recycling potential. The mass loss from recycling plastics from households is, for instance, much larger than the loss from recycling plastic waste from manufacturing industries.

The number of economic sectors within each waste group may differ. Thus, the sectors within a waste group are indexed by $j = 1, 2, ..., J_g$.

2.3 Waste generation and fractions

The waste generated by each sector is sorted into fractions. This sorting effort occurs within the households and firms before the waste is collected for treatment.

When collecting the waste, the main issue is how much waste mass that needs to be collected. The aggregate waste mass of fraction n is given by:

$$\bar{Q}_n = \sum_{g=1}^{G} \sum_{j=1}^{J_g} q_{n,g,j}, \tag{1}$$

where $q_{n,g,j}$ is waste mass of fraction n generated by sector j in waste group g (e.g. plastic waste generated by chemical industries in the waste group for manufacturing industries). In this version of the model, all $q_{n,g,j}$'s are exogenous.

For the recycling potential, the quality of the waste type is crucial. And the quality of a given waste fraction depends heavily on which sector the waste comes from. Dividing the waste into waste groups ensures that (most) of this heterogeneity is captured for each fraction. Thus, the relevant metric when it comes to the recycling potential is the waste fraction mass per waste group:

$$\hat{Q}_{n,g} = \sum_{j=1}^{J_g} q_{n,g,j}. \tag{2}$$

This metric becomes relevant in section 2.6.

2.4 The waste sector and production structure

The waste sector is bifurcated into two sectors: the waste collection sector and the treatment sector. The *waste collection sector* is obligated to collect all the waste generated by the economy. There is a single representative firm in this sector that handles all the different waste fractions. All the collected waste is delivered to the treatment sector.

The *treatment sector* is obligated to handle all the waste delivered to them by the waste collection sector. There is a single representative firm in this sector that handles all the different types of waste.

Waste collection and treatment require labor, capital, and energy inputs. Although these two production processes are very different, they are assumed to follow the same overall production structure. Let the production level, Y_h, be given by

$$Y_h = \left[\gamma_h^{\frac{1}{\epsilon_h}} \left(K_h^{\mathrm{E}}\right)^{\frac{\epsilon_h - 1}{\epsilon_h}} + (1-\gamma_h)^{\frac{1}{\epsilon_h}} L_h^{\frac{\epsilon_h - 1}{\epsilon_h}} \right]^{\frac{\epsilon_h}{\epsilon_h - 1}}, \quad \gamma_h \in (0,1), \quad \epsilon_h > 0, \tag{3}$$

where $h \in \{I, B\}$ denotes the sector (I: waste collection sector; B: treatment sector), K_h^{E} is a capital-energy aggregate, and L_h is the labor input.

The capital-energy aggregate is given by

$$K_h^{\mathrm{E}} = \eta_h^{\mathrm{KE}} \left[(\gamma_h^{\mathrm{KE}})^{\frac{1}{\epsilon_h^{\mathrm{KE}}}} K_h^{\frac{\epsilon_h^{\mathrm{KE}} - 1}{\epsilon_h^{\mathrm{KE}}}} + (1-\gamma_h^{\mathrm{KE}})^{\frac{1}{\epsilon_h^{\mathrm{KE}}}} E_h^{\frac{\epsilon_h^{\mathrm{KE}} - 1}{\epsilon_h^{\mathrm{KE}}}} \right]^{\frac{\epsilon_h^{\mathrm{KE}}}{\epsilon_h^{\mathrm{KE}} - 1}}, \eta_h^{\mathrm{KE}} > 0, \ 0 < \gamma_h^{\mathrm{KE}} < 1, \ \epsilon_h^{\mathrm{KE}} > 0, \tag{4}$$

where K_h and E_h are the capital and energy inputs, respectively.

Minimizing the cost of obtaining a given production level implies that:

$$\left(\frac{w}{p_h^{\mathrm{KE}}}\right) = \left(\frac{\gamma_h}{1-\gamma_h} \frac{K_h^{\mathrm{E}}}{L_h}\right)^{\frac{1}{\epsilon_h}} \quad \text{and} \quad \left(\frac{r}{p_{E,h}}\right) = \left(\frac{\gamma_h^{\mathrm{KE}}}{1-\gamma_h^{\mathrm{KE}}} \frac{E_h}{K_h}\right)^{\frac{1}{\epsilon_h^{\mathrm{KE}}}}, \tag{5}$$

where w is the wage rate, r is the real interest rate, $p_{E,h}$ is the price of the energy aggregate, and p_h^{KE} is the ideal price index for the capital-energy aggregate given by:

$$p_h^{\mathrm{KE}} = \left[(\gamma_h^{\mathrm{KE}})\, r^{1-\epsilon_h^{\mathrm{KE}}} + (1-\gamma_h^{\mathrm{KE}})\, (p_{E,h})^{1-\epsilon_h^{\mathrm{KE}}} \right]^{\frac{1}{1-\epsilon_h^{\mathrm{KE}}}}. \tag{6}$$

The wage rate, w, the real interest rate, r, and the energy price, $p_{E,h}$, are exogenous in the present model.[1]

2.5 Collecting the waste

Firms in the waste collection sector are obligated to collect the waste mass, $\bar{Q}_n$, for each waste fraction. To do this, firms need to obtain the production level:

$$\sum_{n=1}^{N} \mu_n \bar{Q}_n = \eta_I Y_I(\cdot), \quad \eta_{I,n} > 0, \tag{7}$$

where η_I is a scale parameter measured in waste mass per unit of production, and μ_n is a weight parameter that allows the unit cost of transportation to differ between waste fractions.

[1] These prices will be endogenous in the final Green Reform model.

If only the weight of the waste matters for waste collection, then $\mu_n = 1$ for all n.

Firms in the sector are assigned the task of collecting waste through a public procurement. This ensures that the most productive firms are assigned the task. Essentially, the procurement forces firms to minimize their costs associated with the collection task. The problem of the representative collecting firm is:

$$\min_{K_I, L_I, E_I} L_I w + K_I r + E_I p_{E,I} \quad \text{st.} \quad \sum_{n=1}^{N} \mu_n \bar{Q}_n = \eta_I Y_I \left(K_I, L_I, E_I\right). \tag{8}$$

The first-order conditions associated with this problem are given by (5) and (6).

The treatment sector provides a service to the collection sector when taking over the waste. The collection sector pays a fee $p_n^{\bar{Q}} > 0$ per unit of waste delivered to the treatment sector. There is an additional fee, $p_n^d > 0$, for getting rid of waste that cannot be recycled or used directly for energy generation (typically hazardous waste). The total payment to the treatment sector per waste fraction, Υ_n, is:

$$\Upsilon_n = p_n^{\bar{Q}} \bar{Q}_n + p_n^d \sum_{g=1}^{G} d_{n,g} \hat{Q}_{n,g},$$

where $d_{n,g}$ is an exogenous fraction of the waste unsuited for utilization, cf. below. Note that firms in the treatment sector cannot avoid these fees, as they must deliver all the waste generated by the economy to the treatment sector. Accordingly, the fees do not affect firm behaviour in the sector.

The government imposes a non-profit constraint on firms within the waste collection sector, and thus:

$$\pi_I = 0, \tag{9}$$

where π_I is the profit of the representative firm in the waste collection sector.

Seen in isolation, profits associated with the collection activity are negative, as firms have expenses associated with both collecting and delivering the waste. Yet firms in the waste collection sector also have two sources of income: (1) households must pay a lump-sum transfer to the collection sector for handling the waste, and (2) firms must pay a fee to the collection sector for handling the waste. The government ensures zero profits through the lump-sum transfer and fees. The profit of the representative firm is:

$$\pi_I = T_I^H + T_I^F - L_I w - K_I r - E_I p_{E,I} - \sum_{n=1}^{N} \Upsilon_n,$$

where T_I^H is the lump-sum transfer from households, and T_I^F is the income obtained from collecting waste from the firm (all the sectors covered by the sector groups $g = 2, ..., G$). The latter is given by:

$$T_I^F = \sum_{n=1}^{N} \sum_{g=2}^{G} \sum_{j=1}^{J_g} p_n^I q_{n,g,j},$$

where p_n^I is the fraction specific fee paid by waste generating firms.

The representative firm represents a large number of firms that do not necessarily handle all waste types. But the non-profit condition applies to all individual firms in the sector. Hence it is assumed that the profit associated with each waste fraction is zero. As the production function is constant returns to scale, the production cost associated with a given fraction is simply the total production cost multiplied by the fraction of the weighted waste mass that the waste fraction constitute. Thus, to ensure zero profit for each fraction:

$$\sum_{g=2}^{G} \sum_{j=1}^{J_g} p_n^I q_{n,g,j} - \Upsilon_n - [L_I w + K_I r + E_I p_{E,I}] \, \Gamma_n^I = 0 \;\; \forall n$$

where

$$\Gamma_n^I \equiv \frac{\sum_{g=2}^{G} \sum_{j=1}^{J_g} \mu_n q_{n,g,j}}{\sum_{n=1}^{N} \sum_{g=1}^{G} \sum_{j=1}^{J_g} \mu_n q_{n,g,j}}.$$

The variable Γ_n^I captures the share of total production expenditures that can be assigned to waste fraction n for the groups $g > 1$ (all non-household groups).

2.6 Treatment, disposal, and recycling

The waste that needs treatment includes both the primary waste production and residual waste generated from the recycling processes. The residual waste mass of fraction n belonging to group g is denoted $q_{n,g}^R$. It is costless to dispose the residual waste, as this waste generation occurs within the treatment sector.

A fixed share of each waste fraction cannot be burned or recycled. One example is soft

PVC which is unsuited for recycling and generates toxins when burned. This waste ends up as landfill. The fraction of waste sent to landfill, $d_{n,g}$, is here exogenous, as regulation (the waste hierarchy) generally dictates that waste, if possible, should be burned or recycled.

The waste mass handled by the treatment sector net of landfill, Q_n, is given by:

$$Q_{n,g} = (1 - d_{n,g}) \left(\hat{Q}_{n,g} + q^R_{n,g} \right), \tag{10}$$

where it is (so far) assumed that the same fraction of primary and residual waste must go to landfill.

The waste net of landfill can be utilized in two ways: it can be burned to generate energy, or it can be recycled. However, the waste fractions are not completely homogeneous, implying that some parts of a waste fraction are more suited for recycling than others. To capture this property, the transformation function between energy waste and recycled waste is given by the modified CET (constant elasticity of transformation) functions:

$$Q_{n,g} = \Omega_{n,g} \left[\gamma_{T,n,g}^{\frac{1}{\epsilon_{T,n,g}}} M_{T,n,g}^{\frac{\epsilon_{T,n,g}-1}{\epsilon_{T,n,g}}} + (1 - \gamma_{T,n,g})^{\frac{1}{\epsilon_{T,n,g}}} A_{T,n,g}^{\frac{\epsilon_{T,n,g}-1}{\epsilon_{T,n,g}}} \right]^{\frac{\epsilon_{T,n,g}}{\epsilon_{T,n,g}-1}} + \xi_{n,g} A_{T,n,g}, \tag{11}$$

where T indicates that the parameter or variable belongs to the transformation function, $M_{T,n,g}$ is the mass of new materials obtained from the recycling process, and $A_{T,n,g}$ is the mass of energy waste sent to the energy sector.

All the waste can, in principle, be burned. In that case, there is no generation of new raw materials implying that $M_{T,n,g} = 0$. To ensure that all waste mass is accounted for in the CET function when all waste mass is burned, it must hold that:

$$Q_{n,g} = \left(\Omega_{n,g} (1 - \gamma_{T,n,g})^{\frac{1}{\epsilon_{T,n,g}-1}} + \xi_{n,g} \right) \underbrace{A^{\mathbf{MAX}}_{T,n,g}}_{=Q_{n,g}} \Leftrightarrow 1 = \Omega_{n,g} (1 - \gamma_{T,n,g})^{\frac{1}{\epsilon_{T,n,g}-1}} + \xi_{n,g},$$

where $A^{\mathbf{MAX}}_{T,n,g}$ is the maximum mass of energy waste that can be generated which equals the total waste mass available, $Q_{n,g}$.

Figure 1 presents an example, where the waste mass, $Q_{n,g}$, is normalized to one. The entire waste mass can be sent to the energy sector, where it may be burned. If some part of the waste mass is recycled then only some of this mass ends up as new raw materials, as the recycling process generates residual waste and pollution emissions. The horizontal distance between any point on the CET function and the physical constraint reflects the combined pollution emission and residual waste mass generated from recycling. Mathematically,

$$\bar{M}_{T,n,g} = \hat{q}^R_{n,g} + e_{T,n,g} + M_{T,n,g} = Q_{n,g} - A_{T,n,g}, \tag{12}$$

where $\bar{M}_{T,n,g}$ is the waste mass sent to recycling, $\hat{q}^R_{n,g}$ is the residual waste mass generated from recycling, and $e_{T,n,g}$ is waste mass exiting the system as emissions from the recycling process. The first equation states that the waste mass entering the recycling process is transformed into new raw materials, emissions, and residual waste. The second equality states that the materials sent to recycling equals the waste mass net of landfill, $Q_{n,g}$, subtracting waste mass used for energy purposes.

Notice that the recycling process is more efficient in terms of transforming waste into new raw materials the smaller the distance between the CET function and the physical constraint. The curvature of the CET function reflects the heterogeneity of the waste mass: some parts of a given waste mass are more suited for recycling than others.

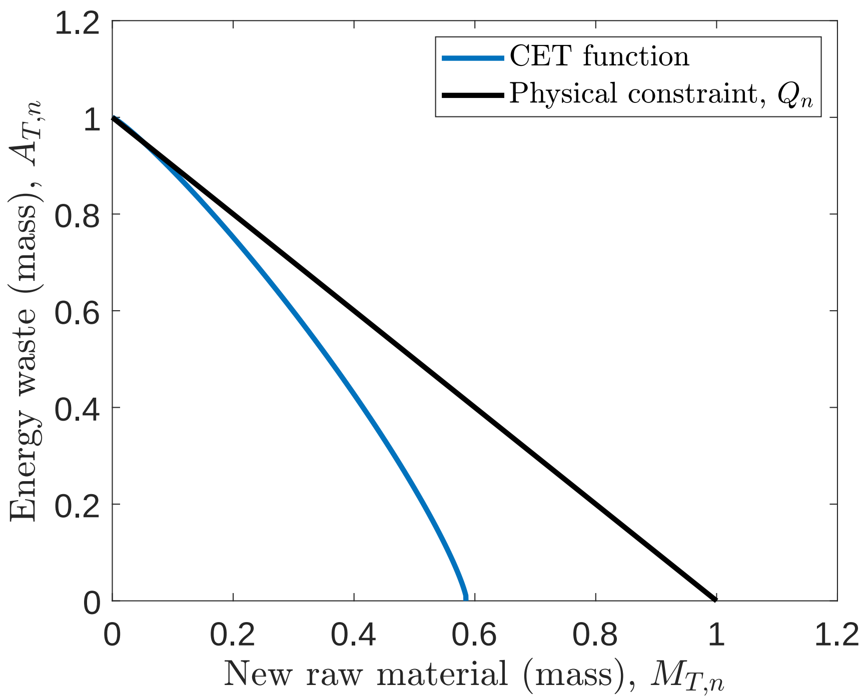

FIGURE 1: Illustration of modified CET function.

One technical detail is worth emphasizing. The modified CET function is homogeneous of degree one (i.e. constant returns to scale). This ensures that the transformation possibilities are independent of the amount of waste that needs to be handled. The implicit assumption is that the composition of waste is independent of scale given both the waste fraction and waste group.

The production level required in the treatment sector is

$$\eta_B Y_B = \sum_{n=1}^{N} \sum_{g=1}^{G} \left\{ \left(Q_{n,g} - A_{T,n,g}\right) c_{n,g}^{M} + A_{T,n,g} c_{n,g}^{A} + D_{n,g} c_{n,g}^{D} \right\}, \tag{13}$$

where η_B is a scale parameter measured in waste mass per unit of production, $D_{n,g}$ is the amount of waste displaced as landfill, while $c_{n,g}^{M}$, $c_{n,g}^{A}$ and $c_{D,n}$ are cost parameters.

The choice between recycling and burning is restricted by legislation through the waste hierarchy. The hierarchy dictates that recycling is preferable to energy utilization. As a point of departure, it is assumed that the waste hierarchy places a binding constraint on firms in the treatment sector such that

$$\chi_{n,g} Q_{n,g} = \bar{M}_{T,n,g}, \quad \chi_{n,g} \in (0,1). \tag{14}$$

The condition states that the treatment sector must recycle a constant fraction, $\chi_{n,g}$, of the waste mass of a given fraction for each waste group. The restriction tells us how much material that enters the recycling process, while the CET function tells us how much new material that is generated.

The profit function of the representative firm in the treatment sector is given by:

$$\pi_B = \sum_{n=1}^{N} \sum_{g=1}^{G} \left\{ p_{M,n} M_{T,n,g} + p_{E,n} A_{T,n,g} \right\} + \sum_{n=1}^{N} \Upsilon_n - L_B w - K_B r - E_B p_{E,B}, \tag{15}$$

where $p_{M,n}$ is the price of the recycled material, and $p_{E,n}$ is the price of the energy waste.[2] The treatment sector sells the recycled material to the remaining part of the economy, and it sells the energy waste to the energy sector. Prices differ between waste fractions. The price of the energy waste is determined by its heating value, while the type of material generated from recycling is fraction specific.

The problem of each representative firm in the treatment sector is to maximize the profit function (15) subject to the transformation function (11), the production constraint (13), and the waste hierarchy constraint (14).

The waste hierarchy constraint (14) dictates how much waste the representative firm

[2]These two prices are exogenous in this partial equilibrium model, but they will be endogenous in the final Green Reform model.

allocates to energy and recycling purposes. To see this, combine (12) and (14) to obtain:

$$A_{T,n,g} = (1 - \chi_{n,g})Q_{n,g}.$$

Hence $A_{T,n,g}$ is a constant fraction of the total waste mass that needs treatment. Substituting this expression into the modified CET function (11) results in one equation with one unknown, $M_{T,n,g}$, which has a unique solution. Hence, the waste hierarchy determines both $A_{T,n,g}$ and $M_{T,n,g}$.

As the waste hierarchy constraint eliminates an active choice between energy utilization and recycling, the problem of the representative firm boils down to a cost minimization problem:

$$\min_{K_B,L_B,E_B} \quad L_B w + K_B r + E_B p_{E,B}$$
$$\text{st.} \quad \eta_B Y_B = \sum_{n=1}^{N}\sum_{g=1}^{G}\left\{\left(Q_{n,g} - A_{T,n,g}\right)c^M_{n,g} + A_{T,n,g}c^A_{n,g} + D_{n,g}c^D_{n,g}\right\},$$

where $Q_{n,g}$, $D_{n,g}$, and $A_{T,n,g}$ are taken as given. The first-order conditions associated with these problems are given by (5) and (6).

As the treatment sector operates under a non-profit constraint, the price for delivering waste to the treatment sector, $p^{\bar{Q}}_n$, is endogenously determined such that profits for each fraction is zero. The arguments are similar to those given above for the non-profit constraints of the waste collection sector. The condition ensuring zero profit per waste fraction is:

$$\sum_{g=1}^{G}\left\{p_{M,n}M_{T,n,g} + p_{E,n}A_{T,n,g}\right\} + \Upsilon_n = \left(L_B w + K_B r + E_B p_{E,B}\right)\Gamma^B_n,$$

where

$$\Gamma^B_n \equiv \frac{\sum_{g=1}^{G}\left\{\left(Q_{n,g} - A_{T,n,g}\right)c^M_{n,g} + A_{T,n,g}c^A_{n,g} + D_{n,g}c^D_{n,g}\right\}}{\sum_{n=1}^{N}\sum_{g=1}^{G}\left\{\left(Q_{n,g} - A_{T,n,g}\right)c^M_{n,g} + A_{T,n,g}c^A_{n,g} + D_{n,g}c^D_{n,g}\right\}}.$$

The variable Γ^B_n captures the fraction of factor input expenditures associated with waste fraction n. Hence the equation states that the revenue generated from handling a waste fraction must equal the cost of doing so.

Finally, it is assumed that the additional fee for delivering landfill waste is proportional to $p_n^{\bar{Q}}$:

$$p_n^d = \kappa p_n^{\bar{Q}}, \quad \kappa > 0.$$

2.7 Residual waste generation

The residual waste mass generated in a particular recycling process is divided into waste fractions and waste groups. These waste masses then enters the recycling processes of the relevant waste fractions and waste groups.

The total mass of residual waste generated from the recycling of fraction n in waste group g must equal the sum of fraction-group residual waste masses:

$$\hat{q}_{n,g}^R = \sum_{\hat{n}=1}^{N} \sum_{\hat{g}=1}^{G} \hat{q}_{n,g,\hat{n},\hat{g}}^R,$$

where $\hat{q}_{n,g,\hat{n},\hat{g}}^R$ is the residual waste mass of fraction $\hat{n}$ in group $\hat{g}$ generated from the recycling of fraction n in group g.

The total mass of residual waste of fraction n in group g that the treatment sector must handle equals:

$$q_{n,g}^R = \sum_{\hat{n}=1}^{N} \sum_{\hat{g}=1}^{G} \hat{q}_{\hat{n},\hat{g},n,g}^R.$$

The fractional waste masses generated from recycling are assumed to be a constant share of the total residual waste mass generated:

$$\hat{q}_{n,g,\hat{n},\hat{g}}^R = \omega_{\hat{n},\hat{g}} \hat{q}_{n,g}^R, \quad \sum_{\hat{n}=1}^{N} \sum_{\hat{g}=1}^{G} \omega_{\hat{n},\hat{g}} = 1. \tag{16}$$

Further, the emission masses are constant fractions of the material masses entering the recycling process:

$$e_{T,n,g} = \phi_{n,g} \bar{M}_{T,n,g}, \quad 0 < \phi_{n,g} < 1. \tag{17}$$

2.8 Physical constraints

The First Law of Thermodynamics implies that the mass entering the treatment process equals the sum of masses after treatment:

$$\hat{Q}_{n,g} + q^R_{n,g} = D_{n,g} + A_{T,n,g} + M_{T,n,g} + e_{T,n,g} + \hat{q}^R_{n,g}, \tag{18}$$

where $D_{n,g} = d_{n,g}\left(\hat{Q}_{n,g} + q^R_{n,g}\right)$ is the waste mass that are allocated to landfill.

Summing over all fractions and waste groups:

$$\sum_{n=1}^{N}\sum_{g=1}^{G}\hat{Q}_{n,g} + \sum_{n=1}^{N}\sum_{g=1}^{G}q^R_{n,g} = \sum_{n=1}^{N}\sum_{g=1}^{G}\left(D_{n,g} + A_{T,n,g} + M_{T,n,g} + e_{T,n,g}\right) + \sum_{n=1}^{N}\sum_{g=1}^{G}\hat{q}^R_{n,g} \quad \Leftrightarrow$$
$$\sum_{n=1}^{N}\sum_{g=1}^{G}\hat{Q}_{n,g} = \sum_{n=1}^{N}\sum_{g=1}^{G}\left(D_{n,g} + A_{T,n,g} + M_{T,n,g} + e_{T,n,g}\right).$$

This equation is important for the intuition. It shows that the total waste mass generated by the economy must equal the sum of four masses: mass utilized for energy generation (sent to the energy sector), mass transformed into new materials, mass displaced as landfill, and mass ending up as emissions. Hence, even though the recycling process generates residual waste, each unit of waste mass will - in the end - be sent to the energy sector, transformed into new useful materials, displaced as landfill, or end up as emissions.

The cancellation of the residual waste terms may be rationalized the following way. The secondary waste input is given by

$$\sum_{n=1}^{N}\sum_{g=1}^{G}q^R_{n,g} = \sum_{n=1}^{N}\sum_{g=1}^{G}\sum_{\hat{n}=1}^{N}\sum_{\hat{g}=1}^{G}\hat{q}^R_{n,g,\hat{n},\hat{g}} = \sum_{\hat{n}=1}^{N}\sum_{\hat{g}=1}^{G}\underbrace{\sum_{n=1}^{N}\sum_{g=1}^{G}\hat{q}^R_{n,g,\hat{n},\hat{g}}}_{=q^R_{\hat{n},\hat{g}}}.$$

In the first equation it is used that the residual waste input of fraction n in group g equals the sum of residual waste masses generated by each of the $n \times g$ recycling processes within the treatment sector. The second equality follows from a general math rule saying that for finite sums, one may change the order of summation. The equation clearly shows that the two residual waste masses in (18) cancel out when summing over the N waste fractions and G waste groups.