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The Waste Sector in the GreenREFORM model: A Tentative Static Partial Equilibrium Model

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Economic memo
3 March 2020

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March, 2020

1 Introduction

This paper describes a tentative static version of the waste sector module to the Green Reform model. We expect the main structure described below to carry through to the final version of the module, while the level of detail increases.

The final waste module of the Green Reform model will differ from the present model in at least three important aspects. First, the model presented below is static, and thus, describes the waste sector in a particular year. Capital is fully adjustable within this period. The final Green Reform model will be dynamic, and capital will adjust endogenously over time. Second, the final Green Reform model will be able to project economy-wide waste generation over the coming decades. Thus, the final Green Reform model will feature endogenous waste generation processes. Meanwhile, the present model takes the waste generated by the economy as given.

2 Model

2.1 Overview

The economy generates waste from production and consumption. The waste is sorted by households and firms before collection and treatment.

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The present model takes waste generation as given, while waste collection and treatment are explicitly modelled. These activities require labor, capital, and energy input. Waste can be burned, transformed into new raw materials, or end up as landfill. The first activity is not explicitly modelled. Instead, firms handling waste treatment sell waste as a fuel input to the electricity and heating sector.

The chosen waste treatment for each waste fraction depends on economic trade-offs and regulation. Regulation requires that hazardous waste is deposited as landfill or undergoes special treatment. In addition, the waste hierarchy dictates that recycling is preferable to energy utilization. Finally, the government impose a non-profit constraint on firms within the waste sector. This constraint implies that firms in the sector obtain zero profits after paying the factors of production (including capital).

2.2 Waste generation and fractions

There are $j = 1, 2, ..., J$ sectors in the economy generating waste. Sector $j = 1$ is the household sector, while the renaming $J - 1$ sectors are production sectors. The waste generated by each sector is sorted into $n = 1, 2, ..., N$ waste fractions. This sorting effort occurs within the households and firms before the waste is collected for treatment.

Aggregate waste of fraction n is given by:

$$\bar{Q}_n = \sum_{j=1}^{J} q_{n,j}, \tag{1}$$

where $q_{n,j}$ is waste of fraction n generated by sector j. In this version of the model, all $q_{n,j}$'s are exogenous.

2.3 The waste sector and production structure

The waste sector is bifurcated into two sectors: the waste collection sector and the treatment sector. The *waste collection sector* is obligated to collect all the waste generated by the economy. There are N representative firms in this sector each handling a different waste fraction. All the collected waste is delivered to the treatment sector.

The *treatment sector* is obligated to handle all the waste delivered to them by the waste collection sector. There are N representative firms in the sector each handling a different waste fraction.

Waste collection and treatment require labor, capital, and energy input. Although these two production processes are very different, they are assumed to follow the same overall production structure. Let the production level, $Y_{j,n}$, be given by

$$Y_{j,n} = \left[\gamma_{j,n}^{\frac{1}{\epsilon_{j,n}}} \left(K_{j,n}^{\mathrm{E}} \right)^{\frac{\epsilon_{j,n}-1}{\epsilon_{j,n}}} + (1-\gamma_{j,n})^{\frac{1}{\epsilon_{j,n}}} L_{j,n}^{\frac{\epsilon_{j,n}-1}{\epsilon_{j,n}}} \right]^{\frac{\epsilon_{j,n}}{\epsilon_{j,n}-1}}, \quad \gamma_{j,n} \in (0,1), \quad \epsilon_{j,n} > 0, \tag{2}$$

where $j \in \{I, B\}$ denotes the sector (I: waste collection sector; B: treatment sector), $K_{I,n}^{\mathrm{E}}$ is a capital-energy aggregate, and $L_{I,n}$ is the labor input.

The capital-energy aggregate is given by

$$K_{j,n}^{\mathrm{E}} = \eta_{j,n}^{\mathrm{KE}} \left[\left(\gamma_{j,n}^{\mathrm{KE}} \right)^{\frac{1}{\epsilon_{j,n}^{\mathrm{KE}}}} K_{j,n}^{\frac{\epsilon_{j,n}^{\mathrm{KE}}-1}{\epsilon_{j,n}^{\mathrm{KE}}}} + \left(1-\gamma_{j,n}^{\mathrm{KE}} \right)^{\frac{1}{\epsilon_{j,n}^{\mathrm{KE}}}} E_{j,n}^{\frac{\epsilon_{j,n}^{\mathrm{KE}}-1}{\epsilon_{j,n}^{\mathrm{KE}}}} \right]^{\frac{\epsilon_{j,n}^{\mathrm{KE}}}{\epsilon_{j,n}^{\mathrm{KE}}-1}}, \eta_{j,n}^{\mathrm{KE}} > 0, \; 0 < \gamma_{j,n}^{\mathrm{KE}} < 1, \; \epsilon_{j,n}^{\mathrm{KE}} > 0, \tag{3}$$

where $K_{j,n}$ and $E_{j,n}$ are the capital and energy inputs, respectively.

Minimizing the cost of obtaining a given production level implies that:

$$\left(\frac{w}{p_{j,n}^{\mathrm{KE}}} \right) = \left(\frac{\gamma_{j,n}}{1-\gamma_{j,n}} \frac{K_{j,n}^{\mathrm{E}}}{L_{j,n}} \right)^{\frac{1}{\epsilon_{j,n}}} \quad \text{and} \quad \left(\frac{r}{p_{E,j,n}} \right) = \left(\frac{\gamma_{j,n}^{\mathrm{KE}}}{1-\gamma_{j,n}^{\mathrm{KE}}} \frac{E_{j,n}}{K_{j,n}} \right)^{\frac{1}{\epsilon_{j,n}^{\mathrm{KE}}}}, \tag{4}$$

where w is the wage rate, r is the real interest rate, $p_{E,I,n}$ is the price of the energy aggregate, and $p_{j,n}^{\mathrm{KE}}$ is the ideal price index for the capital-energy aggregate given by:

$$p_{j,n}^{\mathrm{KE}} = \left[\left(\gamma_{j,n}^{\mathrm{KE}} \right) r^{1-\epsilon_{j,n}^{\mathrm{KE}}} + \left(1-\gamma_{j,n}^{\mathrm{KE}} \right) \left(p_{E,j,n} \right)^{1-\epsilon_{j,n}^{\mathrm{KE}}} \right]^{\frac{1}{1-\epsilon_{j,n}^{\mathrm{KE}}}}. \tag{5}$$

The wage rate, w, the real interest rate, r, and the energy price, $p_{E,I,n}$, are exogenous in the present model.[1]

2.4 Collecting the waste

Firms in the waste collection sector are obligated to collect the waste mass, $\bar{Q}_n$, for each waste fraction. To do this, firms need to obtain the production level:

$$\bar{Q}_n = \eta_{I,n} Y_{j,n}, \quad \eta_{I,n} > 0, \tag{6}$$

[1] These prices will be endogenous in the final Green Reform model.

where $\eta_{I,n}$ is a scale parameter measured in waste mass per unit of production value.

Firms in the sector are assigned the task of collecting waste through a public procurement. This ensures that the most productive firms are assigned the task. Essentially, the procurement forces firms to minimize their costs associated with the collection task:

$$\min_{K_{I,n},L_{I,n},E_{I,n}} L_{I,n}w + K_{I,n}r + E_{I,n}p_{E,I,n} \quad \text{st.} \quad \bar{Q}_n = \eta_{I,n}Y_{I,n}\left(K_{I,n}, L_{I,n}, E_{I,n}\right). \tag{7}$$

The first-order conditions associated with this problem are given by (4) and (5).

The treatment sector provides a service to the collection sector when taking over the waste. The collection sector pays a fee $p_{\bar{Q}_n} > 0$ per unit of waste delivered to the treatment sector. There is an additional fee, $p_{d,n} > 0$, for getting rid of waste that cannot be recycled or used directly for energy generation (typically hazardous waste). The total payment to the treatment sector is $(p_{\bar{Q}_n} + d_n p_{d,n})\bar{Q}_n$, where d_n is an exogenous fraction of the waste unsuited for utilization, cf. below. Note that firms in the treatment sector cannot avoid these fees, as they must deliver all the waste generated by the economy to the treatment sector. Accordingly, the fees do not affect firm behaviour in the sector.

The government imposes a non-profit constraint on firms within the waste collection sector, and thus:

$$\pi_{I,n} = 0, \tag{8}$$

where $\pi_{I,n}$ is the profit of the representative firm for waste fraction n.

Profits associated with the collection activity is negative, as firms have expenses associated with both collecting and delivering the waste. The government ensures zero profits through a lump-sum transfer, according to

$$\pi_{I,n} = T_{I,n} - L_{I,n}w - K_{I,n}r - E_{I,n}p_{E,I,n} - \left(p_{\bar{Q}_n} + d_n p_{d,n}\right)\bar{Q}_n, \tag{9}$$

where $T_{I,n}$ is a fraction specific lump-sum transfer.

2.5 Treatment, disposal, and recycling

The waste that needs treatment includes both the primary waste production, $\bar{Q}_n$ and secondary waste. The secondary waste mass, $q_{S,n}$, is a waste product generated in the recycling process of other waste fractions. It is costless to dispose the secondary waste, as this secondary waste generation occurs within the treatment sector.

A fixed share of each waste fraction cannot be burned or recycled. One example is soft PVC which is unsuited for recycling and generates toxins when burned. This waste ends up as landfill. The fraction of waste sent to landfill, d_n, is here exogenous, as regulation (the waste hierarchy) generally dictates that waste, if possible, should be burned or recycled.

The waste mass handled by the treatment sector net of landfill, Q_n, is given by:

$$Q_n = (1 - d_n)\left(\bar{Q}_n + q_{S,n}\right), \tag{10}$$

where it is (so far) assumed that the same fraction of primary and secondary waste must go to landfill.

The waste net of landfill can be utilized in two ways: it can be burned to generate energy, or it can be recycled. However, the waste fractions are not completely homogeneous, implying that some parts of a waste fraction are more suited for recycling than others. To capture this property, the transformation function between energy waste and recycled waste is given by the modified CET (constant elasticity of transformation) functions:

$$Q_n = \Omega_n \left[\gamma_{T,n}^{\frac{1}{\epsilon_{T,n}}} M_{T,n}^{\frac{\epsilon_{T,n}-1}{\epsilon_{T,n}}} + (1-\gamma_{T,n})^{\frac{1}{\epsilon_{T,n}}} A_{T,n}^{\frac{\epsilon_{T,n}-1}{\epsilon_{T,n}}} \right]^{\frac{\epsilon_{T,n}}{\epsilon_{T,n}-1}} + \xi_n A_{T,n}, \tag{11}$$

where T indicates that the parameter or variable belongs to the transformation function, $M_{T,n}$ is the mass of new materials obtained from the recycling process, and $A_{T,n}$ is the mass of energy waste.

All the waste can, in principle, be burned. In that case, there is no generation of new raw materials implying that $M_{T,n} = 0$. To ensure that all waste mass is accounted for in the CET function when all waste mass is burned, it must hold that:

$$Q_n = \left(\Omega_n (1-\gamma_{T,n})^{\frac{1}{\epsilon_{T,n}-1}} + \xi_n \right) \underbrace{A_{T,n}^{\mathbf{MAX}}}_{=Q_n} \quad \Leftrightarrow \quad 1 = \Omega_n (1-\gamma_{T,n})^{\frac{1}{\epsilon_{T,n}-1}} + \xi_n, \tag{12}$$

where $A_{T,n}^{\mathbf{MAX}}$ is the maximum mass of energy waste that can be generated which equals the total waste mass available, Q_n.

Figure 1 presents an example, where the waste mass, Q_n, is normalized to one. The entire waste mass can be sent to the energy sector, where it may be burned. If some part of the waste mass is recycled then only some of this mass ends up as new raw materials, as the recycling process generates secondary waste and pollution emission. The horizontal distance between any point on the CET function and the physical constraint reflects the combined pollution emission and secondary waste mass generated from recycling. Mathematically,

$$\bar{M}_{T,n} = \hat{q}_{S,n} + e_{T,n} + M_{T,n} = Q_n - A_{T,n}, \tag{13}$$

where $\bar{M}_{T,n}$ is the waste mass sent to recycling, $\hat{q}_{S,n}$ is the secondary waste mass generated from recycling, and $e_{T,n}$ is waste mass exiting the system as emissions from the recycling process. The first equation states that the waste mass entering the recycling process is transformed into new raw materials, emission, and secondary waste. The second equality states that the materials sent to recycling equals the waste mass net of landfill, Q_n, subtracting waste mass used for energy purposes.

Notice that the recycling process is more efficient in terms of transforming waste into new raw materials the smaller the distance between the CET function and the physical constraint. The curvature of the CET function reflects the heterogeneity of the waste mass: some parts of a given waste mass are more suited for recycling than others.

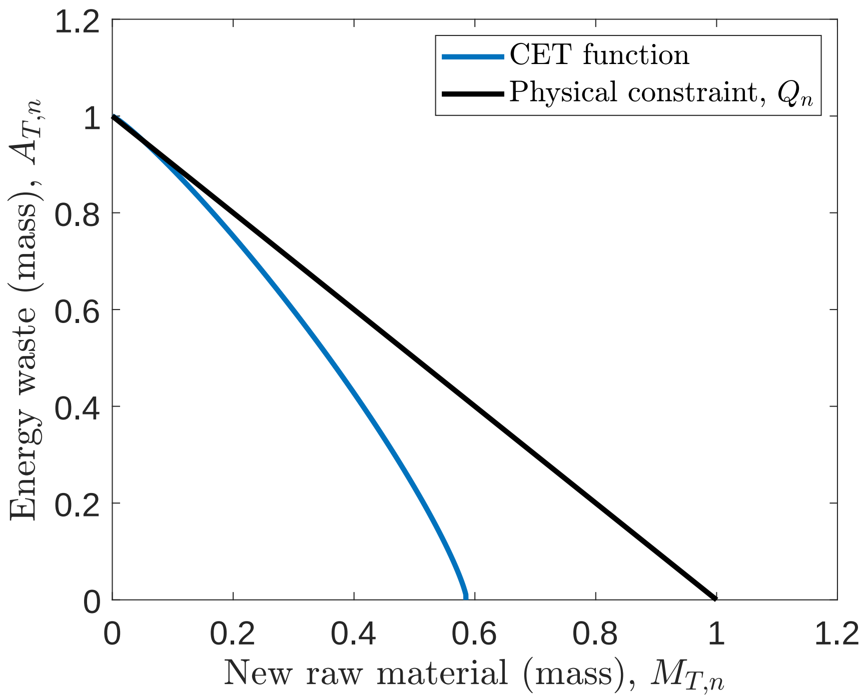

FIGURE 1: Illustration of modified CET function.

One technical detail is worth emphasizing. The modified CET function is homogeneous of degree one (i.e. constant returns to scale). This ensures that the transformation possibilities are independent of the amount of waste that needs to be handled.

The production level required in the treatment sector is

$$Q_n + c_{A,n}A_{T,n} + c_{D,n}D_n = \eta_{B,n}Y_{B,n}, \tag{14}$$

where $\eta_{B,n}$ is a scale parameter measured in waste mass per unit of production value, D_n is the amount of waste displaced as landfill, while $c_{A,n}$ and $c_{D,n}$ are cost parameters. The two terms $c_{A,n}A_{T,n}$ and $c_{D,n}D_n$ ensure that there may be a cost difference between treatment methods. Here $c_{A,n}$ is the additional per unit cost of delivering waste for burning in the energy sector compared to recycling it. As it is probably more costly to recycle a given amount of waste, one may expect that $c_{A,n}$ is negative. The interpretation of $c_{D,n}$ is different as the waste mass D_n is not included in Q_n. If $c_{D,n}$ equals one, the two treatments, landfill and recycling, are equally costly. Landfill is more costly for $c_{D,n} > 1$ and vice versa.

The choice between recycling and burning is restricted by legislation through the waste hierarchy. The hierarchy dictates that recycling is preferable to energy utilization. As a point of departure, it is assumed that the waste hierarchy places a binding constraint on firms in the treatment sector such that

$$\chi_n Q_n = \bar{M}_{T,n}, \quad \chi \in (0,1). \tag{15}$$

The condition states that the treatment sector must recycle a constant fraction, χ, of the waste mass net of landfill, Q_n. The restriction tells us how much material that enters the recycling process, while the CET function tells us how much new material that is generated.

The profit function of each representative firm in the treatment sector is given by:

$$\pi_{B,n} = p_{M,n}M_{T,n} + p_{E,n}A_{T,n} + \left(p_{\bar{Q}_n} + d_n p_{d,n}\right)\bar{Q}_n - L_{B,n}w - K_{B,n}r - E_{B,n}p_{E,B,n}, \tag{16}$$

where $p_{M,n}$ is the price of the recycled material, and $p_{E,n}$ is the price of the energy waste.[2] The treatment sector sells the recycled material to the remaining part of the economy, and it sells the energy waste to the energy sector. Prices differ between waste fractions. The price of the energy waste is determined by its heating value, while the type of material generated from recycling is fraction specific.

The problem of each representative firm in the treatment sector is to maximize the profit

[2] These two prices are exogenous in this partial equilibrium model, but they will be endogenous in the final Green Reform model.

function (16) subject to the transformation function (11), the production constraint (14), and the waste hierarchy constraint (15).

The waste hierarchy constraint (15) dictates how much waste each representative firm allocates to energy and recycling purposes. To see this, combine (13) and (15) to obtain:

$$A_{T,n} = (1 - \chi)Q_n.$$

Hence $A_{T,n}$ is a constant fraction of the total waste mass that needs treatment. Substituting this expression into the modified CET function (11) results in one equation with one unknown, $M_{T,n}$, which has a unique solution. Hence, the waste hierarchy determines both $A_{T,n}$ and $M_{T,n}$.

As the waste hierarchy constraint eliminates an active choice between energy utilization and recycling, the problems of the representative firms boil down to cost minimization problems:

$$\begin{aligned} &\min_{K_{B,n},L_{B,n},E_{B,n}} \quad L_{B,n}w + K_{B,n}r + E_{B,n}p_{E,B,n} \\ &\text{st.} \quad Q_n + c_{A,n}A_{T,n} + c_{D,n}D_n = \eta_{B,n}Y_{B,n}\left(K_{B,n}, L_{B,n}, E_{B,n}\right), \end{aligned}$$

where Q_n, D_n, and $A_{T,n}$ are taken as given. The first-order conditions associated with these problems are given by (4) and (5).

As the treatment sector operates under a non-profit constraint, the price for delivering waste to the treatment sector, $p_{\bar{Q}_n}$, is endogenously determined such that:

$$\pi_{B,n} = 0. \tag{17}$$

It is assumed that the additional fee for delivering landfill waste is proportional to $p_{\bar{Q}_n}$:

$$p_{d,n} = \kappa p_{\bar{Q}_n}, \quad \kappa > 0. \tag{18}$$

2.6 Secondary waste generation

The secondary waste mass is transferred directly to the relevant subsectors within the treatment sector. In this regard, the total mass of secondary waste must equal the sum of fractional secondary waste masses:

$$\hat{q}_{S,n} = \sum_{\hat{n}=1}^{N} \hat{q}_{S,n}^{\hat{n}}, \tag{19}$$

where $\hat{q}_{S,n}^{\hat{n}}$ is the secondary waste of fraction $\hat{n}$ generated from recycling of fraction n.

The fractional waste masses generated from recycling are assumed to be a constant share of the total secondary waste mass generated:

$$\hat{q}_{S,n}^{\hat{n}} = \omega_{\hat{n}} \hat{q}_{S,n}, \quad \sum_{\hat{n}=1}^{N} \omega_{\hat{n}} = 1. \tag{20}$$

Further, the emission masses are constant fractions of the material mass entering the recycling process:

$$e_{T,n} = \phi_n \bar{M}_{T,n}, \quad 0 < \phi_n < 1. \tag{21}$$

2.7 Physical constraints

The First Law of Thermodynamics implies that the mass entering the treatment process equals the sum of masses after treatment:

$$\bar{Q}_n + q_{S,n} = D_n + A_{T,n} + M_{T,n} + e_{T,n} + \hat{q}_{S,n}, \tag{22}$$

where $D_n = d_n \left(\bar{Q}_n + q_{S,n} \right)$.

Summing over all fractions:

$$\sum_{n=1}^{N} \bar{Q}_n + \sum_{n=1}^{N} q_{S,n} = \sum_{n=1}^{N} \left(D_n + A_{T,n} + M_{T,n} + e_{T,n} \right) + \sum_{n=1}^{N} \hat{q}_{S,n} \quad \Leftrightarrow$$
$$\sum_{n=1}^{N} \bar{Q}_n = \sum_{n=1}^{N} \left(D_n + A_{T,n} + M_{T,n} + e_{T,n} \right).$$

This equation is important for the intuition. It shows that the total waste mass generated by the economy must equal the sum of four masses: mass utilized for energy generation (sent to the energy sector), mass transformed into new materials, mass displaced as landfill, and mass ending up as emissions. Hence, even though the subsectors within the recycling sector generate secondary waste, each unit of waste mass will - in the end - be sent to the energy sector, transformed into new useful materials, displaced as landfill, or end up as emissions.

The cancellation of the secondary waste terms may be rationalized the following way. The secondary waste input is given by

$$\sum_{n=1}^{N} q_{S,n} = \sum_{n=1}^{N}\sum_{\hat{n}=1}^{N} \hat{q}^{n}_{S,\hat{n}} = \sum_{\hat{n}=1}^{N}\sum_{n=1}^{N} \hat{q}^{n}_{S,\hat{n}}.$$

In the first equation it is used that the secondary waste input of fraction n equals the sum of secondary waste masses generated by each of the n subsectors within the treatment sector. The second equality follows from a general math rule saying that for finite double sums, one may change the order of summation.

The secondary waste generated from recycling is given by:

$$\sum_{n=1}^{N} \hat{q}_{S,n} = \sum_{n=1}^{N}\sum_{\hat{n}=1}^{N} \hat{q}^{\hat{n}}_{S,n}, \tag{23}$$

where it is used that the secondary waste mass generated by subsector n within the treatment sector is obtained by summing over all the waste fractions generated.

The two secondary waste terms are clearly identical, and they, therefore, cancel out when summing over the N fractions in (22).