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Strategy for integrating the energy sector model and the CGE model in GreenREFORM

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Introduction

The purpose of this note is to lay out the strategy for the integration between the general equilibrium model (CGE-model) and the energy sector model of GreenREFORM.

An initial integration project was succesfully completed prior to the writing of this note. However, at that point in time, the CGE-model lacked a number of features necessary to make a full integration possible.

The two models will interact as follows: The energy sector model describes the market clearing and supply decisions of a number of representative district heating and electric power (heat and power) plants. The energy sector model takes into account physical limitations in the transmission systems and in installed capacity as well as fluctuations in intermittent production (wind and solar etc.) and demand on an intra-year basis. The energy sector model determines the level of production, the amount of various inputs in production, trade patterns, and price formation. All this information is passed on to the CGE-model in terms of yearly totals and averages. Likewise, the yearly total demand for power and heat as well as the prices on inputs in production are determined in the CGE-model and passed on to the energy sector model.

Being a complete general equilibrium model for the Danish economy, production and the supply and demand balances of heat and power is already described by the CGE-model, albeit at a higher level of aggregation and with no concern for physical limitations and intra-year variation. However, it is essential that the general equilibrium properties, including national account balances that are built into the CGE-model, are maintained in the integrated model.

To achieve this, the chosen strategy is to make endogenous adjustments to a number of otherwise exogenous parameters in the CGE-model, so as to adapt the model to the descriptions provided by the energy sector model on a year by year basis. As will be discussed, this strategy also has the advantage, that it allows the two models to function independently of each other, and importantly, that the simulated forecast of the model can be expanded beyond the scope of the energy sector model.

In section 1 the interactions between the two models are described and complications in this regard are discussed. In section 2 the above mentioned adaptions of the CGE-model are described (see also appendix A). Section 3 provides a brief overview of future developments regarding the integration between the CGE model and the Energy sector model.

1 Interaction between the models

The energy sector model describes the market clearing and supply decisions of a number of representative heat and power plants on an intra-year basis. The model is based on rich bottom-up data and is calibrated to match yearly statistiscs from 'Energistatistikken' in historical years on production output prices, fuel input quantities etc. Energistatistikken is also an important source of information for the National Accounts. However, since the National Accounts need to respect a variety of other statistics and must uphold a number of accounting balances, and because of differences in definitional delimitations, perfect symmetry cannot be expected. A priori, the two models will therefore not fit seamlesslesly together.

We address this is issue by letting both models stay true to there respective data sources, and adopt an integration strategy by which to allow for any differences in statistics to be maintained during simulations. Minor differences between the statistics of the two models will be regarded as acceptable, whereas major differences will be addressed by explicit modelling at a later stage.

1.1 Notation

In this note the index $md = \{eng, cge\}$ will be used to distuinguish between the energy sector model and the CGE-model. $eo = \{heat, powr\}$will be used to distinguish between heat and power, $e = \{heat, powr, ...\}$ will be used to distinguish between an expanded set of energy types to also cover inputs in production. At present it is assumed that the set defintions of energy types are symmetric in the two models. This will be discussed in section 4 in a future update. As in the CGE-model, all variables will be defined in yearly total flows or yearly average prices identified by index t.

In the CGE-model, production of heat and power is concentrated in two sectors, both of which produce heat as well as power. One sector covers waste-to-energy plants, and the other sector covers all other plant types. In principle the structure of the CGE-model would allow for any sector to produce any type of energy. In practice the input data of the model will dictate which sectors produce what. We use the set $p = \{pwaste, pother\}$to distinguish between the two sectors. The index m is denoted as superscript and the other indexes as supscript, such that $X^{cge}_{eo,p,2010}$denotes a variable of the CGE-model defined on energy outputs and sectors in year 2010.

1.2 Market balances for heat and power

The total yearly production meassured in physcial units of energy content (Joule or Watt-hour) is defined by the variable $qY^{md}_{eo,p,t}$. Similarly domestic demand is defined by $qD^{md}_{eo,t}$, exports qEx^{md}_{eo}, and imports qIm^{md}_{eo}. Assuming that distribution losses are included in domestic demand, both models must satisfy a physical constraint on supply and demand[1]:

$$\sum_p \left(qY^{md}_{eo,p,t}\right) + qIm^{md}_{eo,t} = qD^{md}_{eo,t} + qEx^{md}_{eo,t} \tag{1}$$

In the energy sector model, similar physical constraints are upheld in each intra-year time period for each price region. In each such instance, markets are cleared by a uniform price.

Since each demand and supply component is distributed differently across the year and across the two danish price regions, the yearly average prices of production $pY^{eng}_{eo,p,t}$, of domestic demand $pD^{eng}_{eo,t}$, of exports$pEx^{eng}_{eo,t}$, and of imports $pIm^{eng}_{eo,t}$ will be systimatically different from each other. Even so, a balance of transactions condition will also hold when averaging across the year and across price regions[2], ie:

$$\sum_p \left(pY^{md}_{eo,p,t} * qY^{md}_{eo,p,t}\right) + pIm^{md}_{eo,t} * qIm^{md}_{eo,t} = pD^{md}_{eo,t} * qD^{md}_{eo,t} + pEx^{md}_{eo,t} * qEx^{md}_{eo,t} \tag{2}$$

Similar market balance conditions apply in the CGE-model. The modelling of energy markets in the CGE-model is described in detail in a separate note, but can very briefly be described as follows. The output price are determined from the supply side by technology, market conditions and input prices. On the demand side, consumers do not distinguish between different sources of supply. On the demand side, firms and households are faced with the average yearly output price with the addition of a markup premium specific to the sector of the demand side firm, or households etc. The markup is introduced to comply with the reality of differences in the yearly average prices on the demand side, cf. above. On the supply side, an artificial distributor is modelled, whom channel total demand towards a mix of import and one or more domestic sectors. The balance of transactions condition is met by introduction of an endogenous lumpsum transfer of excess profits to the domestic suppliers. In years covered by data, this excess profit will be zero by default, but during simulations, changes in the composition of total demand will lead to positive or negative excess profits due to the markups.

When the two models are integrated, the energy model will take over the role of the artificial distributor of allocating total demand to suppliers, and the excess profit rate will be fixed to zero.

[1] Transmission losses are assumed part of total domestic demand in equation.

[2] For simplification equation2 does not account for congestion rents on the interconnector between the two domestic price regions.

1.3 Linkage equations for power

As described in the introduction, the CGE-model will determine domestic demand, while the energy sector model will determine imports, exports, production, and the respective average yearly price of each of these including that of domestic demand. Because of possible statistical discrepancies and differences in definitions, we allow for differences in the levels, and instead link by introducing equations, that dictate identical growth rates in each of the above mentioned variables across time. J_-terms with a prori values of zero are included to allow for freedom of calibration to data in statistical years. When ever possible we isolate the variable, which we think of as being determined by an equation, on the left hand side of equations. We thus formulate the linkage equations as follows:

$$qD^{eng}_{powr,t} = \frac{qD^{cge}_{powr,t}}{qD^{cge}_{powr,t-1}} * qD^{cge}_{powr,t-1} + J\_qD_{powr,t} \tag{3}$$

$$qY^{cge}_{powr,p,t} = \frac{qY^{eng}_{powr,p,t}}{qY^{eng}_{powr,p,t-1}} * qY^{cge}_{powr,p,t-1} + J\_qY_{powr,p,t} \tag{4}$$

$$qIm^{cge}_{powr,p,t} = \frac{qIm^{eng}_{powr,p,t}}{qIm^{eng}_{powr,p,t-1}} * qIm^{cge}_{powr,p,t-1} + J\_qIm_{powr,p,t} \tag{5}$$

$$pD^{cge}_{powr,t} = \frac{pD^{eng}_{powr,t}}{pD^{eng}_{powr,t-1}} * pD^{cge}_{powr,t-1} + J\_pD_{powr,t} \tag{6}$$

$$pY^{cge}_{powr,p,t} = \frac{pY^{eng}_{powr,p,t}}{pY^{eng}_{powr,p,t-1}} * pY^{cge}_{powr,p,t-1} + J\_pY_{powr,p,t} \tag{7}$$

$$pIm^{cge}_{powr,p,t} = \frac{pIm^{eng}_{powr,p,t}}{pIm^{eng}_{powr,p,t-1}} * pIm^{cge}_{powr,p,t-1} + J\_pIm^{cge}_{powr,p,t} \tag{8}$$

By allowing for (minor) differences in the levels of variables in the two models, we must allow for a slack variable for each of the two market clearing conditions in the CGE-model. We choose to make these the price and the quantity of exports, ie. $pEx^{cge}_{powr,t}$ and $qEx^{cge}_{powr,t}$:

$$pEx^{cge}_{powr,t} = \frac{\left(\sum_p \left(pY^{cge}_{powr,p,t} * qY^{cge}_{powr,p,t}\right) + pIm^{cge}_{powr,t} * qIm^{cge}_{powr,t}\right) - \left(pD^{cge}_{powr,t} * qD^{cge}_{powr,t}\right)}{qEx^{md}_{eo,t}} + J\_pEx_{powr,t} \tag{9}$$

$$qEx^{cge}_{powr,t} = \left(\sum_p \left(qY^{cge}_{powr,p,t}\right) + qIm^{cge}_{powr,t}\right) - qD^{cge}_{powr,t} + J\_qEx_{powr,t} \tag{10}$$

As long as

1.4 Linkage equations for heat

Exports and imports are zero for heat. But we still have the complexity of two sectors producing heat at different average yearly prices. In order to maintain market balances, we choose to let the price of domestic demand $pD^{cge}_{heat,t}$ be a slack variable in the balance of transactions, and to let the supply from the non waste-combustion sector $qY^{cge}_{heat,pother,t}$ be a slack variable in the physical market constraint. We thus introduce the following equations:

$$qD^{eng}_{heat,t} = \frac{qD^{cge}_{heat,t}}{qD^{cge}_{heat,t-1}} * qD^{eng}_{heat,t-1} + J\_qD_{heat,t} \tag{11}$$

$$pY^{cge}_{heat,p,t} = \frac{pY^{eng}_{heat,p,t}}{pY^{eng}_{heat,p,t-1}} * pY^{cge}_{heat,p,t-1} + J\_pY_{heat,p,t} \tag{12}$$

$$qY^{cge}_{heat,pwaste,t} = \frac{qY^{eng}_{heat,pwaste,t}}{qY^{eng}_{heat,pwaste,t-1}} * qY^{cge}_{heat,pwaste,t-1} + J\_qY_{heat,pwaste,t} \tag{13}$$

$$qY^{cge}_{heat,pother,t} = qD^{cge}_{heat,t} - \left(qIm^{cge}_{heat,t} + qY^{cge}_{heat,pwaste,t}\right) \tag{14}$$

$$pD^{cge}_{heat,t} = \frac{\left(\sum_p \left(pY^{cge}_{heat,p,t} * qY^{cge}_{heat,p,t}\right) + pIm^{cge}_{heat,t} * qIm^{cge}_{heat,t}\right) - \left(pEx^{cge}_{heat,t} * qEx^{cge}_{heat,t}\right)}{qD^{cge}_{heat,t}} \tag{15}$$

1.5 Linkage equations for energy inputs and production costs

The yearly total input of energy is described by $qRE^{md}_{e,p,t}$ and the respective prices by $pRE^{md}_{e,p,t}$. As described in the introduction, the CGE-model will determine prices of inputs, and the energy sector model will determine energy inputs. We thus impose the following equations, which dictate equal growth rates of these variables between the two models.

$$qRE^{cge}_{e,p,t} = \frac{qRE^{eng}_{e,p,t}}{qRE^{eng}_{e,p,t-1}} * qRE^{cge}_{e,p,t-1} + J\_qRE_{e,p,t} \tag{16}$$

$$pRE^{eng}_{e,p,t} = \frac{pRE^{cge}_{e,p,t}}{pRE^{cge}_{e,p,t-1}} * pRE^{eng}_{e,p,t-1} + J\_pRE_{heat,p,t} \tag{17}$$

In the energy sector model, production costs are divided into input of each type of energy and a residual non-energy cost components. In the CGE-model non-energy production costs are described in much more detail. We want the price of non-energy inputs in the energy sector model to be determined by the CGE-model, such that changes in labor costs etc. will spill over on the output price of heat and power. To do so, we define a non energy cost price index by comparing the total costs of production less energy at current and next years prices. We let this index determine the growth in the price of non energy inputs in the energy sector model $pO^{eng}_{p,t}$ by imposing the following equation.

$$pO^{eng}_{p,t} = \frac{\sum_{eo}\left(\frac{pY^{cge}_{eo,p,t}}{1+\alpha_{eo,p,t}} * qY^{cge}_{eo,p,t-1}\right) - \sum_{e}\left(pRE_{e,p,t} * qRE_{e,p,t-1}\right)}{\sum_{eo}\left(\frac{pY^{cge}_{eo,p,t-1}}{1+\alpha_{eo,p,t-1}} * qY^{cge}_{eo,p,t-1}\right) - \sum_{e}\left(pRE_{e,p,t-1} * qRE_{e,p,t-1}\right)} * pO^{eng}_{p,t-1} + J\_pO^{eng}_{p,t} \tag{18}$$

Note that the output price is divided by a sector and product specific markup factor $(1 + \alpha_{eo,p,t})$to resemble the cost price, cf. section 1.6. In practice equation 18 can be simplified by expressing total costs and cost of energy input by respective aggregate quantities and prices of the CET- and CES-production structure. For clarity of exposition, we have formulated it in terms of variables already defined, but for the markup.

In the energy sector model, profits arise when the average price attained in the market is higher than average production costs. It is important that we keep track of those profits in the CGE-model. We already have equations in place controlling the market value of heat and power. The following equation dictates equal growth rates in the total production costs (in each sector) across the two models. As explained further in section 2.1, we will use it to effectively control the total inputs of non-energy materials and services, as energy inputs are determined by the equations above.

$$\frac{\sum_{eo}\left(\frac{pY^{cge}_{eo,p,t}}{1+\alpha_{eo,p,t}} * qY^{cge}_{eo,p,t}\right)}{\sum_{eo}\left(\frac{pY^{cge}_{eo,p,t-1}}{1+\alpha_{eo,p,t-1}} * qY^{cge}_{eo,p,t-1}\right)} = \frac{TC^{eng}_{p,t}}{TC^{eng}_{p,t-1}} + J\_\alpha_{p,t} \tag{19}$$

2 Adjustments in the CGE-model.

2.1 Adjustment of production structure

As explained in section 1.2, the energy model will take over the role of the artificial distributor of allocating total demand to suppliers, and the excess profit rate will be fixed to zero.

Firms are thought of as taking market demand as given and adjusting inputs to maximize profits. Each sector will thus readily adapt production to the restrictions put on output, cf. above.

In order for output prices to adapt to restrictions put on the them above, we let output-sector and product specific markups $(\alpha_{eo,p,t})$ adjust year by year.

In order to seperately adjust production costs of each of the two sectors (and thus profits) as by equation 19, we will endogenize a scale parameter governing total productivity in each of the two sectors. In order to adapt the level of energy inputs to the restrictions above, we will endogenize parameters in the

CES-production function governing the input efficiency of each type of energy. See appendix A for further detail.

2.2 Adjustments to demand prices and market clearing

In the CGE-model the price of domestic demand is not uniform, cf. section 1.2. The aforementioned demand price $pD^{cge}_{powr,t}$ is the weighted average of these prices. In order for $pD^{cge}_{powr,t}$ to comply with the restricton put on it above, we will apply an assumption of fixed relative prices, such that in each period all demand side prices will adjust by the same factor relative to the previous period.

2.3 Connect and disconnect feature

In years covered by data and in years beyond the scope of the energy sector model, the CGE-model must function independently of the energy sector model. In years beyond the scope of the energy sector model, the values of the price terms, the productivity parameters and the markups of the final year of integration should carry over into consecutive years, for example $\alpha^{cge}_{p,t} = \alpha^{cge}_{p,t-1}$. Markets should then clear via standard assumptions for energy goods.

2.4 Calibration

Calibration of the CGE-model to statistical data will be the same for heat and power as any other sector. In forecast years, the total domestic demand for heat and power will be calibrated to external forecasts. This will be done by scale adjustment of parameters governing demand across all sources of domestic demand, in a similar fashion to how demand for energy input in production of heat and power is adjusted.

3 Future development

Investment in production capacity is at present exogenous in the energy sector model. In the CGE-model investment will adjust to meet capital requirements of production by the same assumptions as all other sectors in the CGE-model. We are currently researching how to improve on this.

The modelling of taxes and subsidies and the linkages between the two models in this regard is yet another topic, we need to address.

The energy sector model also needs to be linked to other sector specific models of GreenREFORM. For example the waste management model will determine the domestic supply of waste to be used in the waste to energy plants in the energy sector model, the transportation model will determine the level of electricity used for transportation, and the model for agriculture and land use will determine the domestic supply of biomass. Modelling of emerging Power2X technologies will only lead to deeper integration between the sector-specific models.

Appendix

A Calibration and adjustment of production structure in the CGE-model

GreenREFORM is a CGE model with an integrated energy supply system. The energy supply system gives a correct microfounded description of the production of electricity and heat. The energy supply system will only be active for the first 20-30 years of the model projection. After this, a simplified classic CES/CET-system will take over as is usually done in CGE models. For the first 20-30 years, the production of electricity and heat is described both by the microfounded energy supply system and the CES/CET-system. This is done by adjusting the CES/CET-system to the energy supply system.The advantage of this approach is that the parameters of the CES/CET-system are thereby determined in the first 20-30 years, thus providing a good basis for projecting the production of electricity and heat *after* the first 20-30 years.

We demonstrate the method used with two examples. First, an example is shown where an energy product is produced with two inputs. Then this example is extended to describe the situation where two energy outputs (think of it as electricity and heat) are produced with two inputs. Finally, a description of the practical implementation of the method in a CGE model is given.

A.1 CES-system: Calubration and adjustment

Assume a simplified energy-producing sector has one energy output and two inputs (energy X_t and non-energy Z_t). The demand for the two inputs is given by the CES demand system:

$$X_t = \mu_t^X \left(\frac{p_t^X}{P_t^O}\right)^{-E} Y_t \tag{20}$$

$$Z_t = \mu_t^Z \left(\frac{p_t^Z}{P_t^O}\right)^{-E} Y_t \tag{21}$$

$$P_t^O Y_t = P_t^X X_t + P_t^Z Z_t \tag{22}$$

The output price p_t is defined based on the cost-determined price P_t^O and a markup m_t:

$$p_t = (1 + m_t) P_t^O \tag{23}$$

A.1.1 Calibration

If we have historical data, we can calibrate the model parameters.Suppose we have data for $\left(X_t, Z_t, Y_t, p_t^X, p_t^Z, p_t\right)$. We can then calculate

$$P_t^O = \frac{P_t^X X_t + P_t^Z Z_t}{Y_t}$$

and then calculate

$$\mu_t^X = \frac{X_t}{Y_t}\left(\frac{p_t^X}{P_t^O}\right)^E$$
$$\mu_t^Z = \frac{Z_t}{Y_t}\left(\frac{p_t^Z}{P_t^O}\right)^E$$
$$1 + m_t = \frac{p_t}{P_t^O} = \frac{p_t Y_t}{P_t^X X_t + P_t^Z Z_t}$$

We could alternatively use the model's equations. If we exogenize $\left(X_t, Z_t, Y_t, p_t^X, p_t^Z, p_t\right)$ in (20)-(23) and endogenizes $\left(\mu_t^X, \mu_t^Z, P_t^O, m_t\right)$ we get the calibrated model.

A.1.2 When the model is running normally

When the model runs like a regular CGE model, $\left(X_t, Z_t, P_t^O, p_t\right)$ is determined in (20)-(23). The variables $\left(p_t^X, p_t^Z, Y_t\right)$ are endogenous, but are determined elsewhere in the model. We work with constant returns to scale. Therefore, prices are cost-determined and quantities are demand-determined. Therefore, $\left(p_t^X, p_t^Z\right)$ is determined by what it costs to produce X and Z (which has nothing to do with what the industry (20)-(23) describes) and Y_t is determined by the economy's total demand for the energy product we describe in (20)-(23). The parameters $\left(\mu_t^X, \mu_t^Z, m_t\right)$ are exogenous.

A.1.3 Adjustment to the energy supply model

Suppose in a projection we get data from the energy supply model for the energy variables $\left(X_t, Z_t, Y_t, p_t^X, p^Z, p_t, C_t\right)$. As we realized in the last section, only three of these variables (X_t, Z_t, p_t) are determined in (20)-(23) when the model is running. The variables $\left(Y_t, p_t^X, p^Z\right)$ are determined elsewhere in the model (and therefore must be adjusted in other CES nests). Adjustment is therefore done as follows: (X_t, Z_t, p_t) is exogenized and $\left(\mu_t^X, \mu_t^Z, m_t\right)$ are endogenized.

A.2 A CES/CET-system: Calibration and adjustment

Let's extend the model with a CET split at the top. The industry produces two energy goods Y_t^j, $j = 1, 2$. This gives rise to a new extended system[3]:

$$Y_{jt} = \mu_{jt}^Y \left(\frac{P_{jt}^O}{P_t^O}\right)^{-F} Y_t,\ j = 1, 2,\ F < 0, \tag{24}$$

$$P_t^O Y_t = \sum_j P_{jt}^O Y_{jt}, \tag{25}$$

$$p_{jt} = (1 + m_{jt})\, P_{jt}^O,\ j = 1, 2 \tag{26}$$

where p_{jt} are the two energy types' market prices. The rest of the system is still(20)-(22).

[3]See appendix B

A.2.1 Calibration

Suppose we have data $\left(X_t, Z_t, Y_{jt}, p_t^X, p_t^Z, p_{jt}\right)$ and wants to calibrate the model. In calibration, we want to calculate values for 10 parameters and variables $\left(\mu_t^X, \mu_t^Z, \mu_{jt}^Y, m_{jt}, Y_t, P_t^O, P_{jt}^O\right)$. But we only have 8 equations in the total system (20)-(22), (24)-(26). The problem is that we basically cannot determine any differences in markups simply by looking at data $\left(X_t, Z_t, Y_{jt}, p_t^X, p_t^Z, p_{jt}\right)$. It is not possible to see if one output is subject to more competition than the other. We must therefore add some restriction on the markups. The simplest restriction is to assume that the markup is the same for the two energy products:

$$m_{jt} = m_t, \; j = 1, 2 \tag{27}$$

If we additionally assume that

$$P_t^O = \overline{P}_t^O$$

has 8 equations with 8 unknowns, after which calibration is well defined. We can either assume that $\overline{P}_t^O = 1$ or is given by a chain price index.

A.2.2 When the model is running normally

When the model runs like a regular CGE model, $\left(X_t, Z_t, P_t^O\right)$ is determined in (20)-(22) and $\left(P_{jt}^O, Y_t, p_{jt}\right)$ are determined in (24)-(26). Note that the model contains the possibility of different markups (it is only in the calibration that we assumed they were the same). Prices $\left(p_t^X, p_t^K\right)$ are (cost) determined elsewhere in the model, and Y_{jt} is determined from the demand side.

A.2.3 Adjustment to the energy supply model

Suppose in a projection we get data from the supply model for the energy variables $\left(X_t, Z_t, Y_{jt}, p_t^X, p_t^Z, p_{jt}\right)$. As we realized in the last section, only four of these variables (X_t, Z_t, p_{jt}) are determined in (20)-(22), (24)-(26) when the model is running. The variables (Y_{jt}, p_t^X, p_t^Z) are determined elsewhere in the model (and must therefore be adjusted in another way). Adjustment is therefore done as follows: (X_t, Z_t, p_{jt}) are exogenized and $(\mu_t^X, \mu_t^Z, m_{jt})$ are endogenized. Note that μ_{jt}^Y is not affected. Only the demand side μ's and the markups are affected by the adjustments.

One could imagine that an alternative strategy might be to adjust the supply side parameters μ_t^Y, but this would not work. Suppose the markups are given. For given output prices p_{jt} we would then be able to calculate P_{jt}^O from (26). We would then be able to calculate $P_t^O Y_t$ from the sum in (25). But this sum would typically not equal the total cost defined in (22). Endogenous markups are needed to capture the new cost structure of the energy supply system.

A.3 Practical implementation

In practice, the energy input good X_t can be thought of as a CES-aggregate of several energy inputs. The principle described above of exogenizing X_t and endogenizing μ_t^X can be extended for lower aggregate goods with no further complications.

For non-energy inputs Z_t this is not so. The energy sector model only describes non energy inputs as an aggregate cost compontent much like Z_t above, while in the CGE-model non-energy inputs is divided into various types of inputs, which are not aggregated together in a compund aggregate good seperate from energy inputs like Z_t. At the top nest of production structure for example, an aggregate of materials is nested together with an aggregate of energy, capital and labour[4].

For this reason, it is useful to write down the problem in a slightly different way. Suppose the production function is given by

$$
\begin{aligned}
Y_t &= A_t \left[\left(\mu_t^X\right)^{\frac{1}{E}} X_t^{\frac{E-1}{E}} + \left(\mu_t^Z\right)^{\frac{1}{E}} Z_t^{\frac{E-1}{E}} \right]^{\frac{E}{E-1}} \\
&= \left[\left(A_t^{E-1}\mu_t^X\right)^{\frac{1}{E}} X_t^{\frac{E-1}{E}} + \left(A_t^{E-1}\mu_t^Z\right)^{\frac{1}{E}} Z_t^{\frac{E-1}{E}} \right]^{\frac{E}{E-1}}
\end{aligned}
$$

where A_t is total-productivity.

The overall system will now look like this

$$
\begin{aligned}
X_t &= A_t^{E-1}\mu_t^X \left(\frac{p_t^X}{P_t^O}\right)^{-E} Y_t \\
Z_t &= A_t^{E-1}\mu_t^Z \left(\frac{p_t^Z}{P_t^O}\right)^{-E} Y_t \\
P_t^O Y_t &= P_t^X X_t + P_t^Z Z_t \\
Y_{jt} &= \mu_{jt}^Y \left(\frac{P_{jt}^O}{P_t^O}\right)^{-F} Y_t,\ j = 1, 2,\ F < 0, \\
P_t^O Y_t &= \sum_j P_{jt}^O Y_{jt}, \\
p_{jt} &= \left(1 + m_{jt}\right) P_{jt}^O,\ j = 1, 2
\end{aligned}
$$

Now suppose instead that in a projection we get data from the energy supply model for the energy variables $\left(X_t, Y_{jt}, p_t^X, p_{jt}, C_t\right)$, where C_t is the total cost calculated by the supply model (instead of Z_t and p_t^Z). We must thus add an equation:

$$C_t = P_t^X X_t + P_t^Z Z_t$$

Then the alignment is as follows: (X_t, C_t, p_{jt}) are exogenized and (μ_t^X, A_t, m_{jt}) is endogenized. This is a more general principle, based on the total cost rather than a specific input Z_t.

[4] See the note "Production technology in the CGE-model" for a describtion of the production structre in the CGE-model

B A CET supply system with markup's

This note gives an example of how markup's are introduced in a CET system. Markups are essential if you want to calibrate for real data. The method used is known from Dixit & Stiglitz (1977)[5], which is standard in macroeconomic models.

We consider a sector that produces 2 goods with 1 good as input. There are many identical firms in the sector. The individual firm produces an aggregate y with the production function:

$$y = \phi x \tag{28}$$

The firm produces 2 output goods y_i with the CET function

$$y = \left[\sum_i (\mu_i^y)^{\frac{1}{F}} y_i^{\frac{F-1}{F}} \right]^{\frac{F}{F-1}}, \; F < 0 \tag{29}$$

The firm faces 2 iso-elastic demand curves

$$y_i = \left(\frac{p_i^y}{P_i^y} \right)^{-V_i} Y_i, \; V_i > 1 \tag{30}$$

where p_i^y is the firms price of output commodity i, P_i^y is the average price of commodity i in the sector and Y_i is the average output in the sector.

The firm maximizes profits

$$\pi = \sum_i p_i^y y_i - p^x x$$

with (28), (29) og (30) as conditions.

You can rewrite the demand function (30) to:

$$p_i^y y_i = P_i^y y_i^{1-\frac{1}{V_i}} Y_i^{\frac{1}{V_i}}$$

such that

$$\pi = \sum_i P_i^y y_i^{1-\frac{1}{V_i}} Y_i^{\frac{1}{V_i}} - \frac{p^x}{\phi} y$$

We set up the Lagrange function

$$L = \sum_i P_i^y y_i^{1-\frac{1}{V_i}} Y_i^{\frac{1}{V_i}} - \frac{p^x}{\phi} y - \lambda \left(\left[\sum_i (\mu_i^y)^{\frac{1}{F}} y_i^{\frac{F-1}{F}} \right]^{\frac{F}{F-1}} - y \right)$$

The first-order conditions are:

$$\lambda = \frac{p^x}{\phi}$$

[5]Avinash K. Dixit and Joseph E. Stiglitz (1977), Monopolistic Competition and Optimum Product Diversity. The American Economic Review, Vol. 67, No. 3 (Jun., 1977), pp. 297-308.

and

$$\left(1-\frac{1}{V_i}\right) P_i^y {y_i}^{-\frac{1}{V_i}} Y_i^{\frac{1}{V_i}} = \lambda \frac{\partial}{\partial y_i} \left[\sum_i \left(\mu_i^y\right)^{\frac{1}{F}} y_i^{\frac{F-1}{F}} \right]^{\frac{F}{F-1}}$$

In a symmetric equilibrium where $y_i = Y_i$, we have:

$$\frac{P_i^y}{(1+m_i)\,P^O} = \frac{\partial}{\partial Y_i} \left[\sum_i \left(\mu_i^y\right)^{\frac{1}{F}} Y_i^{\frac{F-1}{F}} \right]^{\frac{F}{F-1}} \tag{31}$$

where the markup's m_i are defined by:

$$1+m_i \equiv \frac{V_i}{V_i-1}$$

and the "optimizing price" P^O is defined by

$$P^O \equiv \frac{p^x}{\phi}$$

The first order condition (31) will exactly be satisfied in the CET supply system:

$$Y_i = \mu_i^y \left(\frac{P_i^y}{(1+m_i)\,P^O} \right)^{-F} Y,\ i = 1, 2 \tag{32}$$

$$P^O Y = \sum_i \frac{P_i^y}{1+m_i} Y_i \tag{33}$$

This can be shown by inserting (32) i (31) and use (33).

Alternatively, this system can be written as:

$$Y_i = \mu_i^y \left(\frac{P_i^O}{P^O} \right)^{-F} Y,\ i = 1, 2$$

$$P^O Y = \sum_i P_i^O Y_i$$

$$P_i^y = (1+m_i)\,P_i^O,\ i = 1, 2$$

Also note that if (32) is inserted into (33) it can be realized that:

$$\begin{aligned} P^O &= \left[\sum_i \mu_i^y \left(\frac{P_i^y}{1+m_i} \right)^{1-F} \right]^{\frac{1}{1-F}} \\ &= \left[\sum_i \mu_i^y \left(P_i^O\right)^{1-F} \right]^{\frac{1}{1-F}} \end{aligned}$$