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Estimating the Constant Elasticity of Substitution when Technical Change is Time-Varying: A Kalman Filtering Approach*

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Abstract

This paper presents a new approach to, simultaneously, provide an estimate of factor elasticities as well as time-varying technical change utilizing the Kalman filter. Using a simulation exercise we show that this approach performs well for different types of non-linear technical change. The method is subsequently applied on Danish macroeconomic data from 1966-2016 for the private sector and at the sectoral level. Using a nested CES production function we find that all inputs in production are gross complements. The estimates suggest that the relevant elasticity of substitution between capital and labor is around 0.5 for the private sector. Moreover, technological change in the private sector has been labor-augmenting in the long run, thus supporting an assumption of Harrod-neutral growth in the long run. However, we find important "medium run" fluctuations and periods where technical change has been capital-augmenting. This may be driven by periods of relatively slow labor productivity growth in the service sector in the early 1990's and after the financial crisis.

Keywords: Biased Technical Change, Medium Run, Factor Substitution, Constant Elasticity of Substitution, Kalman Filter.

JEL: C32, E25, O33.

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1 Introduction

The elasticity of substitution between production factors is a central parameter in macroeconomic models. It affects the speed of output adjustment to shocks when factor prices are affected asymmetrically and the induced price effects of tax or labor market reforms. Over time, it affects the distribution of income, for example between capital and labor as well as cross-sectional employment. In the presence of augmenting (so-called "biased") technical change, the elasticity of substitution determines how firms adjust demand for production factors in response to shifts in their relative efficiency.

Many empirical papers have estimated the elasticity of substitution, typically between labor and capital. Although a central parameter, no consensus value has been established, which may to a large extend be attributed to the identifying assumptions on technical change.[1] In a widely cited paper, Berndt (1976) assumes Hicks-neutral technical change and suggests a unitary value of this elasticity for the US economy, hence providing support for the use of a Cobb-Douglas production function. Later work, for example Antras (2004) and Leon-Ledesma et al. (2015), challenge this and show that the assumption of Hicks-neutrality will bias the estimated elasticity towards one when the true technical change is biased. More recently, it has been acknowledged that technical change, while labor-augmenting in the long run, might be characterized by prolonged periods where it is instead capital augmenting. To allow for dynamics during what Blachard (1997) has dubbed "the medium run", a series of papers (e.g. Klump et al. 2007; 2008 and McAdam and Willman, 2013) depart from the linear trend assumption in earlier work by using a Box-Cox transformation of the growth rates to incorporate time-varying (non-linear) technical change. Although the Box-Cox transformation is able to produce medium run fluctuations, it can only account for accelerating or decelerating growth rates. To get labor augmenting technology in the long run, while allowing for multiple periods of capital augmenting technology, a more flexible approach is needed.

We propose a framework to estimate the constant elasticity of substitution (CES) in a production function with time-varying technical change by using the fact that the problem has a natural state space representation. Thus, the Kalman filter allows us to simultaneously obtain an estimate of the CES elasticity as well as the (unobserved) relative augmenting technical change. We show how the identification is based on a smoothness restriction on technical change which replaces a full parametric specification of the time trends. We also show that our approach nests the constant growth assumption widely applied in empirical work. The approach is fairly easy to implement as an alternative to models with parametric assumptions about the technical change.

The performance of our approach is assessed in a simulation exercise using three different specifications of technical change: One linear case and two nonlinear cases. The Kalman filter

[1] See for example Knoblach and Zwerschke (2016) for a meta regression analysis using 738 estimates from 41 different studies.

performs well: It is able to recover the true elasticity on the median for all three trend specifications. Further, in the cases of non-linear technical change, the distribution of the estimates is much more narrowly centered around the true value, compared to a model which (incorrectly) use a linear trend assumption. The simulation exercise also shows that when the true process of technical change is a Box-Cox transformation the Kalman filter is able to reproduce the true estimates on the median. Finally, the simulation exercise suggests that our approach is robust to misspecification with regards to the degree of smoothing.

After the methodological contribution of the paper, we subsequently show how the method can be applied empirically. Using Danish aggregated and sectoral data for the period 1966-2016, the elasticity of substitution is estimated while allowing for time-varying technical change. We use a nested CES production function in which energy, capital (machinery, transportation and inventory), labor, buildings and materials, respectively, enter as inputs in production.[2] To summarize the results, all factors enter the production function with an elasticity on the unit interval, meaning that they are gross complements rather than substitutes. Specifically, the point estimates of the elasticity of substitution between the capital-energy nest and labor are in the range 0.2-0.5. Moderate substitution between energy and capital is found, whereas buildings and materials are approximately Leontief factors in the production. These general conclusions hold both at the aggregate private sector level and for the two main sectors in Denmark (service and manufacturing). The results shows that, while technical change in the private sector has been labor-augmenting in the long run, there has been periods where it has been capital, during the 1990's and after the financial crisis in 2008. The former is in accordance with the results in Klump et al. (2008) who in part ascribes this structural shift to the IT boom. Interestingly, the latter coincides with a substantial increase in markups following the crisis. Overall, our findings support the assumption that technical change is labor augmenting in the long run, but that there are prolonged periods with capital augmenting technical change.

Since the true signal-to-noise ratio in the data is unknown this begs the question whether the chosen smoothness of technical change is appropriate. As a starting point, we estimate both variances freely with maximum likelihood. However, in some cases this leads to a misspecified model, based on a normalized innovations squared and an autocorrelation test. When this is the case, we perform a grid search for different degrees of smoothing and choose the model that maximizes the likelihood value conditional on being well specified. We show that the estimated elasticities are generally robust to moderate variations in the signal-to-noise ratio around our benchmark specification, however they tend to be increasing in the degree of smoothness. Thus, this paper reiterate the findings in Chirinko and Mallick (2017), that the elasticity is bound to the

[2]The nesting structure is based on the one used in the Danish macroeconomic model MAKRO and does not affect the methodological contribution of this paper.

filtering assumptions made. However, while their filtering assumptions are made on the smoothing of prices prior to estimation, we smooth technical change as part of the estimation.

The remainder of this paper is structured as follows: In Section 2 we take a closer look at some stylized facts for the Danish economy and motivate the use of time-varying technical change. The firm problem is presented in Section 3 along with the econometric framework. The performance of the framework is tested in a simulation exercise in Section 4 and later applied on Danish aggregate data in Section 5, where we also perform a range of diagnostics tests and analyze the appropriateness of the identification strategy. Section 6 concludes.

2 The case for time-varying technical change: A first look at Danish data

As emphasized in Klump et al. (2008), the assumption of constant growth rates in technical change, often assumed in the empirical literature, might not be appropriate for the Euro Area. In this section we present some suggestive evidence that this holds true for Denmark as well. To borrow a term from Blachard (1997), there might be important “medium run” properties in the data which we want our subsequent estimation to reflect.

Figure 1 shows a set of Danish macroeconomic variables. The Danish unemployment rate was generally increasing in the 1970’s and 80’s but shows a declining trend from the early 1990’s until the financial crisis in 2008 (panel a). Similar observations are made for the European countries in Klump et al. (2008) who argue that the increasing unemployment rate might be subscribed to an increasing unit labor cost, i.e. failure of wages to adjust to the productivity growth. A similar argument is put forward by Blachard (1997) who refers to this as a wage push shock. Panel b shows the growth rate of the wage relative to the user cost in Denmark. The average relative growth rate was 5.5% for the period 1969-1987 and only 1.9% in the following period, hence showing a somewhat similar pattern as in the Euro Area.

How do these observations relate to technical change? Acemoglu (2002, 2003) analyses the properties of an endogenous growth model where firms can invest in factor-augmenting technologies. He shows that while technology is Harrod-neutral along the balanced growth path, profit-maximizing behavior leads to transition periods where technology can be capital-augmenting if the cost of capital is relatively high or labor is an abundant resource. The data in Figure 1 seems to corroborate with this theory: In the first part of the sample (1969-1987), labor productivity in the private sector grew at a relatively high rate, 4.2% on average (panel c) whereas that of capital was low or even decreasing. In the period after 1987 (where the growth rate in the wage relative to the user cost was lower), the growth in labor productivity fluctuated around a constant level of

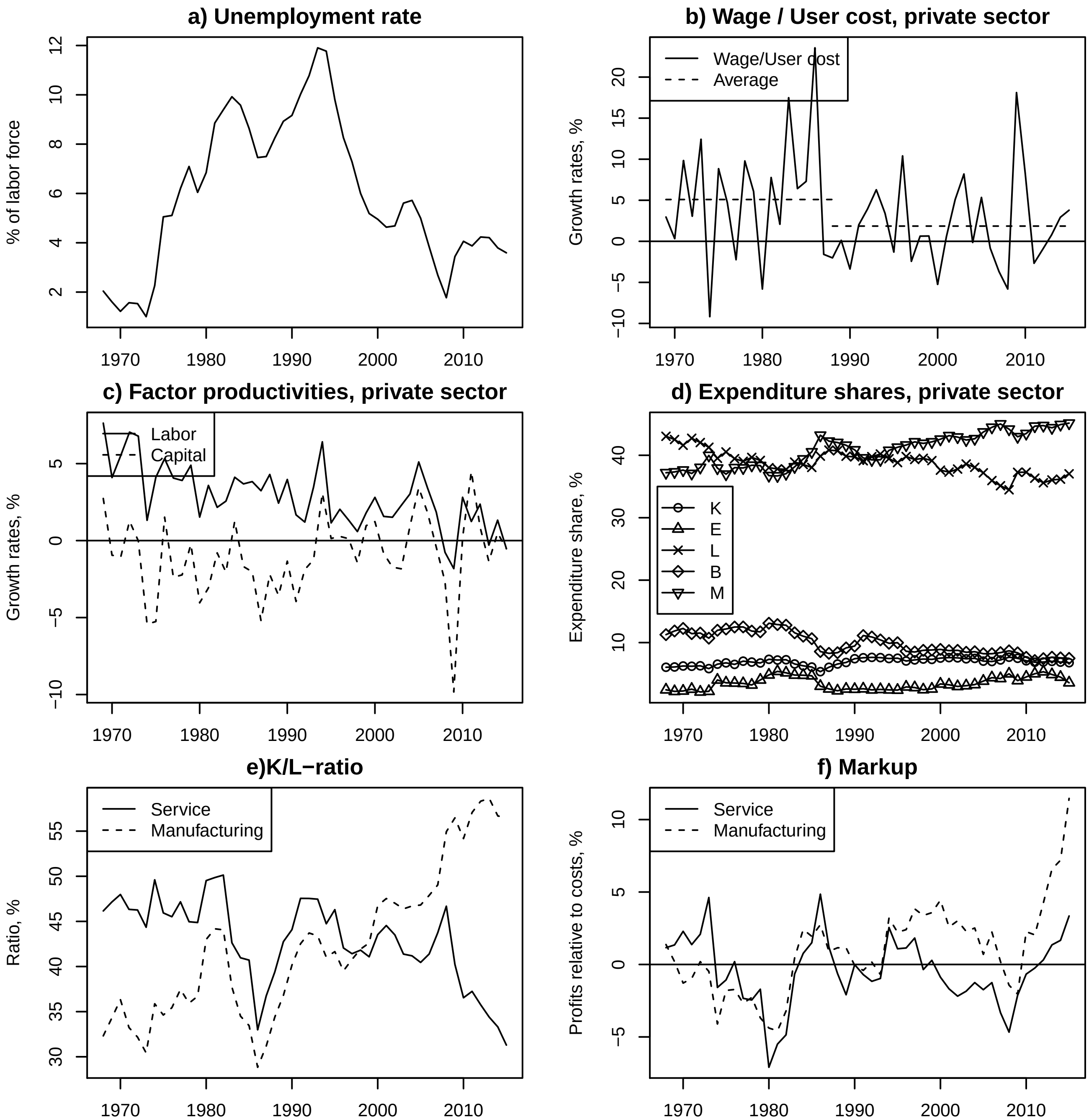

Figure 1: Aggregate Danish data. a) Unemployment rate b) Average wage growth rate relative to growth rate of the user cost of capital c) Factor productivities d) Expenditure shares in gross output. e) K/L ratio measured as share of capital expenditure relative to labor expenditure. f) Markups, calculated assuming constant returns to scale.

approximately 2.5% until the financial crisis, whereas the growth rate of capital productivity was generally increasing.[3] The decreasing ICT costs in the 1990's during the IT-boom is mentioned by several researchers as a specific example of capital-augmenting technical change (Klump et al., 2008; Spiezia et al., 2016).

It is well-known that the unitary elasticity of the Cobb-Douglas production function leads to constant factor shares. In addition, the direction of technical change would not affect total factor income. This assumption is however hard to reconcile with Figure 1 (panel d), which exhibits persistent fluctuations in factor expenditure shares. Specifically, increasing shares of (machinery) capital and materials are observed, while the opposite is true for labor and buildings. A similar decrease in the labor share is seen in several other European economies as well, see e.g. Arpaia et al. (2009) and Karabarbounis and Neiman (2014). The latter find that around 50% of the decrease in the labor share across countries can be subscribed to changes in the relative price, indicating an important role for relative technological change. The rejection of the "stylized fact" of a constant K/L-ratio (panel e) is another case in point, in particular for manufacturing, which has seen an upward trend in the K/L-ratio from approximately 28% in 1986 to 55% in 2015. For services it might be argued that the long run K/L-ratio is constant but with important medium run variations, for an example a large decrease from 1980-1986 and after the financial crisis. This pattern is hard to reconcile with a constant growth rate of technical change as we shall see below.

Finally, while somewhat noisy, the markups in both sectors show considerable persistency over time (panel f).[4] The increase in markups after the financial crisis is particularly pronounced for Denmark but echoed in several other economies (De Loecker and Eeckhout, 2018). De Loecker et al. (2019) examine the macroeconomic implications of rising markups and find a negative relation with the labor share at the firm level. Hence, to the extend that markups affects the relative factor demands, the persistency in the former observed in Figure 1 might also lead to a violation of the linear trend assumption.

To summarize, it is obviously hard to determine what exactly drives the direction of technical change (and we do not attempt to do so in the above). However, when inspecting the data, at least one conclusion can be drawn: When estimating factor demands it is potentially important to allow the econometric model to reflect medium run fluctuations through time-varying changes in the direction of technical change. This motivates the model presented in the next section where technical change is allowed to grow non-linearly while still being specified in a flexible, data driven way.

[3]One might have expected the growth rate of labor productivity to have picked up following the persistent decline in unemployment decreased after 1993. However, Denmark has undergone a series of labor market reforms which are widely believed to have lowered the structural unemployment rate significantly (see for example Andersen and Svarer, 2008). As a result, the unemployment rate might not fully reflect the degree of labor market slack.

[4]The markup is calculated as profits relative to expenditures. This approximation only holds if there is constant returns to scale in production, which is assumed here for simplicity.

3 Model specification and identification strategy of technical change

3.1 The firm problem and the bias of technical change

It is assumed that the firm produces in accordance with a nested CES production function and substitute between two factors in each nest. This specification has the advantage that it allows for different elasticities of substitution between the factors in the production process. First, the firm chooses between capital (K) and energy (E). Secondly, the firm chooses between the aggregate of K and E (KE) against labor (L). Thirdly, how much to spend on buildings. Finally, the firm chooses how much to produce itself (KELB) and how many materials to buy from other firms (M). In short, the firm produces in a KELBM structure, which is the notation used in the rest of the paper. The production in each nest is given by:

$$Y_t = \left[(\Gamma_{1t} X_{1t})^{\frac{\sigma-1}{\sigma}} + (\Gamma_{2t} X_{2t})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where Y_t is output, X_{1t} and X_{2t} are two production factors in a given nest, and σ is the constant elasticity of substitution. Γ_{1t} and Γ_{2t} represent augmenting technical change of the first and second factor, respectively. The ratio $\frac{\Gamma_{1t}}{\Gamma_{2t}}$ is the relative augmenting technical change of the two factors of production. Let $g(\Gamma_{1t})$ and $g(\Gamma_{2t})$ be the growth rates of the factor augmenting technologies. If $g(\Gamma_{1t}) > g(\Gamma_{2t})$ technical change is augmenting the first production factor, whereas $g(\Gamma_{1t}) < g(\Gamma_{2t})$ implies that technical change is augmenting the second factor. Assuming profit maximizing firms the relative expenditure shares for the two production factors derived from the firms cost minimization problem is:[5]

$$log\left(\frac{P_{1t}X_{1t}}{P_{2t}X_{2t}}\right) = (\sigma-1)\, log\left(\frac{\Gamma_{1t}}{\Gamma_{2t}}\right) + (1-\sigma)\, log\left(\frac{P_{1t}}{P_{2t}}\right), \tag{2}$$

where P_{1t} and P_{2t} are the prices of the two factors. Equation (2) illustrates how biased technical change and changes in the relative price interact with the elasticity of substitution: When $\sigma < 1$ the production factors are gross complements and when $\sigma > 1$ they are gross substitutes. Two special cases emerge when $\sigma \to 1$ where production is Cobb-Douglas, resulting in constant relative expenditure shares, and $\sigma = 0$ where production is Leontief and there is no substitution between the production factors due to price changes (perfect complements). Note that Hicks neutral technical change, affecting both technology factors in the same way, will not affect the *relative* technology level in (2) and hence does not affect the *relative* expenditure shares. Thus, we are only able to

[5]Using this formulation has the advantage that it is less plausible that there are correlated measurement errors on both the right and left hand side, which could bias the elasticity estimate (Kemp, 1962).

identify biased technical change in the relative series. As an example, consider the second nest with KE as the first factor and L as the second factor: If $g(\Gamma_{KEt}) > g(\Gamma_{Lt})$, technical change is augmenting KE. If $\sigma < 1$, technical change is biased towards L, whereas technical change is biased towards KE when $\sigma > 1$. This highlights the well-known result that the bias of technical change depends on the elasticity of substitution and therefore, they have to be estimated jointly.

3.2 Obtaining the state space representation of the model

In order to apply the Kalman filter the model needs to be specified as a linear state space model. Equation (2) is a static equation as it does not contain any dynamics. Thus, it is explicitly assumed that the economy is in a long run equilibrium and is the static long run solution from the Engle-Granger two-step procedure. However, in macroeconomic modeling, adjustment costs are often imposed to ensure that there are lags in the response of quantities to relative price changes (e.g. Christiano et al., 2005 and Smets and Wouters, 2007). Consequently, using (2) might result in a small sample bias due to the relatively short sample used in our application. Therefore, an error-correction model is applied to allow for short run dynamics in factor shares.[6] Specifically, we estimate the equation:

$$\Delta s_t = \alpha\left(s_{t-1} - \beta p_{t-1} - \mu_{t-1}\right) + \sum_{i=0}^{k} \kappa_i \Delta p_{t-i} + \sum_{i=1}^{k} \omega_i \Delta s_{t-i} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Sigma^{\varepsilon}) \tag{3}$$

where $s_t \equiv log\left(\frac{P_{1t}X_{1t}}{P_{2t}X_{2t}}\right)$ is the relative expenditure share, $p_t \equiv log\left(\frac{P_{1t}}{P_{2t}}\right)$ is the relative (log)prices and $\mu_t \equiv (\sigma - 1)\, log\left(\frac{\Gamma_{1t}}{\Gamma_{2t}}\right)$ captures the relative technology level. The speed of adjustment to the long run equilibrium is determined by α, $\sigma = \beta + 1$ is the long run elasticity and κ_i and ω_i are the short run elasticities with respect to prices and expenditure shares, respectively. The optimal lag length (k) is determined such that there is no autocorrelation in the measurement errors, ε_t. The variance Σ^{ε} is estimated by a recursively application of the maximum likelihood estimator.

Next, the dynamics of μ_t is specified to satisfy three requirements: First, since the relative factor price contains a trend in many of the estimations, a trend should be allowed (implicitly to allow for $\sigma \neq 1$). Second, we deviate from the linear trend assumption to account for the medium-run fluctuations in factor expenditures. Third, μ_t should be a slow moving process, meaning it cannot reflect the year-to-year errors between the data and the model. This last characteristic is indeed how we define and identify movements in factor efficiency.[7] Specifically, we assume the

[6] An early adoption of the error-correction approach is found in Caballero (1994).

[7] Previous empirical work on Danish data has generalized this assumption by using higher-order polynomials (Gustafsson, 2014) or logistic functions (Thomsen, 2015).

following I(2)-process for μ_t:

$$\Delta\mu_t = \Delta\mu_{t-1} + \eta_t, \qquad \eta_t \sim N(0, \Sigma^\eta). \tag{4}$$

where the variance of η_t is $\Sigma^\eta = \Sigma^\varepsilon/\lambda$. Thus, ε_t captures temporary deviations from the long run equilibrium, whereas η_t captures structural changes in the relative augmenting technical change. The trend specification in (4) is a special case of the Local Trend Model where the variance of the drift term has been set to zero and the result is a model with a smooth trend which is how we identify technical change. The degree of smoothness of technical change is determined by (the inverse of) the signal-to-noise ratio, λ. Very low degrees of smoothing would ascribe almost the entirety of unexplained year-to-year fluctuations in factor demand to changes in technology[8], whereas very high degrees of smoothing imposes a linear trend assumption on the model.

As a starting point, this parameter is estimated with maximum likelihood (25 different initial values in the numerical optimization is used to avoid local minima). However, for some nests, the resulting model is not well-specified, based on a normalized innovations squared (NIS) test and a Breusch-Godfrey test for autocorrelation. Typically, too low degrees of smoothing lead to filter inconsistency (overfitting) whereas too much smoothing results in autocorrelated residuals (trend is too restrictive). In this case, a grid of more than 50 different values of λ ranging from 10 to 1,000 is tried and the one that maximizes the likelihood *conditional* on being well specified is chosen.[9] While the specification in (3) and (4) has the significant advantage that we do not need to impose a functional parametric form of technical, the above discussion highlights that an identifying restriction still needs to be imposed. The sensitivity of our results with respect to λ is examined in Section 5.3.

The elasticity and adjustment parameter are specified as unobservables with a zero variance in the state space representation. Therefore, no standard errors are immediately obtained when using the Kalman smoother. Instead, we use a standard residual-based fixed-design bootstrapping procedure with 1,000 iterations to obtain confidence intervals, which is valid given that the model's innovations are neither autocorrelated nor heterosedastic (this is verified in the empirical application in Section 5).[10] The confidence bands are reported by the 5% and 95% quantiles instead of

[8]This is often referred to as "dynamic calibration" in the realm of CGE-models where parameters are calibrated each year to match the data set.

[9]A 10% significance level is used throughout the paper to acknowledge that we are working with a relatively small sample. As the Kalman filter can be sensitive to starting values in small samples 10 different values of the elasticity parameter σ and the adjustment parameter α are tried (i.e. a grid of a total of 100 combinations of initial parameter values). The starting values that minimize the AIC are chosen. However, for most nests we find the estimation procedure to be relatively stable, meaning that it converges to the same optimum for most of the starting values. In addition, we have restricted the point estimate of the elasticity to be non-negative.

[10]One could also consider a Monte Carlo procedure to obtain confidence intervals as described in Hamilton (1986). However, this procedure only works when λ is estimated freely, as the absolute noise level in the filter does not affect the estimates.

the standard errors, as the distribution of the bootstrapped parameters might be non-normal and not necessarily centered around the estimated parameter values.[11]

4 Simulation evidence

As illustrated in Section 2, technical change in Denmark is unlikely to be linear, thus violating the common linear trend assumption. In this section, we perform a simple simulation study to show that in such cases, the Kalman filter will provide a more accurate measurement of the elasticity of substitution than the linear trend assumption. Specifically, we analyze the factor shares of two generic factors and try to match the data generating process (DGP) with the moments of capital and labor as well as the expenditure share observed in the datasets used in the empirical analysis. Appendix A includes a detailed description of how the data is simulated.

Technology is specified such that it consists of a deterministic and a stochastic trend. We vary the relative importance of these by adjusting the signal-to-noise ratio ($\tilde{\lambda}^{-1}$) in the DGP while keeping the total variance constant. Three different specifications of the deterministic trend are considered: The first is the constant Harrod-neutral growth rate assumption. The second specification is also a constant growth rate but with a break in the augmenting technology, i.e. Harrod-neutrality in the first half of the sample and Solow-neutrality in the second half. The last specification employs a Box-Cox transformation of the growth rates to account for medium run fluctuations. The parameters are specified such that the growth rate of labor productivity is declining, whereas the growth rate of capital productivity is increasing, similar to what might be observed empirically. 1,000 data series of 50 observations are simulated, consistent with what is used in the empirical analysis.

The estimation methodology is as described in Section 3.2. The smoothing parameter in the Kalman filter is fixed at $\lambda = 100$, irrespective of the true signal-to-noise ratio in the DGP.[12] The results of the first specification are shown in Table 1. On the median, both the linear trend and the Kalman filter suceed in replicating the true elasticities. However, when $\tilde{\lambda}$ is low (implying that the stochastic component in the technology process is dominant) the precision of the Kalman filter is far better than the linear trend assumption based on the distance between the quantiles. When technology approaches a linear trend (i.e. when $\tilde{\lambda}$ is high) the linear trend performs slightly better than the Kalman filter.

Next, consider the second specification with a break after 25 observations (Table 2). In this

[11] The parameter estimates of σ in the bootstrapping procedure is restricted to be in the range between -0.75 and 1.75 to discard extreme outliers, likely due to numerical convergence problems. The "acceptance ratio" obtained was fairly close to 1 anyway.

[12] In the empirical analysis it is confirmed that $\lambda = 100$ do in general lead to a well-specified model across different sectors and nests.

	σ= 0.2	σ= 0.5	σ= 0.9	σ= 1.3
	$\tilde{\lambda}=1$			
Linear trend	0.19 (−0.23;0.68)	0.49 (−0.14;1.19)	0.89 (0.14;1.69)	1.30 (0.57;2.03)
Kalman smoother	0.20 (−0.04;0.49)	0.50 (0.14;0.91)	0.90 (0.48;1.37)	1.30 (0.89;1.70)
	$\tilde{\lambda}=10$			
Linear trend	0.20 (0.04;0.37)	0.50 (0.27;0.75)	0.90 (0.64;1.18)	1.30 (1.05;1.58)
Kalman smoother	0.20 (0.07;0.34)	0.50 (0.31;0.70)	0.90 (0.67;1.13)	1.30 (1.09;1.51)
	$\tilde{\lambda}=100$			
Linear trend	0.20 (0.12;0.29)	0.50 (0.38;0.63)	0.90 (0.76;1.05)	1.30 (1.17;1.44)
Kalman smoother	0.20 (0.09;0.31)	0.50 (0.35;0.65)	0.90 (0.72;1.08)	1.30 (1.14;1.46)
	$\tilde{\lambda}=1000$			
Linear trend	0.20 (0.13;0.28)	0.50 (0.40;0.61)	0.90 (0.78;1.03)	1.30 (1.19;1.42)
Kalman smoother	0.20 (0.10;0.30)	0.50 (0.35;0.65)	0.90 (0.73;1.08)	1.30 (1.14;1.46)

Table 1: Estimated (median) elasticities on the simulated data using a constant growth rate specification with Harrod-neutral technology. 1,000 simulations are used and 50 observations. 5% and 95% quantiles are included in paranthesis. $\tilde{\lambda}$ is the inverse of the signal-to-noise ratio used in the data generating process.

	σ= 0.2	σ= 0.5	σ= 0.9	σ= 1.3
		$\tilde{\lambda} = 1$		
Linear trend	0.29 (−3.03;4.58)	0.52 (−0.79;1.79)	0.89 (0.14;1.68)	1.29 (0.32;2.22)
Kalman smoother	0.20 (−0.15;0.57)	0.50 (0.12;0.93)	0.90 (0.48;1.37)	1.31 (0.87;1.72)
		$\tilde{\lambda} = 10$		
Linear trend	0.26 (−2.63;3.05)	0.54 (−0.22;1.27)	0.90 (0.61;1.22)	1.29 (0.83;1.78)
Kalman smoother	0.21 (−0.05;0.49)	0.50 (0.29;0.73)	0.90 (0.68;1.13)	1.30 (1.08;1.51)
		$\tilde{\lambda} = 100$		
Linear trend	0.27 (−2.95;3.38)	0.53 (−0.11;1.17)	0.90 (0.70;1.10)	1.29 (0.91;1.70)
Kalman smoother	0.21 (−0.02;0.46)	0.50 (0.33;0.68)	0.90 (0.72;1.08)	1.30 (1.13;1.48)
		$\tilde{\lambda} = 1000$		
Linear trend	0.27 (−2.49;3.09)	0.54 (−0.12;1.15)	0.91 (0.73;1.08)	1.29 (0.91;1.69)
Kalman smoother	0.20 (−0.02;0.45)	0.51 (0.33;0.67)	0.90 (0.73;1.07)	1.30 (1.13;1.47)

Table 2: Estimated (median) elasticities on the simulated data using a constant growth rate specification with Harrod-neutral technology in the first half of the sample and Solow-neutral in the second half. 1,000 simulations are used and 50 observations. 5% and 95% quantiles are included in paranthesis. $\tilde{\lambda}$ is the inverse of the signal-to-noise ratio used in the data generating process.

	σ= 0.2	σ= 0.5	σ= 0.9	σ= 1.3
	$\tilde{\lambda}=1$			
Linear trend	0.21 (−0.69;1.03)	0.50 (−0.31;1.28)	0.89 (0.12;1.70)	1.31 (0.51;2.10)
Kalman smoother	0.20 (−0.05;0.49)	0.50 (0.14;0.91)	0.90 (0.48;1.37)	1.30 (0.88;1.71)
	$\tilde{\lambda}=10$			
Linear trend	0.22 (−0.29;0.73)	0.50 (0.13;0.90)	0.90 (0.63;1.20)	1.31 (0.99;1.64)
Kalman smoother	0.20 (0.06;0.34)	0.50 (0.31;0.70)	0.90 (0.67;1.13)	1.30 (1.09;1.51)
	$\tilde{\lambda}=100$			
Linear trend	0.22 (−0.23;0.67)	0.51 (0.19;0.81)	0.90 (0.74;1.06)	1.30 (1.08;1.52)
Kalman smoother	0.20 (0.09;0.32)	0.50 (0.35;0.66)	0.90 (0.72;1.08)	1.30 (1.13;1.46)
	$\tilde{\lambda}=1000$			
Linear trend	0.22 (−0.22;0.66)	0.51 (0.22;0.79)	0.90 (0.77;1.04)	1.30 (1.09;1.51)
Kalman smoother	0.20 (0.09;0.31)	0.50 (0.35;0.65)	0.90 (0.73;1.07)	1.30 (1.14;1.46)

Table 3: Estimated (median) elasticities on the simulated data using a Box-Cox transformation of the growth rates of technology. 1,000 simulations are used and 50 observations. 5% and 95% quantiles are included in parenthesis. $\tilde{\lambda}$ is the inverse of the signal-to-noise ratio used in the data generating process.

case, the linear trend assumption is far less accurate on the median than the Kalman filter in all specifications, reflected in wider confidence bands. The difference is most pronounced when the true elasticity is furthest away from a Cobb-Douglas productions function. This is of course not particularly surprising but highlights the fact that a misspecified process for technical change most severely (adversely) affects the estimates when the effect of technical change is large (σ being low).

Finally, we employ a Box-Cox transformation of the growth rates (Table 3). Again, the Kalman filter performs better than the linear trend on the median and on the width of the confidence bands - in particular when $\tilde{\lambda}$ is low. To conclude, the Kalman filter performs almost as accurate as the linear trend assumption when technology is close to linear, and the Kalman filter is far more accurate when technology is non-linear, including the popular Box-Cox transformation. It should be notet that, even when the signal-to-noise ratio used in the filter is not correct, the estimates are still fairly precise. Being convinced of the flexibility and robustness of our method, we move on to the empirical application.

5 Empirical application

In this section, we apply the framework presented in Section 3 on Danish data. In Section 5.1 we describe how the data is generated and our data source. In Section 5.2 we present the main results. Lastly, Section 5.3 includes a detailed analysis and discussion of the validity of the identification strategy.

5.1 Data

The data source is Statistics Denmark from which annual time series for the period 1966-2016 is obtained. We use data from the private sector as a whole (leaving out the public sector), as well as the manufacturing and private services sectors. The variables are defined analogous for all sectors used. As a measure of capital we use two types, which is separated: Machinery, transportation and inventory capital and building capital. The user cost of capital is calculated as shown in Appendix B. This user cost is used to calculate the price of both types of capital. Specifically, a "steady state" user cost (in the absence of installation costs) and a user cost with adaptive expectations are calculated (which includes installation costs). The steady state user cost is used in the long run relation of (3) and the user cost with adaptive expectations is used in the short run relation. This distinction mimics typical specifications used in DSGE models, where adjustment of the capital stock is restricted by some cost function in the short run, while this cost is usually zero in the steady state. As a measure of demand for energy we used the quantity spent on energy and the measure of materials is given as all materials used in production excluding energy. Number of working hours are used as a measure of labor input and the price of labor is the hourly wage.

The price indices in the nests KE, KEL and KELB are calculated as a Paasche index. This index has the advantage that it is close to the CES price index. The quantities are derived through a zero profit condition. In the last nest (the nest between KELB and M), the production volume is used as measure of output and the price is given as the price of total production adjusted by a markup. In this way, the zero profit condition holds given an optimizing price. The time series used for estimation are shown in Figures 3-5 (see Appendix C).

5.2 Results

In this section, we present our results using the Danish datasets. The estimated elasticities for the private sector are shown in Table 4 and the relative augmenting technical change $\left(\frac{\Gamma_{1t}}{\Gamma_{2t}}\right)$ is shown in Figure 2. The elasticity of substitution between K and E is estimated to 0.24 and significantly different from zero and unity. The elasticity of substitution between KE and L is found to be relatively high at 0.49, which can be explained by the fact that the capital stock is very persistent.

The estimate is associated with some uncertainty as the distance between the quantiles is quite large. Consequently, we cannot reject that it is different from unity. The elasticity of substitution between KEL and B is found to be small (0.15) and not significantly different from zero. Therefore, we find that it is weakly identified from an economic perspective. It cannot be rejected that the estimated elasticity of substitution between KELB and M is zero, but it is different from unity. Thus, based on the point estimates buildings and materials might be a Leontief input in production.[13] The detailed results for the manufacturing sector and private services are shown in Appendix D. The elasticity between KE and L is estimated to 0.19 in the manufacturing sector which is somewhat low. It should however be noted that the likelihood function is very flat for all λ in the range of 10-80. Thus, this particular nest is associated with significant uncertainty and the elsaticity could plausibly be in the range of 0.19-0.83. The corresponding estimate for private services is 0.5, close to that of the total private sector in Denmark.

Are these results consistent with the literature? Thomsen (2015) estimate the Danish elasticity of substitution between KE and L in the range 0.32-0.63 in the same sectors that we use. Gustafsson (2014) estimate the elasticity between K and L to 0.25 in manufacturing and 0.33 in services (both papers unfortunately in Danish). Other estimates on Danish data include Muck (2017) who estimates the elasticity in the range of 0.3-0.7 when allowing for non-linear technical change by a Box-Cox transformation and 1.5 when assuming a linear trend. Estimates for the Euro area include Klump et al. (2008) who use a Box-Cox transformation and find the elasticity to be 0.65 and similarly McAdam and Willman (2013) who estimate the elasticity to 0.88. Generally, CES estimates are lower when allowing for non-linear technical change. Finally, the estimates vary widely depending on the assumption of technical change, highlighting the importance of allowing for a flexible data-driven approach to estimate the elasticity and the direction of technical change.

Next we turn to the direction of technical change. Since our estimates of the elasticity of substitution are all below unity, the bias of technical change goes in the opposite direction as the augmenting technology. For all sectors, the augmenting technical change of capital relative to energy has been decreasing (Figure 2 is the private sector, Figure 6 is manufacturing and Figure 7 is private services both shown in Appendix E). Likewise, for all sectors, the augmenting technical change of KE relative to L has been labor augmenting in the long run, but biased towards KE. However, the growth rate of labor augmenting technical change relative to capital augmenting technical change has been decreasing throughout the sample period, consistent with the factor productivities in Section 2. There are however important medium run fluctuations as the relative augmenting technical change in the private sector was increasing in the 90s and again in the end of the sample period where investments are known to be unusually low after the financial crisis.

[13]This is in accordance with Gustafsson (2014), who restrictes both the substitution elasticity between KLE and B and the elasticity between KLEB and M to zero.

	(K)E	(KE)L	(KEL)B	(KELB)M
σ	0.24 (0.12;0.49)	0.49 (0.36;1.51)	0.15 (−0.13;0.81)	0.00 (−0.48;0.60)
α	−0.31 (−0.38;−0.16)	−0.17 (−0.20;−0.06)	−0.24 (−0.33;−0.07)	−0.30 (−0.43;−0.12)
Likelihood	98.67	133.84	128.61	125.48
λ	872	30	85	380
Autocorrelation	[0.94]	[0.14]	[0.10]	[0.10]
Heteroskedasticity	[0.80]	[0.33]	[0.05]	[0.50]
Normality	[0.40]	[0.75]	[0.77]	[0.32]
NIS	0.86	0.73	0.82	0.87

Table 4: Private sector: Estimated results in a (((KE)L)B)M nest structure. Terms in parenthesis are the lower and upper critical values, respectively, on a 10% significance level. Terms in brackets are p-values for the misspecification tests. The critical values of the Normalized Innovations Squared test (NIS) are $[0.68; 1.37]$ on a 10% significance level.

This development offer some support for the motivation in Section 2 of directed technical change.

In manufacturing, technical change was capital augmenting in the 80's, possibly due to an increase in unemployment (Blachard, 1997). In the service sector, technical change was capital augmenting in the 90's, possibly due to the IT boom (McAdam and Willman, 2013)[14], expected to influence service the most. For both manufacturing and services a change in the direction of technical change after the financial crisis can be observed. This might be due to low investments and increasing markups.

The augmenting technical change of KEL relative to B has been steadily increasing with a linear trend until 1990, where a slow-down is observed, possibly due to a declining growth in labor productivity. Lastly, the augmenting technical change of KELB against M has been increasing with a linear trend during the whole sample period implying that technical change is materials biased. In general we observe that our method allows for several periods of medium run fluctuations, declining relative growth rates and an approximately linear trend. These results illustrate the potential gains from the flexibility of our setup which would arguably not be obtained by an a priori parametric restriction on technical change.

5.3 Robustness of the identification strategy

The performance and consistency of the filter relies on the model being well specified. This is secured by construction in our framework as only values of λ that lead to a well specified filter are considered. The filter performance and consistency can further be evaluated graphically by

[14]McAdam and Willman (2013) points out that a non-constant growth of technical change is a key component at understanding Euro-area medium run phenomena.

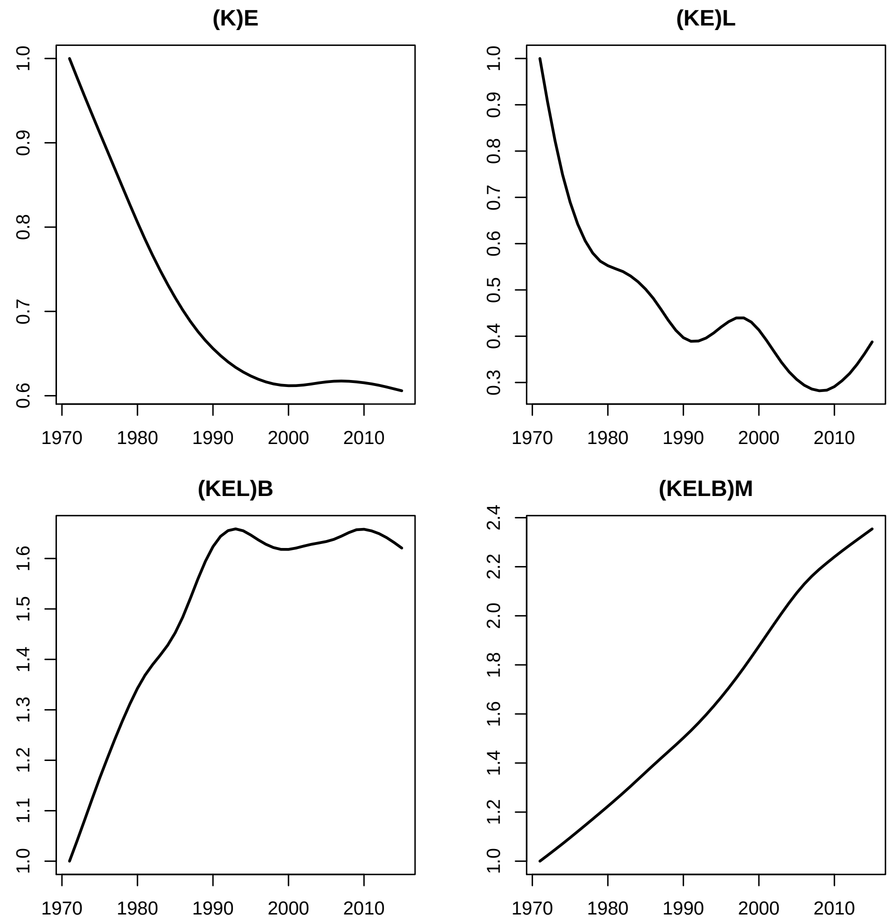

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Figure 2: Private sector: Relative augmenting technical change in a (((KE)L)B)M nest structure. Parenthesis indicates the nested factors. That is, the graph shows the relative augmenting technologies of e.g. capital relative to energy. Index year is 1971.

inspecting the standardized innovations (these are shown in Figures 8-10 in Appendix F). Approximately 10% of the standardized innovations fall outside the 90% confidence intervals and they appear to be drawn from a zero mean and serially uncorrelated distribution. We also perform a more formal test for unbiasedness and misspecification of the overall noise level, using the Normalized Innovations Squared (NIS - see Appendix F for a description). The test statistics for the private sector are shown in Table 4. The Table verifies that the null hypothesis of unit mean cannot be rejected for all nests. We do find that, while within the confidence bands, the test statistic is slightly below one which could indicate that the overall noise level might be too high compared to the parametrization (too little smoothing). Similarly, the fact that we do not find prevailing autocorrelation in our model indicates that the Kalman gain is not suppressed excessively since the state variable is allowed to respond to the forecast errors in the updating step of the filter.

Not surprisingly, the estimate of σ depends on the signal-to-noise ratio, analogous to Klump et al. (2007) who report widely different estimates, depending on the trend assumption. To illustrate this, the estimated elasticities for different values of λ for the private sector are shown in Table 5 and Table 9-10 in Appendix F is the manufacturing and private services sectors, respectively. With few exceptions, the estimated elasticities across sectors and nests are increasing in the level of smoothness. For example, in the KEL nest in the private sector in Table 5 the estimated elasticity is 0.27 when almost no smoothing is imposed on the model ($\lambda = 1$) and increasing towards unity when technical change approximates linear trend. This result is closely related to the one found in Chirinko and Mallick (2017), where the elasticity of substitution was increasing for higher values of the periodicity parameter.[15] This highlights two important points: First, in line with the literature, our findings reiterate the tight link between identifying restrictions on technical change and the elasticity of substitution. Although our model specification is less parametric and seemingly less a priori restrictive we do not escape this fact. Second, since our identification strategy is based on the signal-to-noise ratio in our filter, we can check whether a particular smoothness level results in filter consistency and the absence of autocorrelated residuals.

Since the Kalman filter tends to produce serially correlated innovations if the signal-to-noise ratio becomes too low, we would expect autocorrelation to show up if the model implied excessive smoothing. This is also what is observed for the nest of KELB where the autocorrelation test approaches the critical value as λ increases. When λ is above approximately 1,000 the autocorrelation declines since more lags of the relative expenditure shares and the relative prices are included. This makes the autocorrelation to drop again. When considering the KEL nest in the manufacturing sector, the NIS test exceed the critical value on a 10% significance level for all considered values above $\lambda = 50$ (Table 9 in Appendix F). This suggests that a linear trend in the

[15] One major difference is that we use the unfiltered dataseries and model the technical change explicitely, Chirinko and Mallick (2017) smooths the data with a low-pass filter *prior* to the estimation.

	K(E)			KE(L)			KEL(B)			KELB(M)		
λ	σ	NIS	Auto	σ	NIS	Auto	σ	NIS	Auto	σ	NIS	Auto
1	0.06	39.69	10.32	0.27	91.67	9.85	0.11	0.53	2.35	0.00	0.57	0.41
10	0.09	0.77	0.11	0.52	44.86	8.84	0.11	0.72	0.45	0.00	0.78	0.94
50	0.17	0.82	0.00	0.60	0.74	1.00	0.13	0.80	2.25	0.00	0.84	2.02
100	0.19	0.84	0.00	0.69	0.76	0.08	0.16	0.83	2.81	0.00	0.85	2.28
500	0.22	0.86	0.01	0.68	0.83	0.97	0.28	0.86	3.27	0.00	0.88	2.80
1,000	0.24	0.86	0.01	0.71	0.85	1.54	0.35	0.87	3.30	0.00	0.89	3.03
10,000	0.31	0.87	0.00	0.92	0.88	2.22	0.60	15.22	0.12	0.16	3.37	0.00
100,000	0.34	0.89	0.00	0.99	0.89	2.30	0.65	15.09	0.08	0.21	3.30	0.06

Table 5: Private sector: NIS and Breusch-Godfrey test for autocorrelation for different values of λ. NIS is the Normalized Innovations Squared test and Auto is the Breusch-Godfrey test for autocorrelation. The critical values of the NIS is $[0.68; 1.37]$ on a 10% significance level and the critical value of the Breusch-Godfrey test is 2.71 on a 10% significance level.

augmenting technical change is generally too restrictive to describe the medium run variation in technical change.[16] On the other hand, if our model implied too little smoothing of the relative technical change (λ small) one might expect to reject the NIS test as the filter can be sensitive to tuning of the measurement noise in particular. If λ is too low, the maximum likelihood estimate of σ_ε^2 will either be too small or that of σ_η^2 be too large, leading to the NIS test being above or below 1, respectively. Looking at Table 5 we do find the test for well calibrated innovations to be violated for $\lambda = 1$ based on a too low NIS in all nests. Likewise, we also find $\lambda = 10$ to be too low based on the LM-test for autocorrelation and NIS test in the KEL nest. This indicates that some degree of smoothing of factor augmenting technical change is necessary to obtain a well specified model. Quite encouragingly, moderate variations in the degrees of smoothing around our preferred values generally do not lead to elasticity estimates that are widely different.

6 Concluding remarks

This paper shows how the Kalman filter can be used to simultaneously provide an estimate of the elasticity of substitution and identify time-varying and potentially biased technical change. By exploiting the natural state space representation of the problem, we avoid a full parametric specification of the structural changes in the economy. Instead, potentially asymmetric growth in the augmenting technical change of production factors is identified by a smoothness restriction. We show in a simulation study that our approach performs well in terms of reproducing the true

[16]As comparison to our identification strategy we have estimated (3) by OLS imposing the common linear trend assumption. The estimated elasticity of substitution is almost identical to the elasticity when using $\lambda = 100,000$, no matter the sector or nest considered, confirming that our model nests the common linear trend assumption.

elasticities and performs better than the common linear trend assumption when technology is non-linear. Using Danish data for the period 1966-2016 and our preferred nest structure, we conclude the following for the aggregate and sectoral level: There is little or moderate substitution between capital and energy as a result of price changes. For the nests containing building capital and materials we find the elasticities to be low or even 0. On the other hand, the elasticities between labor and the capital-energy nest are found to be moderate, around 0.5 or slightly lower. Our results for the augmenting technical change show that labor augmenting technical change has been increasing more than capital augmenting technical change in the long run. However, we find that important "medium-run" fluctuations are present where the augmenting technical change of capital has been increasing more than the augmenting technical change of labor. Our analysis at the sectorial level suggests that this result is driven by periods of relatively slow labor productivity in the service sector.

A Simulation evidence - description of the data generating process

The data series are simulated in accordance with the relative expenditure shares in (2), but an error term $\varepsilon_t \sim N(0, \Sigma^\varepsilon)$ is added. As an example, assume that the first factor of production is capital and the second factor is labor. The prices are simulated according to:

$$r_t = r_0 e^{\gamma_r + log(r_{t-1}) + \varepsilon_t^r}, \quad \varepsilon_t^r \sim N(0, \Sigma^r) \tag{5}$$

$$w_t = w_0 e^{\gamma_w + log(w_{t-1}) + \varepsilon_t^w}, \quad \varepsilon_t^w \sim N(0, \Sigma^w) \tag{6}$$

r_t is the user cost of capital and w_t is the real wage rate. The drift parameter $\gamma_r = 0$ is chosen such that the interest rate is a Random Walk, and the parameter γ_w is chosen to reflect an average increase in real wages on 2% per year, similar to what is observed historically. All parameter values are shown in Table 6. We set $r_0 = w_0 = 1$. The variances Σ^r, Σ^w are chosen to match what is observed on average in the data sets used in the empirical application. The augmenting technologies Γ_t^K and Γ_t^L consist of a stochastic and a deterministic part. We vary on the relative importance of these and try three different specifications of the deterministic part:

$$\Gamma_t^K = \Gamma_0^K e^{g(\Gamma^K) + log\left(\Gamma_{t-1}^K\right) + \varepsilon_t^{\Gamma^K}}, \quad \varepsilon_t^{\Gamma^K} \sim N\left(0, \Sigma^{\Gamma^K}\right) \tag{7}$$

$$\Gamma_t^L = \Gamma_0^L e^{g\left(\Gamma^L\right) + log\left(\Gamma_{t-1}^L\right) + \varepsilon_t^{\Gamma^L}}, \quad \varepsilon_t^{\Gamma^L} \sim N\left(0, \Sigma^{\Gamma^L}\right), \tag{8}$$

We assume $\Gamma_0^K = \Gamma_0^L = 1$. The growth rates $g\left(\Gamma^K\right) = \gamma_{\Gamma^K}$ and $g\left(\Gamma^L\right) = \gamma_{\Gamma^L}$ are in the first case are specified such that technology is augmenting labor with a deterministic trend throughout the whole sample period. In the second case, labor is the augmenting factor in the first half of the sample period, whereas capital is the augmenting factor in the second half of the sample period. In the third case we use a Box-Cox transformation of the growth rates given as $g\left(\Gamma^K\right) = \frac{\gamma_{\Gamma^K} t_0}{\lambda_{\Gamma^K}} \left[\left(\frac{t}{t_0}\right)^{\lambda_{\Gamma^K}} - 1\right]$ and $g\left(\Gamma^L\right) = \frac{\gamma_{\Gamma^L} t_0}{\lambda_{\Gamma^L}} \left[\left(\frac{t}{t_0}\right)^{\lambda_{\Gamma^L}} - 1\right]$. γ_{Γ^K} and γ_{Γ^L} are the drift parameters and λ_{Γ^K} and λ_{Γ^L} are the curvature parameters carefully chosen to reflect medium run fluctuations. We set $t_0 = 1$. The variances of the technology parameters $\Sigma^{\Gamma^K}, \Sigma^{\Gamma^L}$ are assumed identical, denoted as Σ^Γ, and specified in conjunction with the measurement error variance Σ^ε according to:

$$\tilde{\lambda} = \frac{\Sigma^\varepsilon}{(\sigma - 1)^2 \Sigma^\Gamma} \tag{9}$$

Parameter	Description	Harrod-Neutral	Harrod- and Solow-Neutral	Box-Cox
$\Sigma^{\Delta s}$	Variance of expenditure shares	0.01	0.01	0.01
Σ^{r}	Variance of the interest rate	0.005	0.005	0.005
Σ^{w}	Variance of real wages	0.005	0.005	0.005
γ_r	Drift parameter in the interest rate	0	0	0
γ_w	Drift parameter in the real wage	0.02	0.02	0.02
$\tilde{\lambda}$	Noise-to-signal ratio in the DGP	$1, 10, 100, 1000$	$1, 10, 100, 1000$	$1, 10, 100, 1000$
σ	Elasticity of substitution	$0.2, 0.5, 0.9, 1.3$	$0.2, 0.5, 0.9, 1.3$	$0.2, 0.5, 0.9, 1.3$
γ_{Γ^K}	Drift parameter in Γ^K	0	0 , first half, 0.02 second half	0.01
γ_{Γ^L}	Drift parameter in Γ^L	0.02	0.05 , first half, 0 second half	0.07
λ_{Γ^K}	Curvature parameter in Γ^K	-	-	0.4
λ_{Γ^L}	Curvature parameter in Γ^L	-	-	-0.9

Table 6: Parameter values used in the simulations

$\tilde{\lambda}$ is the inverse of the signal-to-noise ratio in the data generating process. We try different values of this parameter in the simulations. Jointly, Σ^{ε} and Σ^{Γ} are chosen such that we keep the variance in the expenditure shares (called $\Sigma^{\Delta s}$) constant in all simulations to match the observed variance level in the datasets used in the empirical application. The variance Σ^{Γ} is calculated residually as:

$$\Sigma^{\Gamma} = \frac{\Sigma^{\Delta s} - (1-\sigma)^2 \left(\Sigma^{r} + \Sigma^{w}\right)}{2\left(\sigma - 1\right)^2 \left(1 + \tilde{\lambda}\right)}, \tag{10}$$

B User cost measure

We distinguish between two different types of the user cost. In the short run, we use a measure with installation costs. This user cost is used to estimate the short run elasticity. The long run or "steady state" user cost we define as a user cost in the absence of installation costs. This user cost is used to estimate the long run elasticity. This user cost is similar to the one used in Statistics Denmark. The measure of the user cost of capital[17] p_t^K is derived by solving the firms maximization problem. The data to calculating the user cost is obtained from Statistics Denmark. The first order condition for investments (Tobin's q) is given by:

$$q_t = p_t^I \left(1 - q_t^{book}\right) + p_t^Y \left(1 - t_t^{corp}\right) * \phi_1 \frac{I_t}{K_{t-1}} \tag{11}$$

q_t is Tobins q, p_t^I is the price of investments in machinery, transportation equipment and inventory and I_t is the quantity. p_t^Y is the price of total production volume and t_t^{corp} is the corporate tax rate. K_t denotes the quantity of capital. ϕ_1 measures the marginal effect of installation costs. Thus, higher values of ϕ_1 lead to larger short run fluctuations in the user cost. In the steady state user cost this is set to 0, whereas it is set to 2.5 in the user cost with adaptive expectations. q_t^{book}

[17]The user cost of buildings is derived in the same way.

is the shadow value of the book value of capital:

$$q_t^{book} = \frac{1}{1+r_{t+1}^K}\left(\left(1-\delta^{book}\right)q_{t+1}^{book} + t_{t+1}^{corp}\delta^{book}\right) \tag{12}$$

r_{t+1}^K is the interest rate, which is measured as the banks effective lending rate plus a risk premium on 2%. δ^{book} is the depreciation rate of book capital, which is set to 15%. The terminal condition is given as:

$$q_T^{book} = \frac{t_t^{corp}\delta^{book}}{r_t^K + \delta^{book}} \tag{13}$$

The user cost is then given as:

$$\begin{aligned} p_t^K = \quad & q_t\left(1+r_{t+1}^K\right) - (1-\delta_{t+1})q_t \\ & -t_{t+1}^{corp} * r_{t+1}^K * D^{firm}p_t^I \\ & -\tfrac{1-t_t^{corp}}{1+t_t^Y}p_{t+1}^Y\left(1-t_{t+1}^{corp}\right) * \Phi_{t+1}^K \end{aligned} \tag{14}$$

δ_t is the depreciation rate of capital, which can be calibrated as shown below. D^{firm} is the debt share of firms, which is set to 60%. Φ_{t+1}^K is the quadratic installation costs given by:

$$\Phi_{t+1}^K = \frac{\phi_1}{2}K_t\left(\frac{I_{t+1}}{K_t}\right)^2 \tag{15}$$

$$\delta_t = \frac{K_t + I_t}{K_{t-1}} - 1 \tag{16}$$

C Data

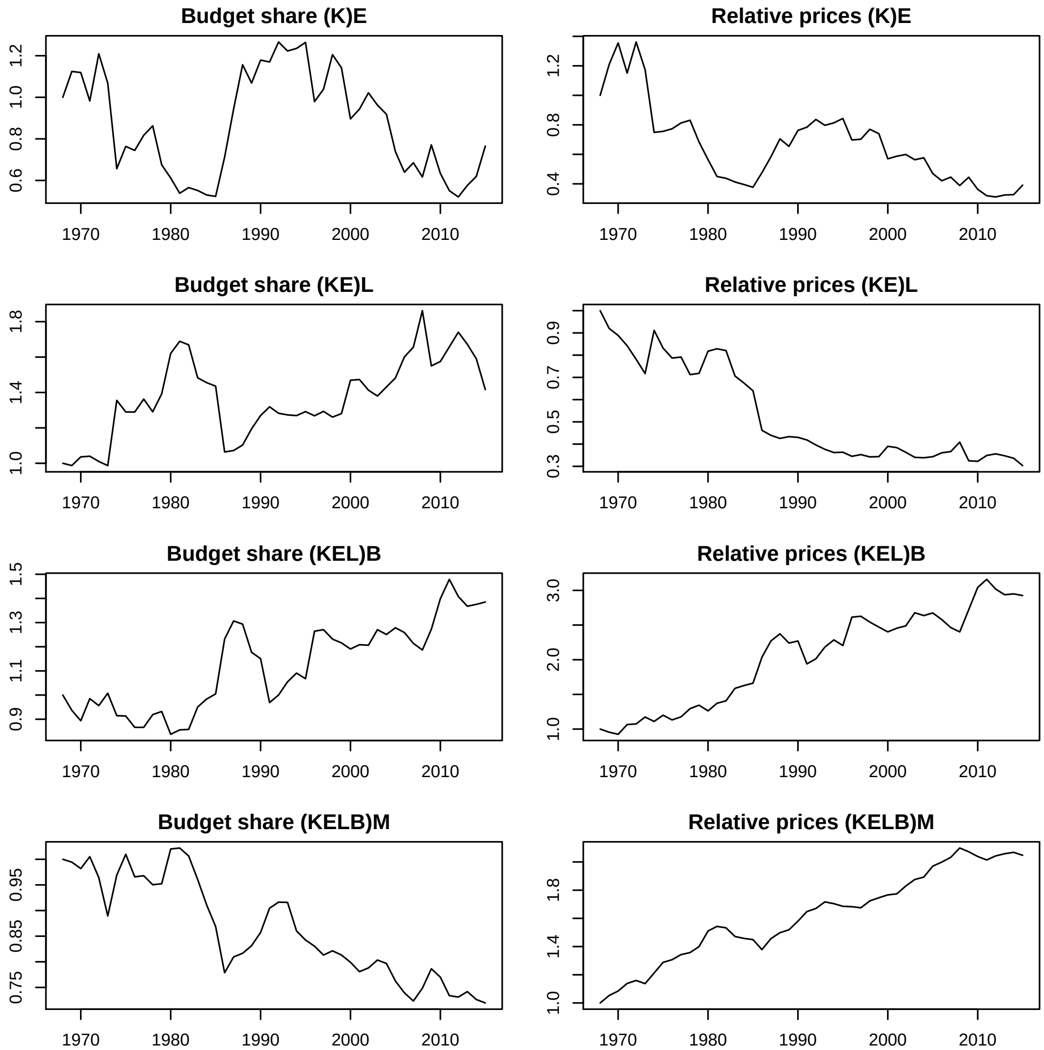

Figure 3: Private sector: Relative prices and budget shares. Parenthesis indicates the nested factors.

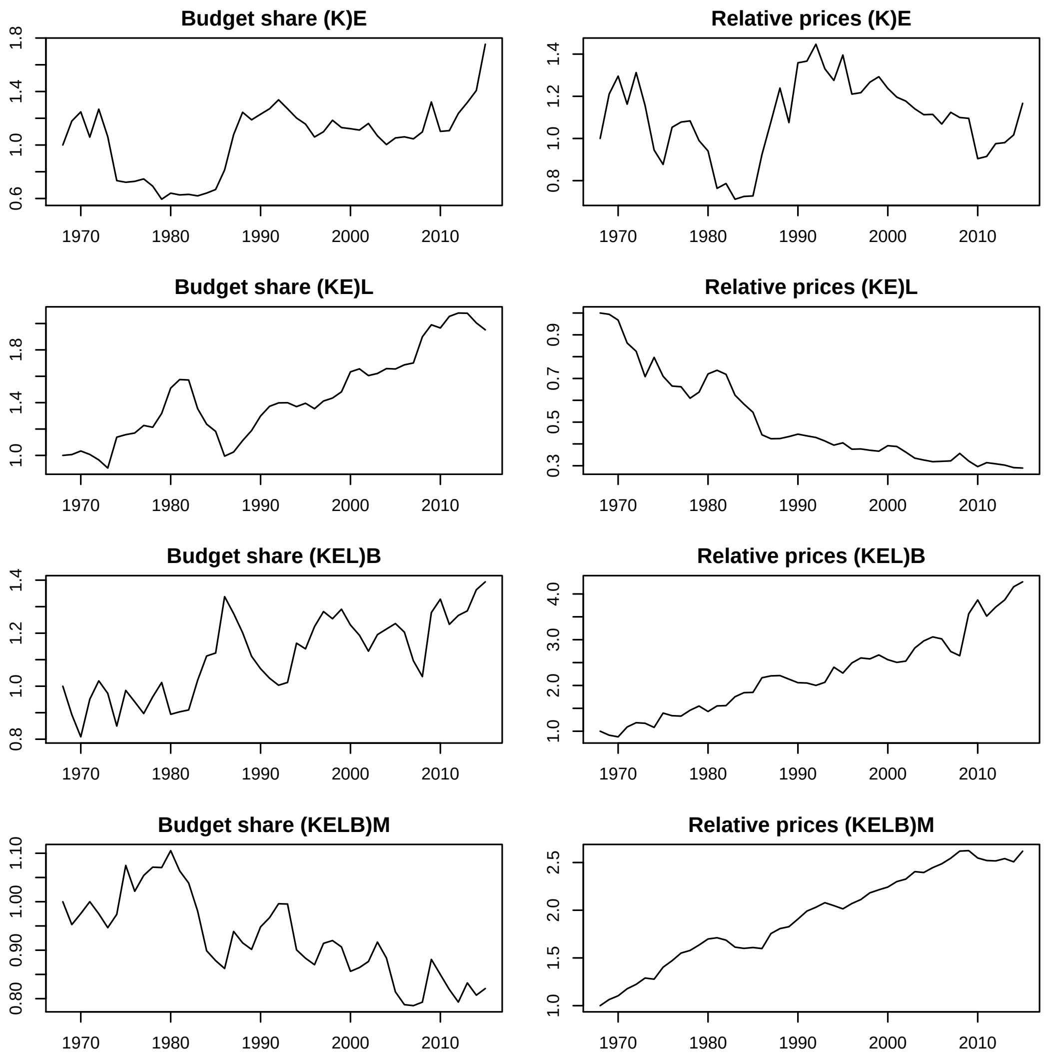

Figure 4: Manufacturing: Relative prices and budget shares. Parenthesis indicates the nested factors.

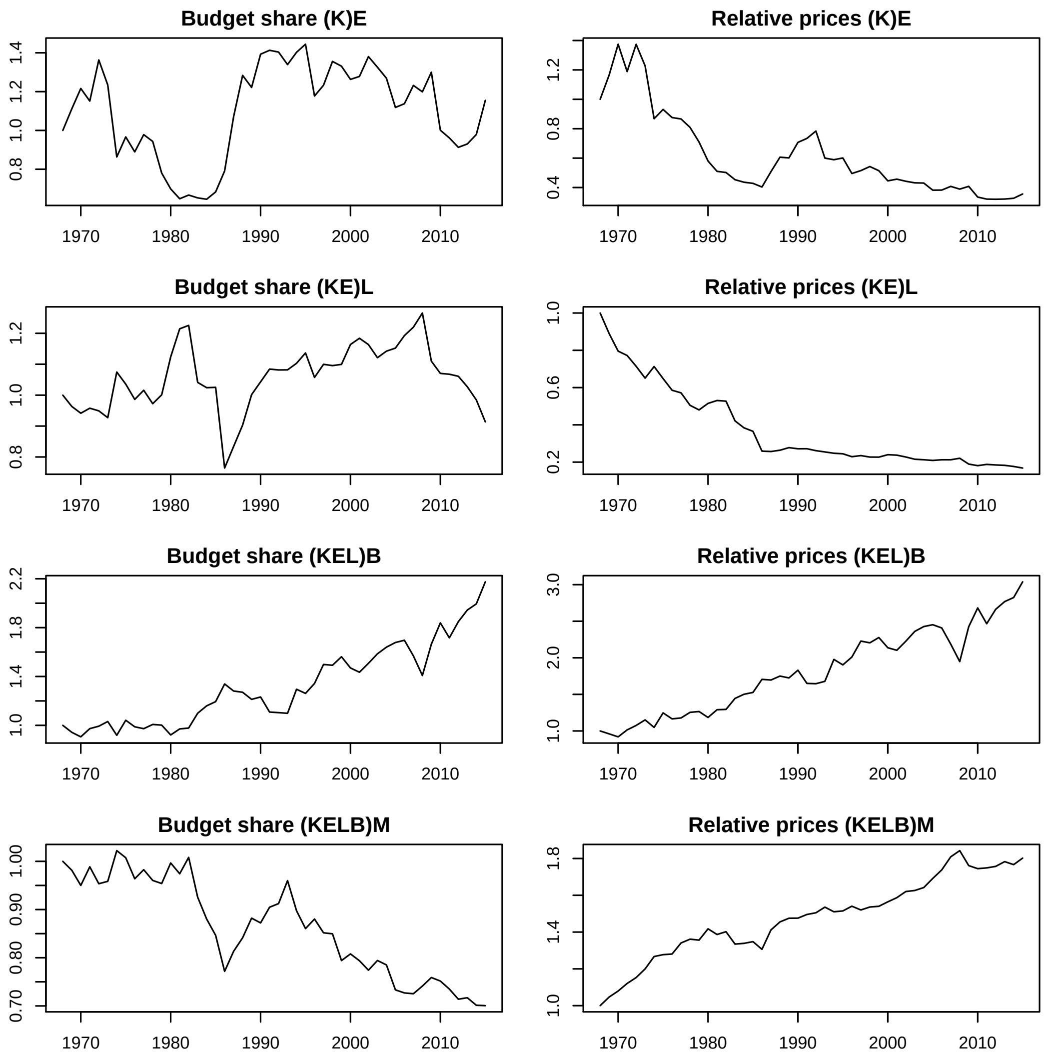

Figure 5: Private service: Relative prices and budget shares. Parenthesis indicates the nested factors.

D Estimation results in manufacturing and private services

	(K)E	(KE)L	(KEL)B	(KELB)M
σ	0.13	0.19	0.00	0.23
	$(-0.08;0.36)$	$(0.05;1.06)$	$(-0.32;0.20)$	$(-0.12;0.67)$
α	-0.80	-0.34	-0.41	-0.47
	$(-0.82;-0.47)$	$(-0.38;-0.12)$	$(-0.47;-0.20)$	$(-0.64;-0.23)$
Likelihood	74.91	106.28	118.77	107.46
λ	8	10	10	1854
Autocorrelation	[0.29]	[0.67]	[0.40]	[0.40]
Heteroskedasticity	[0.75]	[0.32]	[0.15]	[0.23]
Normality	[0.98]	[0.51]	[0.13]	[0.82]
NIS	0.71	0.71	0.71	0.86

Table 7: Manufacturing: Estimated results in a (((KE)L)B)M nest structure. Terms in parenthesis are the lower and upper critical values, respectively, on a 10% significance level. Terms in brackets are p-values for the misspecification tests. The critical values of the Normalized Innovations Squared test (NIS) are $[0.68; 1.37]$ on a 10% significance level.

	(K)E	(KE)L	(KEL)B	(KELB)M
σ	0.36	0.50	0.00	0.00
	$(0.11;0.74)$	$(0.37;1.48)$	$(-0.32;0.17)$	$(-0.57;0.76)$
α	-0.43	-0.19	-0.31	-0.31
	$(-0.54;-0.20)$	$(-0.23;-0.06)$	$(-0.37;-0.14)$	$(-0.48;-0.15)$
Likelihood	84.21	136.70	139.17	115.44
λ	84	55	15	1000
Autocorrelation	[0.98]	[0.12]	[0.21]	[0.22]
Heteroskedasticity	[0.44]	[0.37]	[0.13]	[0.70]
Normality	[0.16]	[0.05]	[0.58]	[0.94]
NIS	0.81	0.80	0.69	0.89

Table 8: Private services: Estimated results in a (((KE)L)B)M nest structure. Terms in parenthesis are the lower and upper critical values, respectively, on a 10% significance level. Terms in brackets are p-values for the misspecification tests. The critical values of the Normalized Innovations Squared test (NIS) are $[0.68; 1.37]$ on a 10% significance level..

E Augmenting technical change in manufacturing and private services

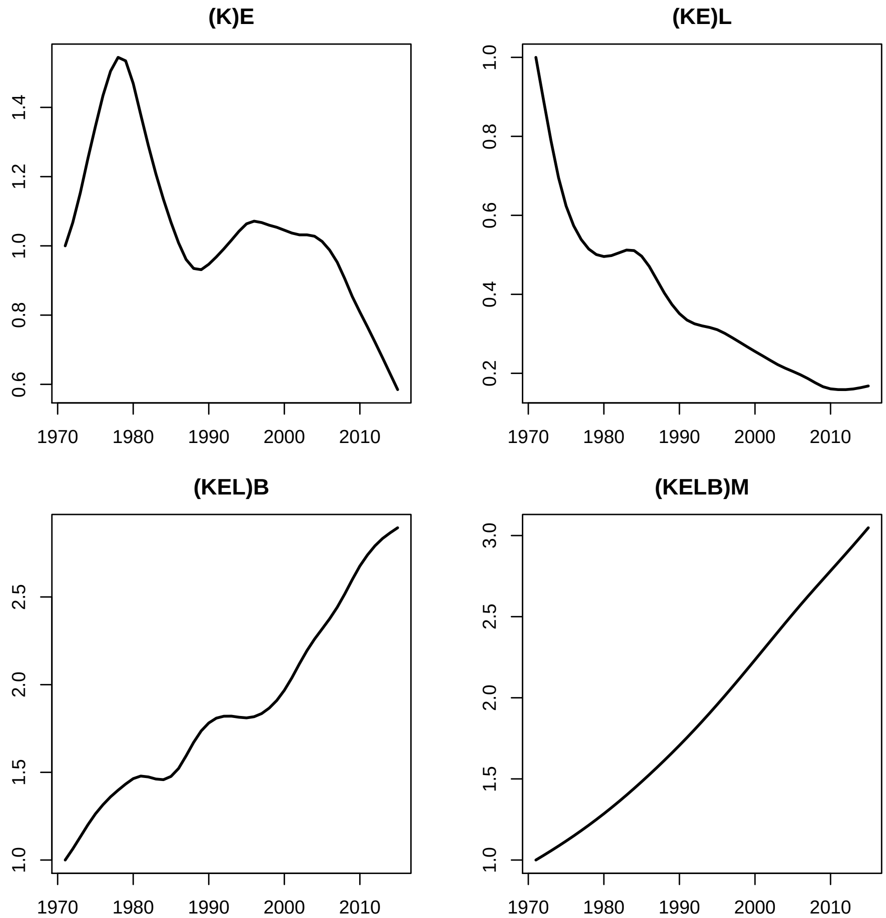

Figure 6: Manufacturing: Relative augmenting technical change in a (((KE)L)B)M nest structure. Parenthesis indicates the nested factors. That is, the graph shows the relative augmenting technologies of e.g. capital relative to energy. Index year is 1971.

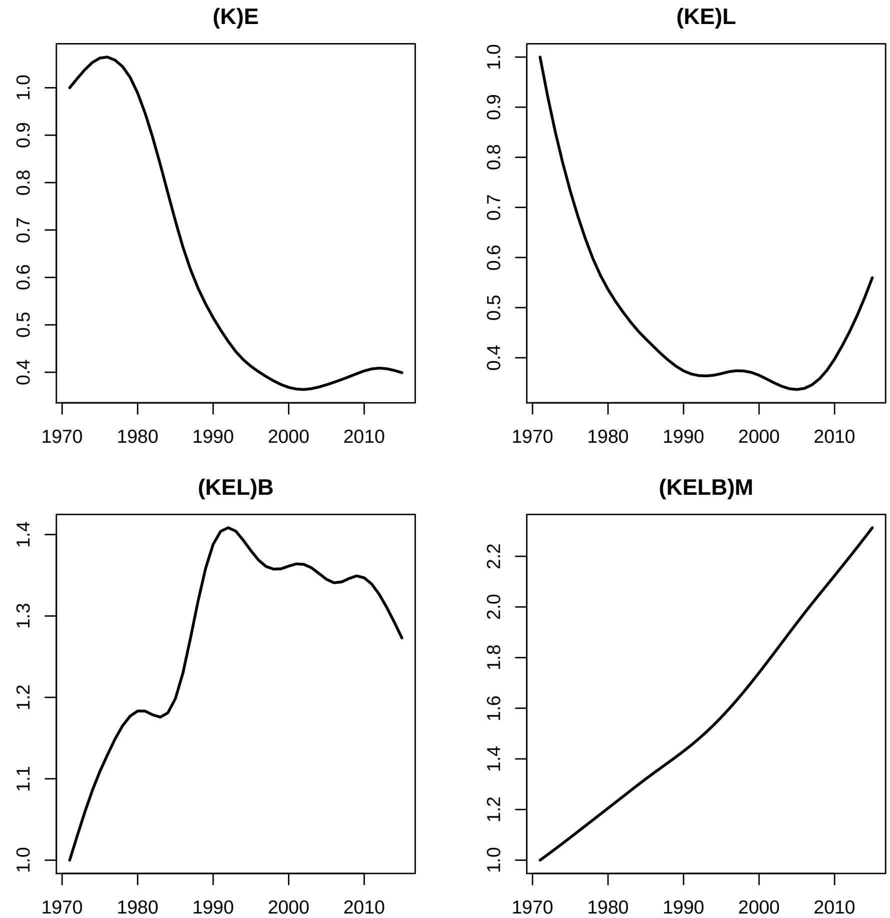

Figure 7: Private services: Relative augmenting technical change in a (((KE)L)B)M nest structure. Parenthesis indicates the nested factors. That is, the graph shows the relative augmenting technologies of e.g. capital relative to energy. Index year is 1971.

F Filter performance, consistency and the identification strategy

To evaluate filter performance we would like to know if the filtered state is a reasonably prediction of the true value. However, as the true state is unknown filter consistency is usually based on information on the innovations in the observation equation. If the filter is consistent, the standardized forecast errors will be a zero-mean and homoskedastic white noise process. This can be evaluated either by graphically inspecting the standardized innovations or (more formally) by considering the Normalized Innovations Squared test (NIS). The NIS test has the following test statistic:

$$m_t = \varepsilon_t^T F_t^{-1} \varepsilon_t, \tag{17}$$

	K(E)			KE(L)			KEL(B)			KELB(M)		
λ	σ	NIS	Auto	σ	NIS	Auto	σ	NIS	Auto	σ	NIS	Auto
1	0.09	0.61	1.92	0.06	0.59	0.87	0.01	45.76	13.80	0.21	0.61	0.86
10	0.13	0.71	1.07	0.19	0.71	0.18	0.00	0.71	0.70	0.06	0.80	0.03
50	0.16	0.75	0.74	0.62	0.79	2.01	0.01	17.54	2.65	0.07	0.83	0.18
100	0.18	0.79	0.55	0.54	7.22	0.92	0.07	18.79	2.54	0.09	0.84	0.25
500	0.33	0.85	0.33	0.67	6.68	0.09	0.18	16.69	0.98	0.16	0.85	0.50
1,000	0.43	0.86	0.28	0.69	6.59	0.01	0.22	16.16	0.66	0.19	0.86	0.62
10,000	0.68	0.88	0.17	0.62	6.62	0.06	0.25	15.49	0.26	0.33	0.88	0.96
100,000	0.72	0.89	0.15	0.58	6.66	0.09	0.20	15.43	0.10	0.40	0.89	1.15

Table 9: Manufacturing: NIS and Breusch-Godfrey test for autocorrelation for different values of λ. NIS is the Normalized Innovations Squared test and Auto is the Breusch Godfrey test for autocorrelation. The critical values of the NIS is $[0.63; 1.45]$ on a 5% significance level and the critical value of the Breusch-Godfrey test is 3.84 on a 5% significance level.

	K(E)			KE(L)			KEL(B)			KELB(M)		
λ	σ	NIS	Auto	σ	NIS	Auto	σ	NIS	Auto	σ	NIS	Auto
1	0.25	10.26	12.15	0.51	74.46	9.10	0.00	82.24	18.10	0.00	1.97	6.76
10	0.35	0.76	0.09	0.71	43.26	4.48	0.00	0.66	0.59	0.00	0.76	0.08
50	0.36	0.80	0.00	0.49	0.79	2.49	0.00	47.50	7.60	0.00	0.82	0.16
100	0.36	0.81	0.00	0.61	0.80	1.79	0.00	44.61	6.29	0.00	0.84	0.39
500	0.32	0.83	0.05	0.93	0.83	0.46	0.05	36.98	5.32	0.00	0.87	1.17
1,000	0.33	0.84	0.08	1.14	0.85	0.19	0.15	34.21	4.54	0.00	0.89	1.48
10,000	0.62	0.87	0.12	1.82	0.88	0.01	0.32	37.85	2.66	0.00	0.91	1.94
100,000	0.91	0.89	0.15	1.99	0.89	0.00	0.45	36.97	2.06	0.00	0.91	2.02

Table 10: Private services: NIS and Breusch-Godfrey test for autocorrelation for different values of λ. NIS is the Normalized Innovations Squared test and Auto is the Breusch Godfrey test for autocorrelation. The critical values of the NIS is $[0.63; 1.45]$ on a 5% significance level and the critical value of the Breusch-Godfrey test is 3.84 on a 5% significance level.

where F_t is the covariance matrix of the innovations. If the assumptions are correct, m_t will be $\chi^2(1)$ distributed, implying that the T period moving average, $\bar{m}_T$, has a $T\chi^2(T)$ distribution (applying the ergodic property of the innovations). Hence, the null hypothesis is $E[m] = 1$ and can be tested by computing the moving average of (17) recursively for an increasing sample size and compare the test statistics to the critical values (see Figure 8-10).

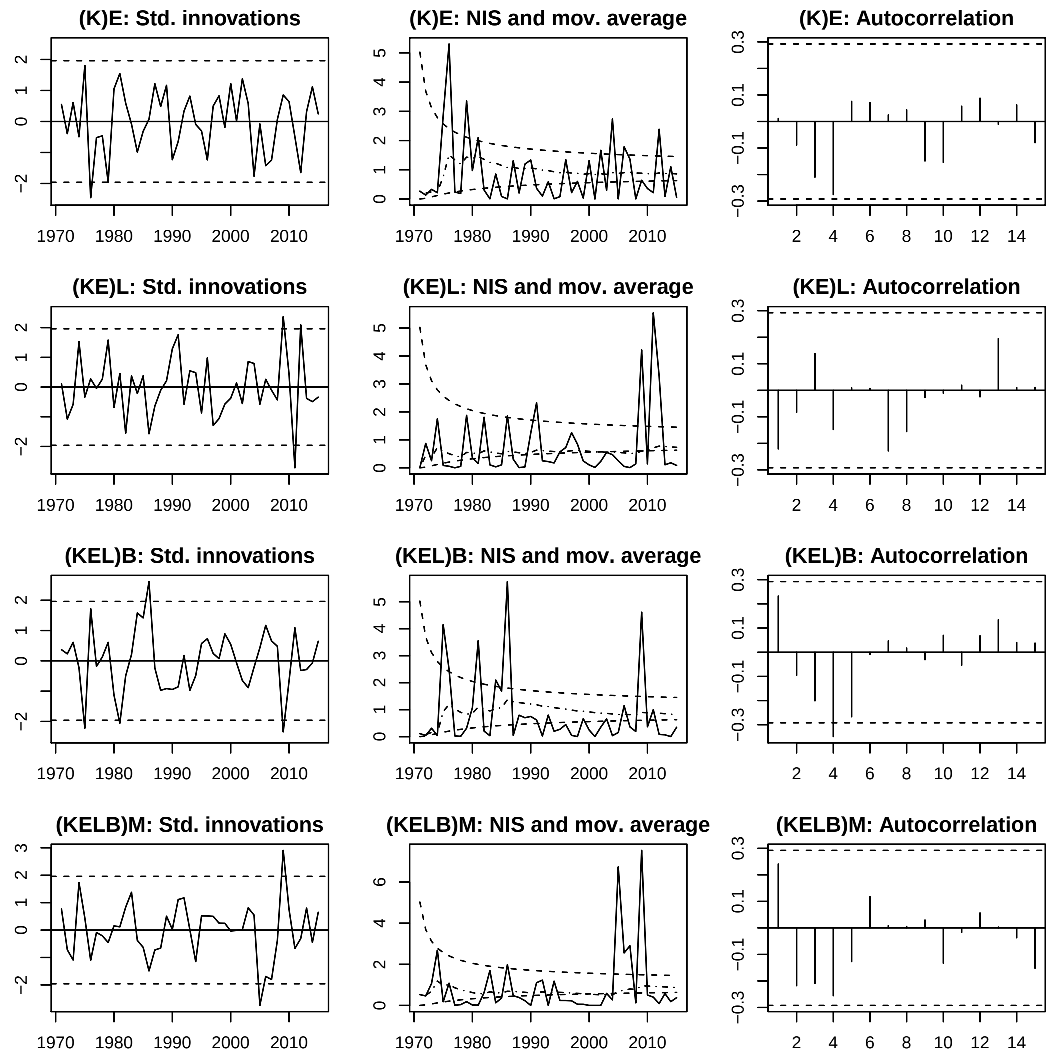

Figure 8: Private sector: Diagnostics where a (((KE)L)B)M structure is applied. Paranthesis indicates the nested factors.

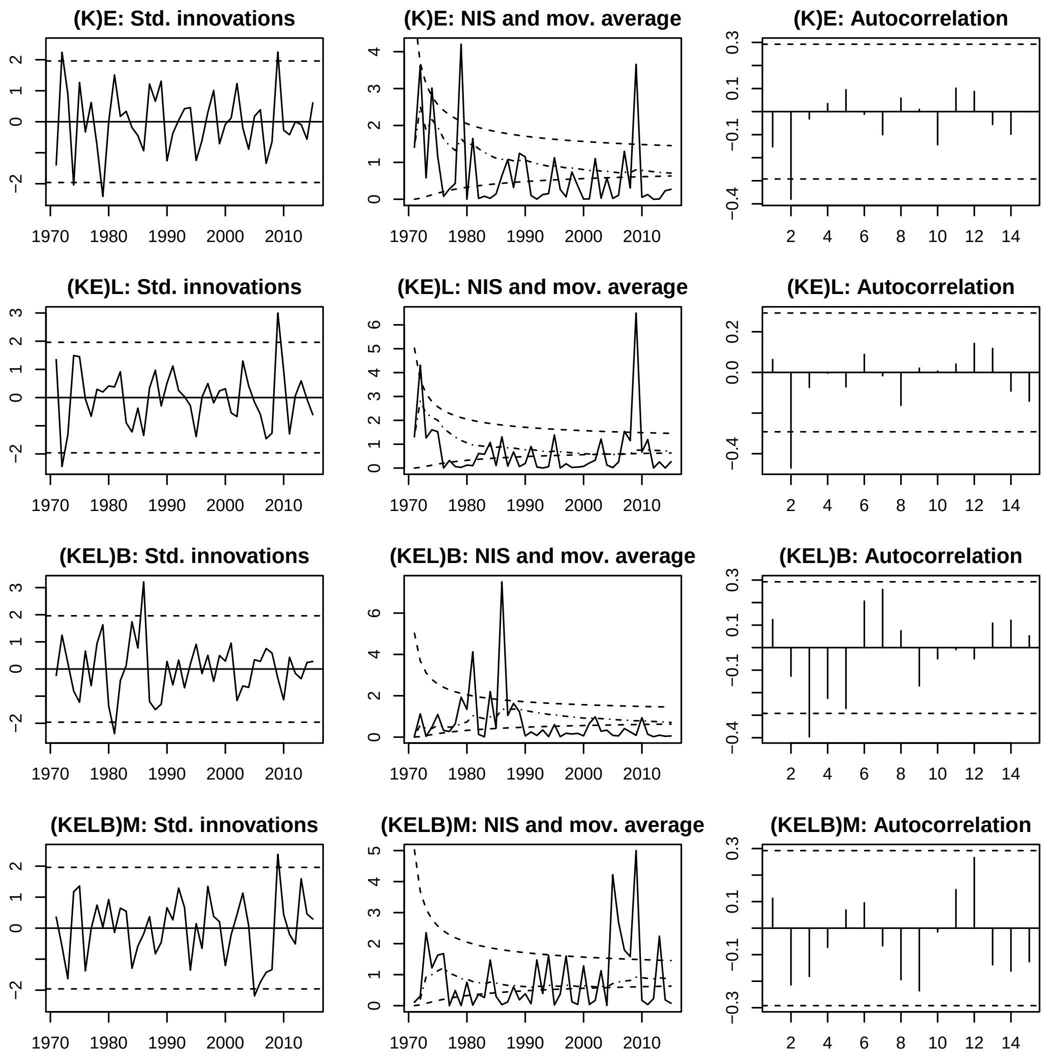

Figure 9: Manufacturing: Diagnostics where a (((KE)L)B)M structure is applied. Parenthesis indicates the nested factors.

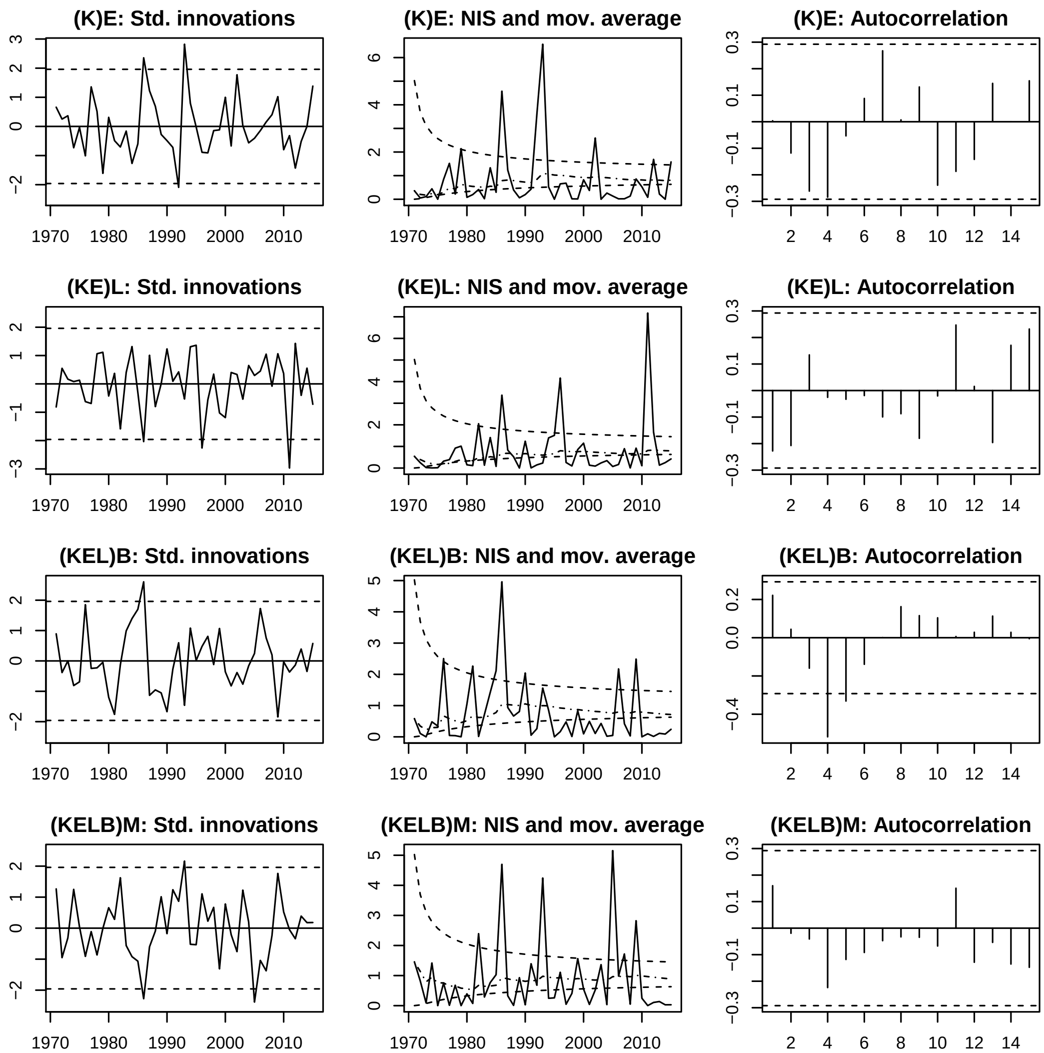

Figure 10: Private services: Diagnostics where a (((KE)L)B)M structure is applied. Parenthesis indicates the nested factors.

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